DECIVIL
DEPARTAMENTO DE ENGENHARIA CIVIL, ARQUITETURA E AMBIENTE
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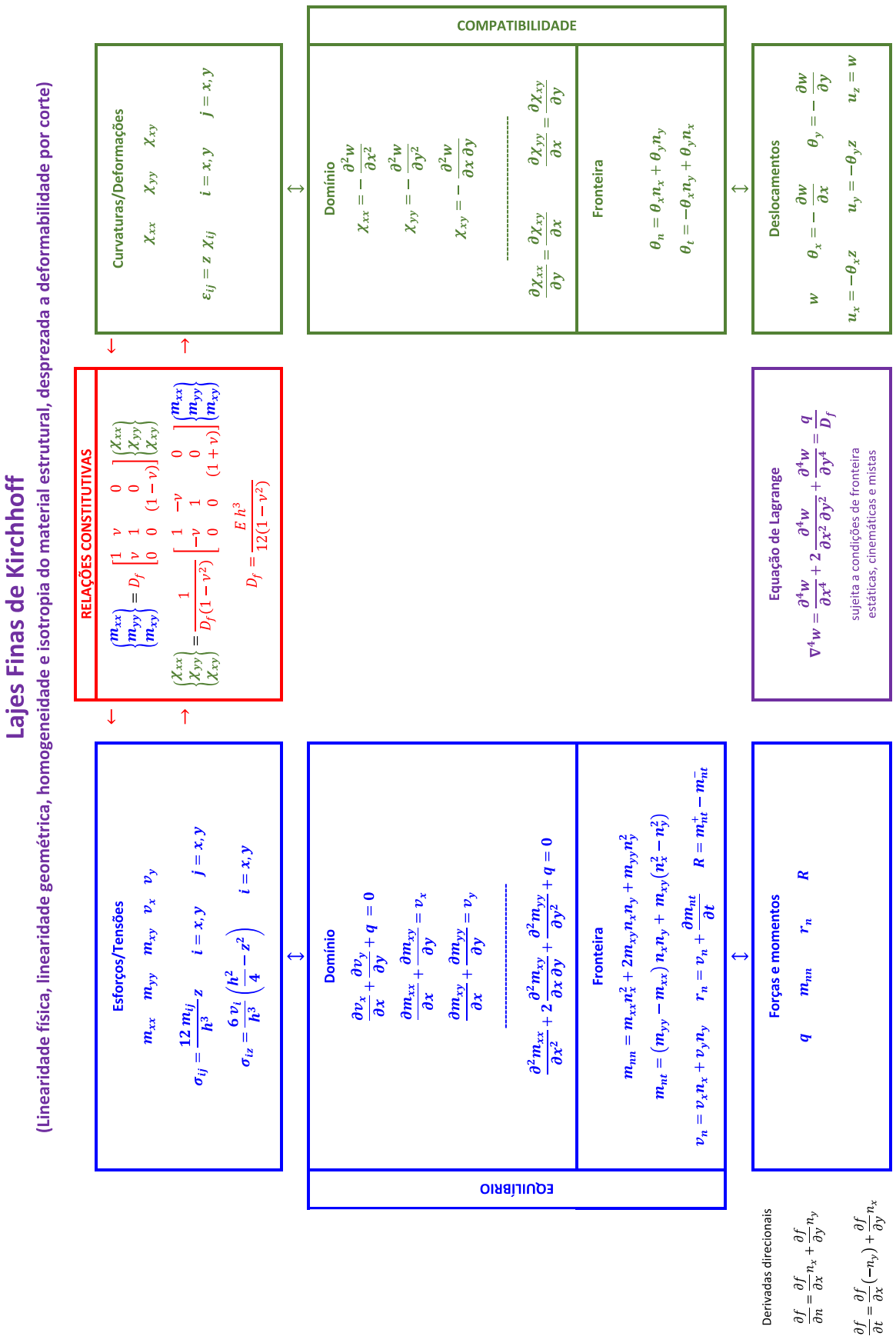
ANÁLISE DE ESTRUTURAS

Tabelas de Análise de Estruturas

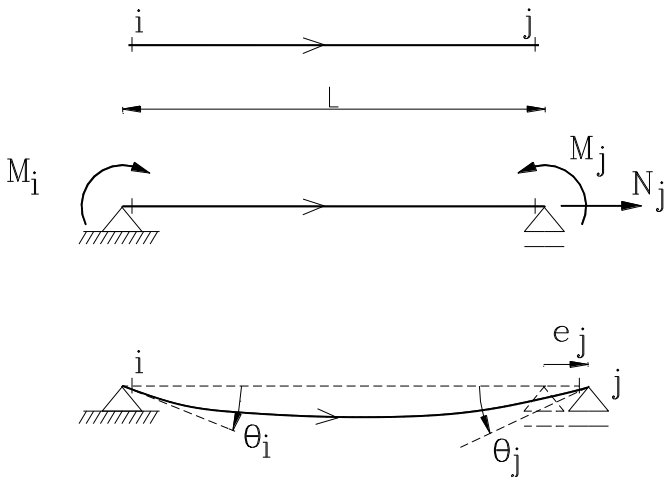
Grupo de Análise de Estruturas
IST, 2025

Formulário de Lajes



Formulário

Elemento de barra (viga)



Esforços e deformações independentes

$$\underline{X} = \begin{bmatrix} M_i \\ M_j \\ N_j \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} \theta_i \\ \theta_j \\ e_j \end{bmatrix}$$

Matriz de flexibilidade elementar

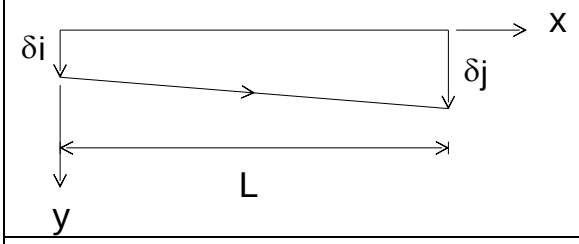
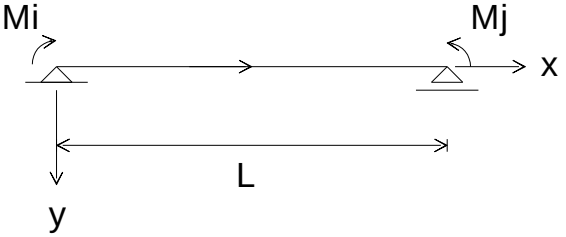
$$\underline{F} = \frac{L}{6EI} \begin{bmatrix} 2 & 1 & \\ 1 & 2 & \\ & & 6I/A \end{bmatrix}$$

Relações deformações-esforços

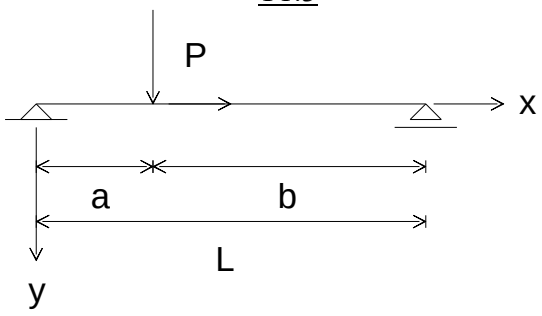
$$\underline{u} = \underline{F}\,\underline{X} + \underline{\bar{u}}$$

Viga simplesmente apoiada

Deslocamentos transversais $y(x)$ e momentos flectores $M(x)$

T1.1 	$M(x) = 0$ $y(x) = \delta_i + \frac{\delta_j - \delta_i}{L} x$
T1.2 	$M(x) = M_i + \frac{M_j - M_i}{L} x$ $y(x) = \frac{L}{6EI}\left[\left(2M_i + M_j\right)x - \frac{3M_i}{L}x^2 + \frac{M_i - M_j}{L^2}x^3 \right]$

T1.3



Caso particular: *a=b=L/2*

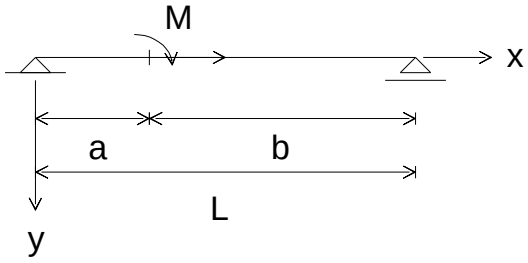
$$M(x=\frac{L}{2})=\frac{PL}{4}$$

$$y(x=\frac{L}{2})=\frac{PL^3}{48EI}$$

$$M(x)=\begin{cases} P\dfrac{b}{L}x & 0\le x\le a \\ P\left[a-\dfrac{a}{L}x\right] & a\le x\le L \end{cases}$$

$$y(x)=\begin{cases} \dfrac{P}{6LEI}\left[ab(2b+a)x-bx^3\right] & 0\le x\le a \\ \dfrac{P}{6LEI}\left[-a^3(b+a)+a\left(2b^2+4ab+3a^2\right)x-3a(b+a)x^2+ax^3\right] & a\le x\le L \end{cases}$$

T1.4



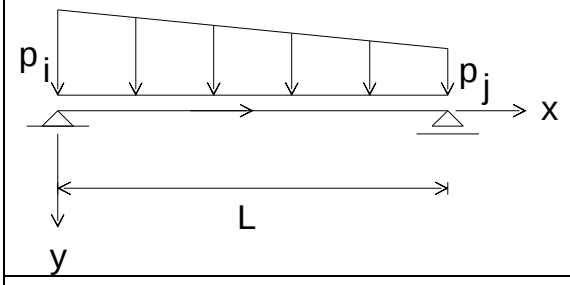
Caso particular: momento a meio-vão

$$y(x)=\begin{cases} \dfrac{M}{6LEI}\left[-\dfrac{L^2}{4}x+x^3\right] & 0\le x\le \dfrac{L}{2} \\ \dfrac{M}{6LEI}\left[-\dfrac{3L^3}{4}+\dfrac{11L^2}{4}x-3Lx^2+x^3\right] & \dfrac{L}{2}\le x\le L \end{cases}$$

$$M(x)=\begin{cases} -\dfrac{M}{L}x & 0\le x\le a \\ M-\dfrac{M}{L}x & a\le x\le L \end{cases}$$

$$y(x)=\begin{cases} \dfrac{M}{6LEI}\left[\left(2b^2-2ba-a^2\right)x+x^3\right] & 0\le x\le a \\ \dfrac{M}{6LEI}\left[-3a^2(a+b)+\left(5a^2+2b^2+4ba\right)x-3(a+b)x^2+x^3\right] & a\le x\le L \end{cases}$$

T1.5



Caso particular: *p* uniforme

$$M(x)=\frac{p}{2}\left(Lx-x^2\right)$$

$$y(x)=\frac{p}{EI}\left[\frac{L^3}{24}x-\frac{L}{12}x^3+\frac{1}{24}x^4\right]$$

$$y(x=\frac{L}{2})=\frac{5pL^4}{384EI}$$

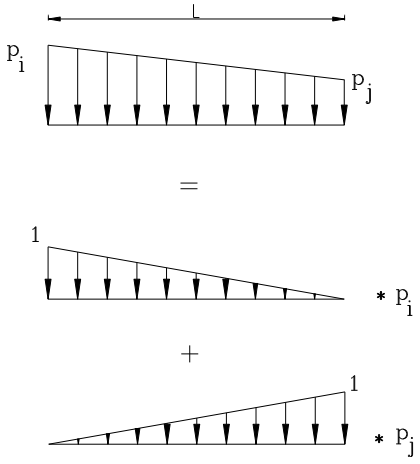
$$M(x)=\left[\left(p_i\frac{L}{3}+p_j\frac{L}{6}\right)x-p_i\frac{1}{2}x^2+\left(p_i-p_j\right)\frac{1}{6L}x^3\right]$$

$$y(x)=\frac{1}{EI}\left[\left(p_i\frac{8L^3}{360}+p_j\frac{7L^3}{360}\right)x-\left(p_i\frac{L}{18}+p_j\frac{L}{36}\right)x^3+p_i\frac{1}{24}x^4-\left(p_i-p_j\right)\frac{1}{120L}x^5\right]$$

Utilização das tabelas

Sendo válida a sobreposição é conveniente decompôr as acções (ou os seus efeitos) em parcelas mais simples. Seja, por exemplo, a acção representada na tabela anterior, Tabela 1.5.

Esta carga trapezoidal pode ser representada pela recta $p=mx+b$ com $m=\frac{p_j-p_i}{L}$ e $b=p_i$. É fácil observar que a mesma recta pode ser obtida pela sobreposição de duas rectas particulares (que tomem o valor unitário numa das extremidades e o valor nulo na outra) devidamente escaladas:



$$p=\frac{p_j-p_i}{L}x+p_i=$$
$$=(1-\frac{x}{L})p_i+\frac{x}{L}p_j$$

Nas tabelas seguintes recorrer-se-á a esta decomposição ou à decomposição alternativa em que se separa o termo constante (o b no caso anterior) do termo linear (o mx no caso anterior).

Viga simplesmente apoiada sujeita a variações de temperatura

T1.6

Variações de Temperatura

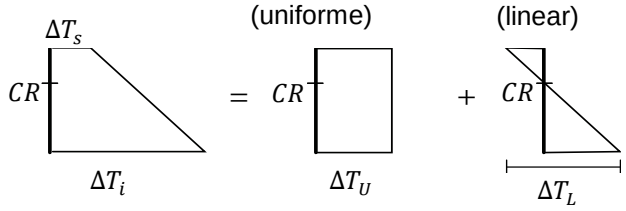
Deve identificar-se claramente:

- a variação de temperatura na altura da secção;
- a variação de temperatura ao longo do vão da viga.

Para a utilização das tabelas, a variação na altura da secção deverá, quando necessário, ser decomposta em:

1. variação **uniforme** na altura da secção, a qual se denota por ΔT_U, associada à deformação axial da secção e dada pela variação de temperatura do centro de rigidez da secção;
2. variação **linear** na altura da secção, associada à curvatura da secção, correspondendo-lhe valores de sinal contrário para a variação de temperatura das fibras inferiores e superiores, e nulos no centro de rigidez. A diferença entre a variação de temperatura da fibra inferior e a variação de temperatura da fibra superior denota-se por ΔT_L.

Graficamente representam-se estes casos de variação na altura da secção:



Exemplo:

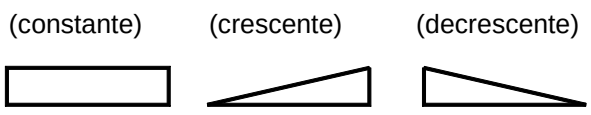
Numa secção transversal, de altura h cujo centro de rigidez se encontra à distância de $h/3$ da fibra superior, se a variação de temperatura na fibra inferior for de 5ºC e a variação de temperatura na fibra superior de 20ºC (ambas positivas), então:

$$\Delta T_U = \Delta T_s + \frac{\Delta T_i - \Delta T_s}{h}\frac{h}{3} = 15^{\circ}\mathrm{C} \qquad \text{e} \qquad \Delta T_L = \Delta T_i - \Delta T_s = -15^{\circ}\mathrm{C}\,.$$

Os 3 casos de variação no vão que constam das tabelas são:

1. variação **uniforme** no vão o que significa que todas as secções da viga têm a mesma variação de temperatura;
2. variação **crescente** no vão, variando a variação de temperatura linearmente desde um valor zero, no início da viga, até ao valor máximo no final da viga;
3. variação **decrescente** no vão, variando a variação de temperatura linearmente desde um valor máximo, no início da viga, até ao valor zero no final da viga.

Graficamente representam-se estes casos de variação no vão na forma seguinte:



Distribuição de temperatura linear na secção, ΔT_L :

$M(x) = 0$

Caso de variação constante no vão:

$$y(x) = \frac{\alpha \Delta T_L}{2h}\left[Lx - x^2 \right]$$

Caso de variação crescente no vão:

$$y(x) = \frac{\alpha \Delta T_L}{2h}\left[\frac{1}{3}x - \frac{1}{3L}x^3 \right]$$

Caso de variação decrescente no vão:

$$y(x) = \frac{\alpha \Delta T_L}{2h}\left[\frac{2L}{3}x - x^2 + \frac{1}{3L}x^3 \right]$$

em que α é o coeficiente de dilatação térmica do material e h é a altura da secção

Distribuição de temperatura uniforme na secção, ΔT_U :

$N(x) = 0$

Caso de variação constante no vão:

$$u(L) = \alpha \Delta T_U \, L$$

Caso de variação crescente no vão:

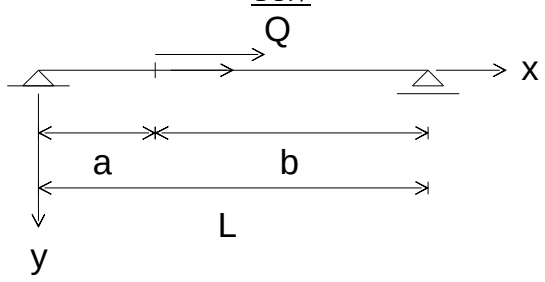
$$u(L) = \alpha \Delta T_U \frac{L}{2}$$

Caso de variação decrescente no vão:

$$u(L) = \alpha \Delta T_U \frac{L}{2}$$

Viga simplesmente apoiada sujeita a cargas axiais

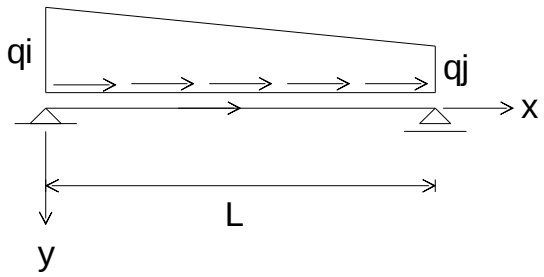
T1.7



$$N(x)=\begin{cases} Q & 0 \le x \le a \\ 0 & a \le x \le L \end{cases}$$

$$u(x)=\begin{cases} \dfrac{Qx}{EA} & 0 \le x \le a \\ \dfrac{Qa}{EA} & a \le x \le L \end{cases}$$

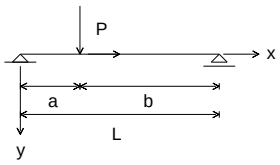
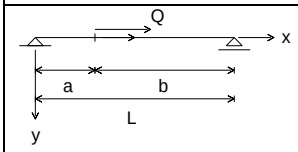
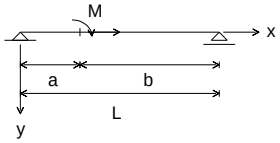
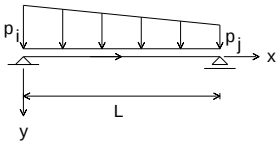
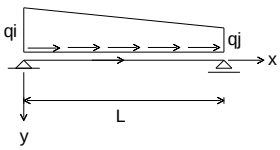
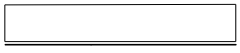
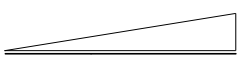
T1.8



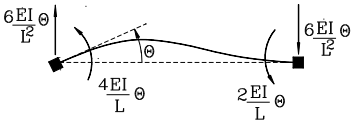
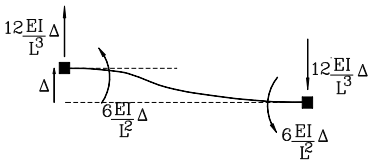
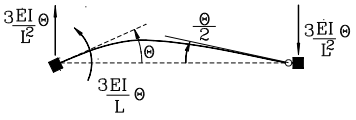
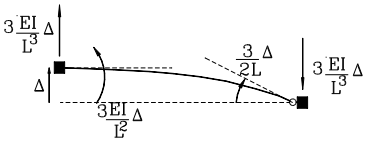
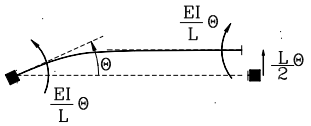
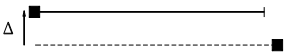
$$N(x)=\left[\left(q_i+q_j\right)\frac{L}{2}-q_i x+\left(q_i-q_j\right)\frac{1}{2L}x^2\right]$$

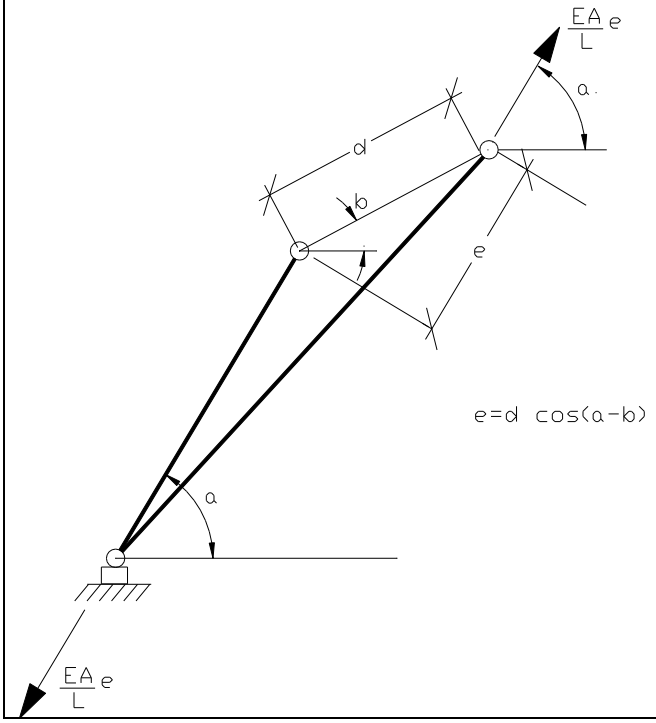
$$u(x)=\frac{1}{EA}\left[\left(q_i+q_j\right)\frac{L}{2}x-q_i\frac{x^2}{2}+\left(q_i-q_j\right)\frac{x^3}{6L}\right]$$

T2 Deformações independentes em barras com cargas de vão - Termos correctores

Carga de vão	$\overline{\theta_i}$	$\overline{\theta_j}$	$\overline{e_j}$
	$\frac{Pab(L+b)}{6LEI}$ Caso particular a=L/2 $\frac{PL^2}{16EI}$	$\frac{Pab(L+a)}{6LEI}$ Caso particular a=L/2 $\frac{PL^2}{16EI}$	0
	0	0	$\frac{Qa}{EA}$
	$\frac{M(3b^2-L^2)}{6LEI}$ Caso particular a=L/2 $-\frac{ML}{24EI}$	$\frac{M(L^2-3a^2)}{6LEI}$ Caso particular a=L/2 $\frac{ML}{24EI}$	0
	$\frac{1}{EI}\left[p_i\frac{8L^3}{360}+p_j\frac{7L^3}{360}\right]$ Caso particular $p_i=p_j$ $\frac{p_iL^3}{24EI}$	$\frac{1}{EI}\left[p_i\frac{7L^3}{360}+p_j\frac{8L^3}{360}\right]$ Caso particular $p_i=p_j$ $\frac{p_iL^3}{24EI}$	0
	0	0	$\frac{q_iL^2}{6EA}+\frac{2q_jL^2}{6EA}$
ΔT 	$\frac{\alpha\Delta T_L L}{2h}$	$\frac{\alpha\Delta T_L L}{2h}$	$\alpha\Delta T_U L$
ΔT 	$\frac{\alpha\Delta T_L L}{6h}$	$\frac{\alpha\Delta T_L L}{3h}$	$\frac{1}{2}\alpha\Delta T_U L$
ΔT	$\frac{\alpha\Delta T_L L}{3h}$	$\frac{\alpha\Delta T_L L}{6h}$	$\frac{1}{2}\alpha\Delta T_U L$

T.3 Deformadas para deslocamentos impostos

Tipo de barra	Imposição de rotação à esquerda	Imposição de deslocamento transversal
bi-encastrada		
encastrada-rotulada		
encastrada-enc desliz.		



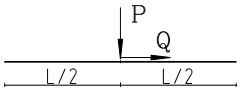
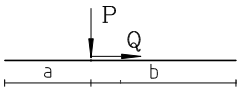
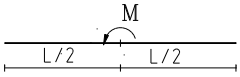
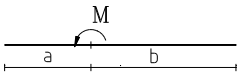
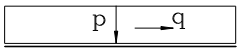
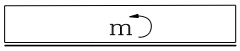
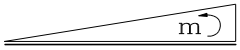
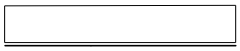
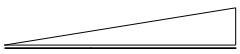
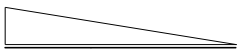
Deformada da barra sujeita apenas a esforço normal

NOTA:

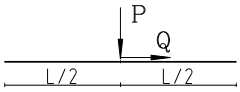
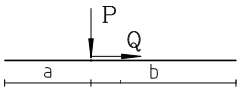
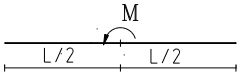
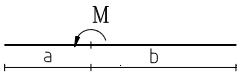
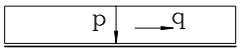
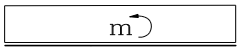
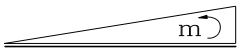
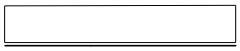
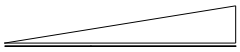
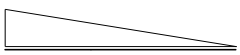
A deformada final da barra é sempre obtida considerando a sobreposição dos diversos efeitos nomeadamente:

- a(s) rotação(ões) independentes;
- o deslocamento transversal relativo entre extremidades;
- o deslocamento axial relativo entre extremidades;
- o efeito das solicitações de vão.

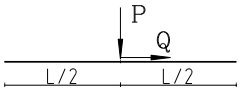
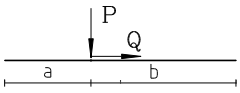
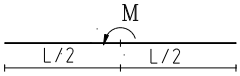
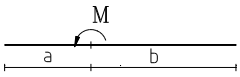
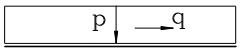
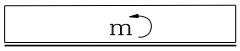
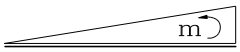
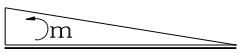
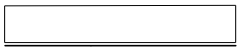
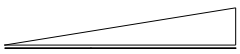
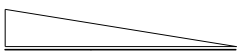
Forças de fixação devidas a cargas de vão na barra bi-encastrada	

	M_A	M_B	V_A	V_B	N_A	N_B
	$\frac{PL}{8}$	$-\frac{PL}{8}$	$\frac{P}{2}$	$\frac{P}{2}$	$-\frac{Q}{2}$	$-\frac{Q}{2}$
	$\frac{Pab^2}{L^2}$	$-\frac{Pba^2}{L^2}$	$\frac{Pb^2(3a+b)}{L^3}$	$\frac{Pa^2(a+3b)}{L^3}$	$-\frac{Qb}{L}$	$-\frac{Qa}{L}$
	$\frac{M}{4}$	$\frac{M}{4}$	$\frac{3M}{2L}$	$-\frac{3M}{2L}$	0	0
	$\frac{Mb(2a-b)}{L^2}$	$\frac{Ma(2b-a)}{L^2}$	$\frac{6Mab}{L^3}$	$-\frac{6Mab}{L^3}$	0	0
	$\frac{pL^2}{12}$	$-\frac{pL^2}{12}$	$\frac{pL}{2}$	$\frac{pL}{2}$	$-\frac{qL}{2}$	$-\frac{qL}{2}$
	$\frac{pL^2}{30}$	$-\frac{pL^2}{20}$	$\frac{3pL}{20}$	$\frac{7pL}{20}$	$-\frac{qL}{6}$	$-\frac{qL}{3}$
	$\frac{pL^2}{20}$	$-\frac{pL^2}{30}$	$\frac{7pL}{20}$	$\frac{3pL}{20}$	$-\frac{qL}{3}$	$-\frac{qL}{6}$
	0	0	m	$-m$	0	0
	$\frac{mL}{12}$	$-\frac{mL}{12}$	$\frac{m}{2}$	$-\frac{m}{2}$	0	0
	$-\frac{mL}{12}$	$\frac{mL}{12}$	$-\frac{m}{2}$	$\frac{m}{2}$	0	0
ΔT 	$\frac{\alpha \Delta T_L EI}{h}$	$-\frac{\alpha \Delta T_L EI}{h}$	0	0	$\alpha \Delta T_U EA$	$-\alpha \Delta T_U EA$
ΔT 	0	$-\frac{\alpha \Delta T_L EI}{h}$	$-\frac{\alpha \Delta T_L EI}{Lh}$	$\frac{\alpha \Delta T_L EI}{Lh}$	$\frac{\alpha \Delta T_U EA}{2}$	$-\frac{\alpha \Delta T_U EA}{2}$
ΔT 	$\frac{\alpha \Delta T_L EI}{h}$	0	$\frac{\alpha \Delta T_L EI}{Lh}$	$-\frac{\alpha \Delta T_L EI}{Lh}$	$\frac{\alpha \Delta T_U EA}{2}$	$-\frac{\alpha \Delta T_U EA}{2}$

Forças de fixação devidas a cargas de vão na barra encastrada-rotulada

	M_A	θ_B	V_A	V_B	N_A	N_B
	$\frac{3PL}{16}$	$\frac{PL^2}{32EI}$	$\frac{11P}{16}$	$\frac{5P}{16}$	$-\frac{Q}{2}$	$-\frac{Q}{2}$
	$\frac{Pab(L+b)}{2L^2}$	$\frac{Pba^2}{4EIL}$	$\frac{Pb\ (3L^2-b^2)}{2L^3}$	$\frac{Pa^2(3L-a)}{2L^3}$	$-\frac{Qb}{L}$	$-\frac{Qa}{L}$
	$\frac{M}{8}$	$-\frac{ML}{16EI}$	$\frac{9M}{8L}$	$-\frac{9M}{8L}$	0	0
	$\frac{M(L^2-3b^2)}{2L^2}$	$-\frac{Ma(2b-a)}{4EIL}$	$\frac{3Ma(L+b)}{2L^3}$	$-\frac{3Ma(L+b)}{2L^3}$	0	0
	$\frac{pL^2}{8}$	$\frac{pL^3}{48EI}$	$\frac{5pL}{8}$	$\frac{3pL}{8}$	$-\frac{qL}{2}$	$-\frac{qL}{2}$
	$\frac{7pL^2}{120}$	$\frac{pL^3}{80EI}$	$\frac{27pL}{120}$	$\frac{33pL}{120}$	$-\frac{qL}{6}$	$-\frac{qL}{3}$
	$\frac{8pL^2}{120}$	$\frac{pL^3}{120EI}$	$\frac{48pL}{120}$	$\frac{12pL}{120}$	$-\frac{qL}{3}$	$-\frac{qL}{6}$
	0	0	m	$-m$	0	0
	$\frac{mL}{8}$	$\frac{mL^2}{48EI}$	$\frac{5m}{8}$	$-\frac{5m}{8}$	0	0
	$-\frac{mL}{8}$	$-\frac{mL^2}{48EI}$	$\frac{3m}{8}$	$-\frac{3m}{8}$	0	0
ΔT 	$\frac{3\alpha\,\Delta T_L EI}{2h}$	$\frac{\alpha\,\Delta T_L L}{4h}$	$\frac{3\alpha\,\Delta T_L EI}{2Lh}$	$-\frac{3\alpha\,\Delta T_L EI}{2Lh}$	$\alpha\,\Delta T_U EA$	$-\alpha\,\Delta T_U EA$
ΔT 	$\frac{\alpha\,\Delta T_L EI}{2h}$	$\frac{\alpha\,\Delta T_L L}{4h}$	$\frac{\alpha\,\Delta T_L EI}{2Lh}$	$-\frac{\alpha\,\Delta T_L EI}{2Lh}$	$\frac{\alpha\,\Delta T_U EA}{2}$	$-\frac{\alpha\,\Delta T_U EA}{2}$
ΔT 	$\frac{\alpha\,\Delta T_L EI}{h}$	0	$\frac{\alpha\,\Delta T_L EI}{Lh}$	$-\frac{\alpha\,\Delta T_L EI}{Lh}$	$\frac{\alpha\,\Delta T_U EA}{2}$	$-\frac{\alpha\,\Delta T_U EA}{2}$

Forças de fixação devidas a cargas de vão na barra encastrada-encastrada deslizante

	M_A	M_B	V_A	δ_B	N_A	N_B
	$\frac{3PL}{8}$	$\frac{PL}{8}$	P	$\frac{PL^3}{24EI}$	$-\frac{Q}{2}$	$-\frac{Q}{2}$
	$\frac{Pa(b+L)}{2L}$	$\frac{Pa^2}{2L}$	P	$\frac{Pa^2(a+3b)}{12EI}$	$-\frac{Qb}{L}$	$-\frac{Qa}{L}$
	$-\frac{M}{2}$	$-\frac{M}{2}$	0	$-\frac{ML^2}{8EI}$	0	0
	$-\frac{Mb}{L}$	$-\frac{Ma}{L}$	0	$-\frac{Mab}{2EI}$	0	0
	$\frac{pL^2}{3}$	$\frac{pL^2}{6}$	pL	$\frac{pL^4}{24EI}$	$-\frac{qL}{2}$	$-\frac{qL}{2}$
	$\frac{5pL^2}{24}$	$\frac{3pL^2}{24}$	$\frac{pL}{2}$	$\frac{7pL^4}{240EI}$	$-\frac{qL}{6}$	$-\frac{qL}{3}$
	$\frac{3pL^2}{24}$	$\frac{pL^2}{24}$	$\frac{pL}{2}$	$\frac{3pL^4}{240EI}$	$-\frac{qL}{3}$	$-\frac{qL}{6}$
	$-\frac{mL}{2}$	$-\frac{mL}{2}$	0	$-\frac{mL^3}{12EI}$	0	0
	$-\frac{mL}{6}$	$-\frac{mL}{3}$	0	$-\frac{mL^3}{24EI}$	0	0
	$-\frac{mL}{3}$	$-\frac{mL}{6}$	0	$-\frac{mL^3}{24EI}$	0	0
ΔT 	$\frac{\alpha \Delta T_L EI}{h}$	$-\frac{\alpha \Delta T_L EI}{h}$	0	0	$\alpha \Delta T_U EA$	$-\alpha \Delta T_U EA$
ΔT 	$\frac{\alpha \Delta T_L EI}{2h}$	$-\frac{\alpha \Delta T_L EI}{2h}$	0	$\frac{\alpha \Delta T_L L^2}{12h}$	$\frac{\alpha \Delta T_U EA}{2}$	$-\frac{\alpha \Delta T_U EA}{2}$
ΔT 	$\frac{\alpha \Delta T_L EI}{2h}$	$-\frac{\alpha \Delta T_L EI}{2h}$	0	$-\frac{\alpha \Delta T_L L^2}{12h}$	$\frac{\alpha \Delta T_U EA}{2}$	$-\frac{\alpha \Delta T_U EA}{2}$

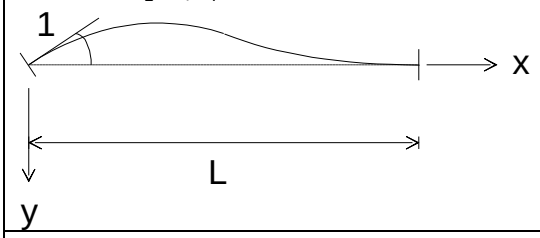
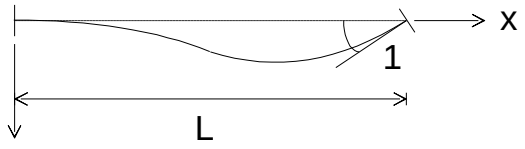
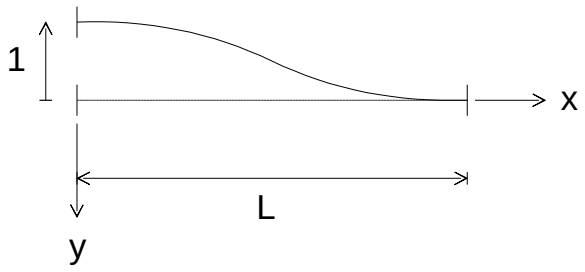
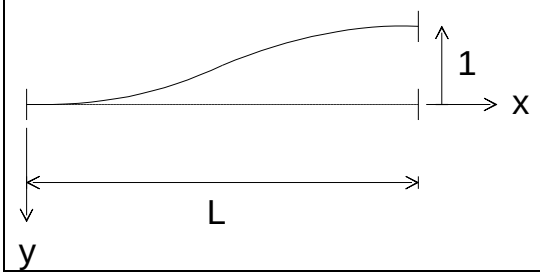
Elemento de barra - deslocamentos prescritos

Deslocamentos independentes a considerar em cada nó

- rotação;
- deslocamento transversal.

A deformada obtém-se por sobreposição das 4 deformadas, correspondentes a cada um dos 4 deslocamentos considerados:

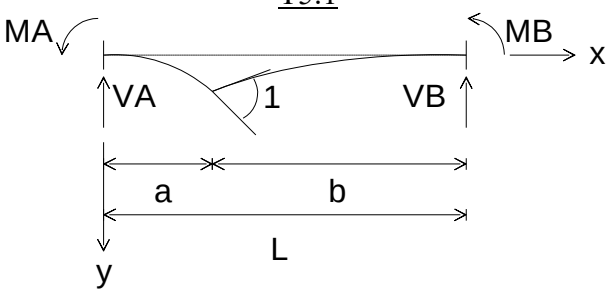
$$y(x) = \sum_{i=1}^{4} \delta_i \varphi_i(x)$$

Efeito de δ_1=1, δ_i=0 $\forall$ i$\neq$ 1 	T4.1 $\varphi_1(x) = -\frac{1}{L^2}\left(L^2 x - 2Lx^2 + x^3\right)$
Efeito de δ_2=1, δ_i=0 $\forall$ i$\neq$ 2 	T4.2 $\varphi_2(x) = -\frac{1}{L^2}\left(-Lx^2 + x^3\right)$
Efeito de δ_3=1, δ_i=0 $\forall$ i$\neq$ 3 	T4.3 $\varphi_3(x) = -\frac{1}{L^3}\left(L^3 - 3Lx^2 + 2x^3\right)$
Efeito de δ_4=1, δ_i=0 $\forall$ i$\neq$ 4 	T4.4 $\varphi_4(x) = -\frac{1}{L^3}\left(3Lx^2 - 2x^3\right)$

Existindo deformação axial também se devem considerar os modos de deformação associados aos deslocamentos independentes axiais. Contudo, estes modos não alteram a função $y(x)$, a distância da corda à posição deformada em cada ponto, que é o que se designa por *deformada*.

Deformadas e forças de fixação na barra bi-encastrada

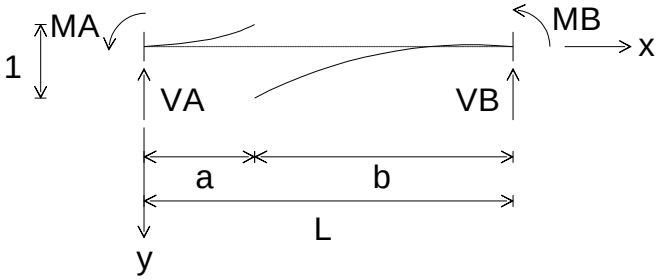
T5.1



$$M_A = -\frac{2EI}{L^2}(a-2b)$$
$$M_B = \frac{2EI}{L^2}(b-2a)$$
$$V_A = \frac{6EI}{L^3}(b-a)$$
$$V_B = \frac{6EI}{L^3}(a-b)$$

$$y(x) = \begin{cases} \left(\frac{2L-3a}{L^2}\right)x^2 + \left(\frac{2a-L}{L^3}\right)x^3 & 0 \le x \le a \\ a - x + \left(\frac{2L-3a}{L^2}\right)x^2 + \left(\frac{2a-L}{L^3}\right)x^3 & a \le x \le L \end{cases}$$

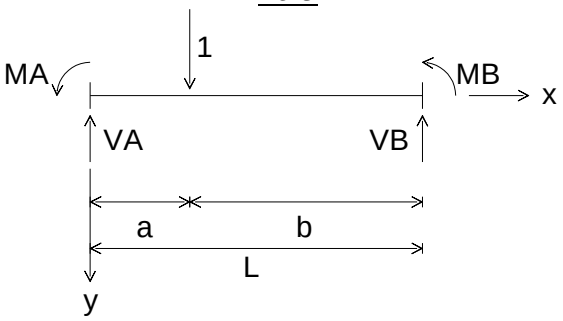
T5.2



$$M_A = -\frac{6EI}{L^2} \qquad M_B = -\frac{6EI}{L^2}$$
$$V_A = -\frac{12EI}{L^3} \qquad V_B = \frac{12EI}{L^3}$$
$$y(x) = \begin{cases} -\frac{3}{L^2}x^2 + \frac{2}{L^3}x^3 & 0 \le x \le a \\ 1 - \frac{3}{L^2}x^2 + \frac{2}{L^3}x^3 & a \le x \le L \end{cases}$$

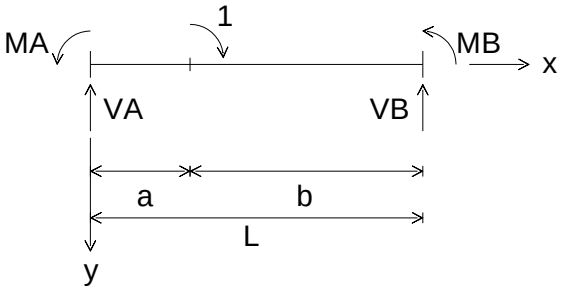
Forças de fixação para força e momento unitários

T5.3



$$M_A = -\frac{-L^2a + 2La^2 - a^3}{L^2}$$
$$M_B = \frac{-La^2 + a^3}{L^2}$$
$$V_A = \frac{L^3 - 3La^2 + 2a^3}{L^3} \qquad V_B = \frac{3La^2 - 2a^3}{L^3}$$

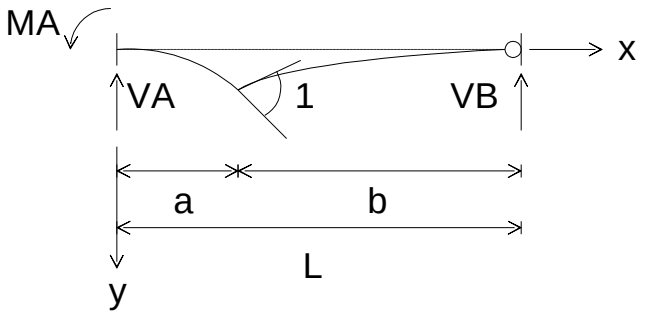
T5.4



$$M_A = -\frac{-L^2 + 4La - 3a^2}{L^2}$$
$$M_B = \frac{-2La + 3a^2}{L^2}$$
$$V_A = \frac{-6La + 6a^2}{L^3} \qquad V_B = \frac{6La - 6a^2}{L^3}$$

Deformadas e forças de fixação na barra encastrada-apoiada

T6.1



Libertação em B:

$$M_A = \frac{3EI}{L^2} b \qquad V_A = \frac{3EI}{L^3} b \quad V_B = -\frac{3EI}{L^3} b$$

Libertação em A:

$$M_B = \frac{3EI}{L^2} a \qquad V_A = -\frac{3EI}{L^3} a \qquad V_B = \frac{3EI}{L^3} a$$

Libertação em B:

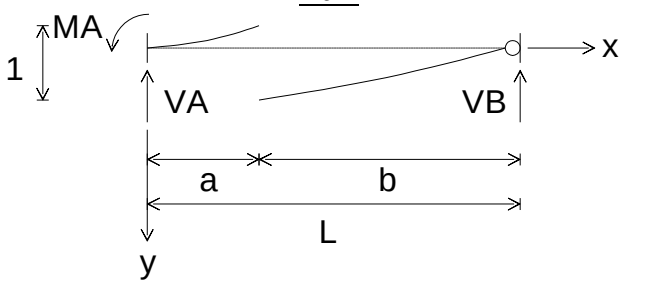
$$y(x) = \begin{cases} \left(\frac{3L-3a}{2L^2}\right)x^2 + \left(\frac{a-L}{2L^3}\right)x^3 & 0 \le x \le a \\ a - x + \left(\frac{3L-3a}{2L^2}\right)x^2 + \left(\frac{a-L}{2L^3}\right)x^3 & a \le x \le L \end{cases}$$

Libertação em A:

$$y(x) = \begin{cases} \left(-\frac{3L-3b}{2L}\right)x + \left(\frac{L-b}{2L^3}\right)x^3 + L - b & a \le x \le L \\ \left(\frac{3b-L}{2L}\right)x + \left(\frac{L-b}{2L^3}\right)x^3 & 0 \le x \le a \end{cases}$$

Notar que *a* (*b*) é a distância desde a extremidade inicial (final) à secção da descontinuidade.

T6.2



Libertação em B:

$$M_A = -\frac{3EI}{L^2} \qquad V_A = -\frac{3EI}{L^3} \qquad V_B = \frac{3EI}{L^3}$$

Libertação em A:

$$M_B = -\frac{3EI}{L^2} \qquad V_A = -\frac{3EI}{L^3} \qquad V_B = \frac{3EI}{L^3}$$

Libertação em B:

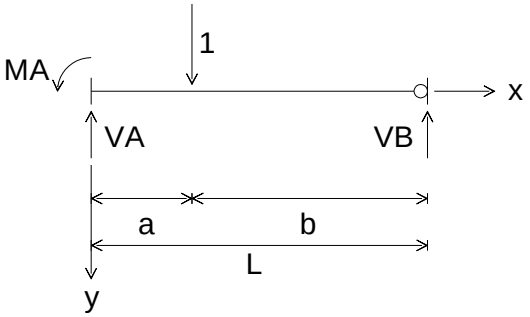
$$y(x) = \begin{cases} -\frac{3}{2L^2}x^2 + \frac{1}{2L^3}x^3 & 0 \le x \le a \\ 1 - \frac{3}{2L^2}x^2 + \frac{1}{2L^3}x^3 & a \le x \le L \end{cases}$$

Libertação em A:

$$y(x) = \begin{cases} 1 - \frac{3}{2L}x + \frac{1}{2L^3}x^3 & a \le x \le L \\ -\frac{3}{2L}x + \frac{1}{2L^3}x^3 & 0 \le x \le a \end{cases}$$

Forças de fixação para força e momento unitários

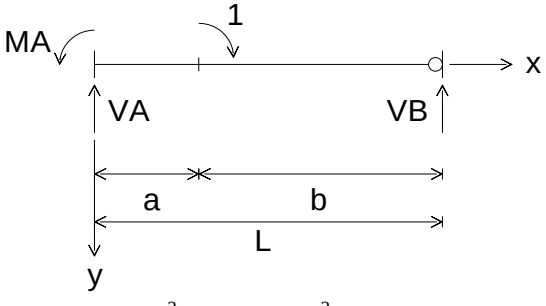
T6.3



$$M_A = -\frac{-2L^2a + 3La^2 - a^3}{2L^2}$$

$$V_A = \frac{2L^3 - 3La^2 + a^3}{2L^3} \qquad V_B = \frac{3La^2 - a^3}{2L^3}$$

T6.4

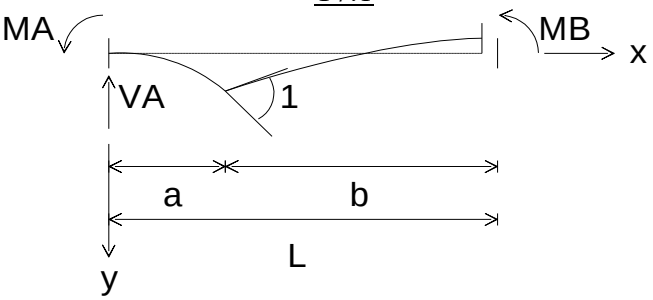


$$M_A = -\frac{-2L^2 + 6La - 3a^2}{2L^2}$$

$$V_A = \frac{-6La + 3a^2}{2L^3} \qquad V_B = \frac{6La - 3a^2}{2L^3}$$

Deformadas e forças de fixação na barra encastrada-encastrada deslizante

T7.1



$M_A = \frac{EI}{L}$ $M_B = -\frac{EI}{L}$ $V_A = 0$

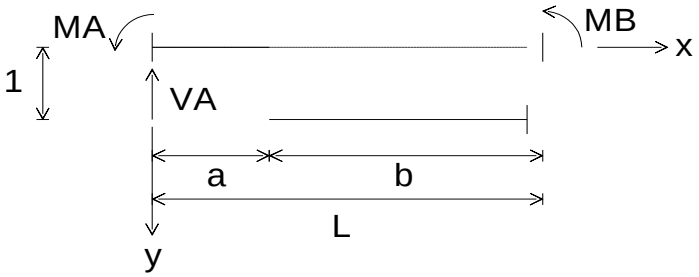
Libertação em B:

$$y(x) = \begin{cases} \frac{1}{2L}x^2 & 0 \le x \le a \\ a - x + \frac{1}{2L}x^2 & a \le x \le L \end{cases}$$

Libertação em A:

$$y(x) = \begin{cases} \frac{L}{2} - x + \frac{1}{2L}x^2 & a \le x \le L \\ -\frac{L}{2} + b + \frac{1}{2L}x^2 & 0 \le x \le a \end{cases}$$

T7.2



$M_A = 0$ $M_B = 0$ $V_A = 0$

Libertação em B:

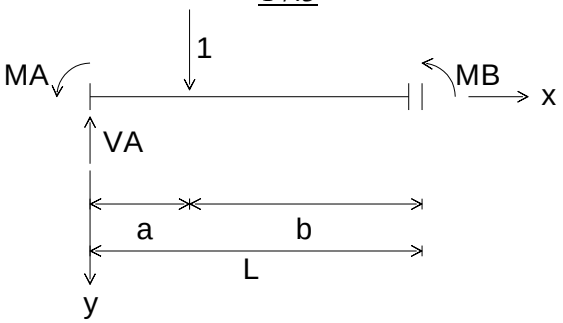
$$y(x) = \begin{cases} 0 & 0 \le x \le a \\ 1 & a \le x \le L \end{cases}$$

Libertação em A:

$$y(x) = \begin{cases} -1 & 0 \le x \le a \\ 0 & a \le x \le L \end{cases}$$

Forças de fixação para força e momento unitários

T7.3

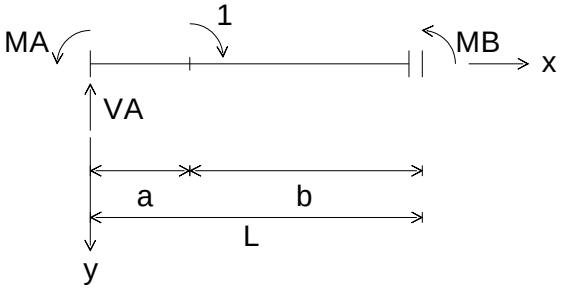


$$M_A = -\frac{-2La + a^2}{2L}$$

$$M_B = \frac{a^2}{2L}$$

$$V_A = 1$$

T7.4



$$M_A = \frac{L - a}{L}$$

$$M_B = \frac{a}{L}$$

$$V_A = 0$$