Flow over rough mobile beds: friction factor and vertical distribution of the longitudinal mean velocity

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Abstract. The objective of the present study is to identify the impacts of bed mobility on the vertical profile of the mean longitudinal velocity and on resistance in flows over water-worked beds of poorly sorted mixtures of sand and gravel. Water-worked beds with sediment transport are explicitly distinguished from immobile beds with imposed sediment feed. Flows with different equilibrium sediment transport rates are generated in a laboratory flume. The initial bed mixtures featured combinations of sand and gravel modes. Data collection included instantaneous velocities measured with Laser Doppler Annemometry. Wall similarity, in the sense of [Townsend, 1976], is assumed. The parameters of the formulae are discussed within three scenarios comprising different definitions of $u_*$ combined with different conceptions of the Von Kármán constant ($\kappa$ flow-independent or flow-dependent). It is shown that the parameters of the formulae that express the velocity profile vary with the Shields parameter and with the initial bed composition. The variation is independent of the adopted scenario, except in what concerns the description of bed smoothening in the presence of sand sizes, which depends on the definition of $k_s$ and the nature of $\kappa$. 
1. Introduction

The empirical characterisation of the mean flow velocity and of flow resistance is a classical theme in open-channel rough-wall hydrodynamics (reviews in [Schlichting & Gersten, 2000], [Castro, 2007], among others). A crucial idealisation, inherited from smooth-wall hydrodynamics, is the partition of the flow column into overlapping *inner* and *outer* regions, valid if (i) gradients in the longitudinal direction, notably the pressure gradient, are small (gradually varied flow in the longitudinal direction), (ii) the aspect ratio is high (time-averaged flow far from the wall is essentially two-dimensional in the vertical plane) and (iii) the relative submersion is high. Under these conditions, the vertical distribution of the mean longitudinal velocity in the overlapping layer can be fitted to the logarithmic law of the wall

\[
\frac{\{ u \} (z)}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z'}{z_0} \right),
\]

where \( \{ u \} \) is an ensemble-averaged longitudinal velocity (generally time-averaged), \( z' \) is the vertical coordinate above the zero of the log-law, \( z_0 \) is the bed’s characteristic roughness height, \( \kappa \) is the von Kármán constant and \( u_* = \sqrt{\tau_0 / \rho(w)} \) defines the kinematic scale for both inner and outer flow variables, where \( \tau_0 \) is the wall shear stress and \( \rho(w) \) is the fluid density.

Below the overlapping layer, the flow within the region influenced by the roughness elements has received growing attention in the past decade with the application of double (time and space) averaging methodologies to river flows, [Nikora et al., 2001], review in [Nikora et al., 2007a]). In the vicinity of the crests of the roughness elements, it has been proposed that the vertical profile of the time and space-averaged longitudinal velocity, \( \langle \bar{u} \rangle (z) \), where \( \langle \cdot \rangle \) and \( \bar{\cdot} \) stand for space averaging and time averaging, respectively, becomes determined by the superposition of the shear layers generated at the wake of the roughness elements. Experimental evidence
smooth-wall analogies ([Nikora et al., 2001]) and mixing layer analogies ([Nikora et al., 2007b]) have backed simple linear formulations such as:

\[
\frac{\langle \bar{u}(z) \rangle - \langle \bar{u}(Z_c) \rangle}{\bar{u}_p} = \frac{z - Z_c}{L_p}
\]

(2)

where \(z\) is the vertical coordinate above some datum, \(Z_c\) is the elevation of the crests of the roughness elements, \(\bar{u}_p\) is a velocity scale and \(L_p\) is a length scale.

A basic premise of this text is that the characterisation of the vertical profile of \(\langle \bar{u} \rangle\) of flows over hydraulically rough beds with bedload can be performed with the same formal apparatus of flow over immobile boundaries, in accordance, if fact, with the current research practice (cf. [Song et al., 1998], [Carbonneau & Bergeron, 2000], [Smart, 1999], [Ferreira et al., 2002], [Campbell et al., 2005] and [Dey & Raikar, 2007], among others).

Especially important in this context is the distinction between (a) porous beds composed of cohesionless poorly-sorted sediment, water-worked under equilibrium bedload transport, and (b) sub-threshold or fixed rough beds with upstream-imposed sediment transport. In the former, under uniform flow and for the same bed mixture, the same slope and the same discharge, there is no degree of freedom between the composition of the bed surface and the quantity and composition of the bedload. In fixed beds, on the contrary, the bedload rate and composition may be arbitrary or defined only as a function of the channel slope and flow discharge, with no relation with the composition of the bed surface (cf. [Carbonneau & Bergeron, 2000] and [Habibzadeh & Omid, 2009]).

This paper will be mostly concerned with water-worked beds. In this case, the strong feedback between the bed surface and the bedload, especially those composed of poorly-sorted sediment...
mixtures, poses specific challenges when it comes to quantify the parameters of equations (1) and (2).

First of all, in porous beds, there is an irredeemable arbitrariness in the definition of the boundary zero. There is no “natural” zero since the elevation of the lowest bed troughs is not definable without ambiguity (it ultimately depends on the equipment used to measure it). As a result, the quantification of the log-law parameters, namely the roughness height $z_0$ and $\kappa$, becomes relative to the adopted definition of the boundary zero and the zero of the log-law.

A second difficulty concerns the calculation of the bed shear stress. While in smooth beds the bed shear stress is unambiguously defined as the momentum transmitted to the boundary, in rough porous beds there are several possible definitions ([cf. Manes et al., 2007]), potentially involving quantities difficult to quantify as vertical momentum fluxes to/from the hyporheic region. Again, the quantification of the parameters of log-law will be relative to the adopted definition of the bed shear stress.

Bearing these issues in mind, the present study is aimed at identifying and discussing the impacts of bed mobility and bedload transport on the vertical profile of the longitudinal velocity and on the friction factor in flows over water-worked beds of poorly sorted mixtures of sand and gravel. It is proposed that the Shields number and the bed composition are the relevant parameters to discuss the influence of bed mobility on the parameters of equations (1) and (2). The discussion is carried out in three scenarios, comprising two definitions of $u^*$ with $\kappa = 0.4$, and one definition of $u^*$ with a flow-dependent $\kappa$.

2. The Physical System

Consider a wide channel with a steady open-channel flow, uniform in the longitudinal direction. The turbulent boundary layer is fully developed over an irregular, porous, mobile bed.
composed of a poorly sorted mixture of cohesionless particles in the sand-gravel range. The
relative submersion of the largest particles is about or larger than 10. The total transport rate of
contact load – the bedload discharge – and the fractional bedload discharges are time-invariant
in a domain whose longitudinal scale is several times the flow depth and no significant bed forms
develop. Under these conditions, the time average of the elevation of the bed is constant, at any
location, and so is the time-averaged bed slope. Furthermore, after enough time is elapsed,
complete mixing is achieved and the space-averaged bed texture becomes time-invariant and
uniform in the longitudinal direction. Additional hypothesis comprise absence of suspended
load, negligible collisional momentum transfer among moving bedload particles and negligible
abrasion.

Figure 1 shows a short longitudinal reach of this idealised stream. The flow is divided in three
regions, the outer region (A), the inner region (B), the pythmenic region (C) and the hyporeic
region (D). There may be overlapping between every two adjacent regions as the phenomena
that characterises each region does not cease to exist abruptly.

In region (A) the turbulent flow is influenced by the free-surface and scales with outer vari-
ables $h$, the flow depth, and $u_\ast$. The flow in the inner region (B) is affected by the characteristics
of the rough wall, directly in the lowermost layers and indirectly, through $u_\ast$, in the uppermost
layers. There may be more than one relevant length scale, expressing the arrangement of the bed
particles, the amplitude of the fluctuation of the bed micro-topography and the bedload transport
rate and composition. The dominant characteristic length scale in region (B) is assumed to be
$k_s$, the roughness scale expressing the thickness of the layer where flow is directly influenced by
the bed micro-topography. In order to ensure compatibility with smooth bed analysis, the lower
boundary of the inner region is set at the elevation of the zero of the log-law, $Z_0$, located $|\Delta|$...
above (or below) the boundary zero ($\Delta$ can be negative). Accepting wall similarity, in the sense of [Townsend, 1976], in the overlapping layer between the inner and outer regions requires that production and dissipation rates should be nearly in equilibrium and that the Reynolds shear stress should vary little. As a consequence, the vertical profile of the longitudinal velocity is assumed logarithmic and expressed by equation (1).

The definitions of the parameters of the log-law require the following remarks:

1. The zero of the vertical scale (the boundary zero) is a matter of convention. It should be set above the elevation $Z_b$, below which there are not important vertical momentum exchanges. In this text it is proposed that the flow depth is $h = Z_s - Z_b$, where $Z_s$ is the elevation of the free surface, independently of the location of the boundary zero.

2. [Monin & Yaglom, 1971] suggested that the boundary zero could be inferred by fitting the data to a logarithmic profile and forcing the slope to be $1/\kappa \approx 1/0.40$. A phenomenological basis to the vertical distance between the boundary zero and the zero of the log-law, $\Delta$, has, nevertheless been sought. [Jackson, 1981], in the wake of [Thom, 1971], postulated (note that he did not prove it, analytically or otherwise) that whatever the origin of the vertical axis, the displacement height adjusts the zero-plane for the log-law so that it coincides with the plane that contains the net drag force over the roughness elements.

[Nikora et al., 2002] proposed that the zero plane of the log-law marks the limit of penetration of large eddies in the rough bed. Using the image of eddy cascade, the linear distribution of Prandtl’s mixing length is $\ell = \kappa(z - \Delta)$, from which one obtains, in the overlapping layer

$$u_s \left( \frac{d\{u\}}{dz} \right)^{-1} = \kappa(z - \Delta)$$

(3)

The value of $\Delta$ is retrieved once the shear-rate and the friction velocity are calculated independently from experimental data. Evidently, equation (3) can be obtained from the derivative...
of equation (1) with no reference to the distribution of the mixing length. If $\kappa = 0.4$, this is equivalent to Monin and Yaglom’s remark.

In this text, no physical explanation of $\Delta$ is endorsed. Equation (3) is employed as a non-ambiguous method of calculation to allow for a discussion of its variation with bed mobility.

3. The assumption that $\kappa$ is universal has long been an object of dispute. One set of arguments ([Krogstad et al., 1992], [Oncley et al., 1996]) is based on the conviction that wall similarity may not hold for all types of rough beds, even if relative submergence is high, rendering the von Kármán constant a function of the roughness Reynolds number $Re_0$.

Other arguments, compatible with wall similarity, are drawn from the fact that $\kappa = 0.4$ is an observational result, not a theoretical one ([Dittrich & Koll, 1997]). In this case, equation (3) would provide a straightforward way to calculate $\kappa$ from the shear rate data in the overlapping layer ([Dey & Raikar, 2007]), or to calculate $\kappa$ and $\Delta$ simultaneously ([Nikora et al., 2002], [Koll, 2006] or [Franca et al., 2008]).

The estimates of $\kappa$ are generally incommensurable across studies due to different criteria for setting the location of the zero of the log-law and for defining $u_*$. In this text, accepting wall similarity, two scenarios for $\kappa$ will be adopted, constant and equal to 0.4 and variable according to equation (3).

4. In spite of research efforts (notably [Jackson, 1981]), there is not a general dynamic model for the roughness height. It is a measure of the boundary roughness and is related to the scale of the roughness elements $k_s$ by

$$\frac{z_0}{k_s} = e^{-\kappa B}$$ (4)
where $B$ is the normalised flow velocity ($\{u\}/u_*$) at the elevation $k_s$ above the zero of the log-law. For simple roughness geometries, including sand-like roughness, and for high roughness Reynolds numbers ($Re_0 \equiv u_0 z_0/\nu > 2.5$) $z_0/k_s = c_r$ where $c_r \approx 0.0333$ (or $c_r \approx 1/32.5$ according to [Schlichting & Gersten, 2000]) and $B = 8.5$. In this case, the bed is called *hydraulically rough*.

The discussion about the effect of bed mobility and bedload transport has been dominated by the idealisation of [Owen, 1964], which postulated that $z_0$ increases with saltation height as it includes the momentum sink due to particle movement (cf. [Bridge & Bennet, 1992], [Wiberg & Rubin, 1989], among others). In this text, this hypothesis is investigated and discussed.

5. The scale $k_s$ is, in this text, not a geometrical property of the bed or a statistic of the bed micro-topography but a function of the latter and of fluid and flow parameters. Presumably it is mainly influenced by bed amplitude $\delta = Z_c - Z_t$, where $Z_c$ and $Z_t$ are the elevations of the planes of the crests and of the troughs, respectively.

6. In fully developed turbulent flows over smooth beds, the friction velocity $u_*$ is unambiguously defined as the viscous shear stress at the elevation of the smooth boundary, expressed in kinematic terms. It renders self-similar both the logarithmic longitudinal velocity profile and the velocity defect law ([Schlichting & Gersten, 2000]). If the bed is rough, $u_*$ is a matter of definition (see [Manes et al., 2007] for a discussion of different definitions). In this text, being $g$ the acceleration of gravity and $S_0$ the bed slope, two definitions are employed:

$$D1 \quad u_* = \sqrt{g S_0 (Z_s - Z_c)},$$
representing the momentum flux at the elevation of the crests of the roughness elements ([Jackson, 1981]), [Manes et al., 2007] among others) and:

$$D2 \quad u_* = \sqrt{\int_{Z_b}^{Z_s} \varphi g S_0 \mathrm{d}z},$$
representing the total momentum transmitted by gravity to the fluid from the free-surface down to $Z_b$. 

As a kinematic scale, the role of $u_*$, for any valid definition, should be to express the bulk effect of the bed roughness on flow variables as it normalises the velocity profile in the overlapping layer between inner and outer regions.

In the pythmenic region (C), the flow is mainly determined by the bed micro-topography. In the overlapping layer between the pythmenic and inner regions, near the crests of the roughness elements, [Raupach et al., 1996]'s idealisation for canopy-free stream momentum exchange has been employed to parametrise the mean (time- and space-averaged) velocity profile ([Katul et al., 2002], [Nikora et al., 2007a]).

Although in gravel-sand rough beds there is little evidence of Kelvin-Helmholtz instabilization at the crests of the roughness elements (fundamental in the original [Raupach et al., 1996]'s model), there is, however, a shear layer of increased vorticity (noticed by [Giménez-Curto & Corniero Lera, 1996] and termed "jet layer") and separation in the wake of the protruding elements, which is not possible in dense canopies (see the Reynolds shear stresses associated a typical shear layer produced at the crest of a roughness element in Figure 2). Vertical exchanges of momentum are the result of the interaction of eddies shed at the wakes of roughness elements, if the distance between protruding elements is small enough (generally verified in gravel-sand beds), and its interaction with separation zones in the lee of those elements. Parameter

$$L_s = \langle \bar{u} \rangle Z_c \left( \frac{d\langle \bar{u} \rangle}{dz} \right)^{-1} Z_c$$

(5)

can be taken as the characteristic length scale of this vertical exchange process at the elevation of the plane of the crests and should represent half the thickness of the shear layer.

Employing a eddy-viscosity model for the total stresses with $L_s$ as a mixing length, assuming that the vertical momentum flux at the plane of the crest is constant and that the longitudinal
mean velocity is small at a depth $L_s$ below the plane of the crests, equation (2) is retrieved with 
$u_p = u_*$, were $u_*$ must be calculated from definition (D1) (see item 6) and $L_p = L_s u_p / \langle \tilde{u} \rangle (Z_c)$. 

In the lower layers of region (C), vertical transfer of fluid momentum by the eddies generated at the shear layer is attenuated and will vanish below a certain elevation ($Z_b$ in Figure 1).

If the amplitude of the bed, $\delta$, is larger than $Z_c - Z_b$, it is proposed that the bed is termed “deep”. Below $Z_b$ Reynolds shear stresses $-\rho^{(w)} \langle u' w' \rangle$ ($u'$ and $w'$ are the time-fluctuating components of the longitudinal and vertical velocities and $\rho^{(w)}$ is the fluid density) are negligible while form-induced stresses are likely to remain non-zero but isotropic. Hence, below $Z_b > Z_t$, the flow should be determined by the same variables that determine the flow in the hyporheic region (D), the permeability (or the void function), the hydraulic gradient, the fluid viscosity and density and the acceleration of gravity.

If the amplitude of the bed, $\delta$, is smaller than $Z_c - Z_b$, it is proposed that the bed is termed “fully active”, in the sense that vertical momentum fluxes penetrate down to the elevation of the lowest troughs.

The distinction between “fully active” and ”deep” beds has a major application in the calculation of $u_*$ by criterion (D2). If the bed is fully active, $Z_b = Z_t$, which simplifies the calculation. If the bed is deep, the active part of the pythmenic layer must be previously determined.

3. Laboratory Work

3.1. Description of the Laboratory Tests

Data collection took place in a 11 m long and 40 cm wide prismatic recirculating tilting flume of the Laboratory of Fluid Mechanics of the University of Aberdeen. The side-walls were made of glass, enabling visualisation and laser measurements. For each test, the flume floor was covered with a 6 cm deep sediment layer.
A total of 17 subcritical and nearly-uniform flow experimental tests were carried out. Their main characteristics are summarised in Table 1. The non-previously defined variables and parameters in this table are the (steady) flow discharge, \( Q \), the depth-averaged mean flow velocity, \( U \), the aspect ratio, \( b_f/h \) (\( b_f \) is the channel width), the Froude number, \( Fr = U/\sqrt{gh} \), the width of the mesh that retains 50 and 10%, respectively, in weight, of a sediment sample of the bed substrate (below the lowest bed troughs) \( d_{50}^{(sb)} \) and \( d_{90}^{(sb)} \), the same relatively to the bed surface, \( d_{50}^{(sr)} \) and \( d_{90}^{(sr)} \), the Shields parameter, \( \theta_{50}^{(sb)} = u_*^2 / \left( g(s-1)d_{50}^{(sb)} \right) \), the boundary Reynolds number, \( Re_s = k_s u_*/\nu \) and the non-dimensional bedload discharge, \( \phi_s = G_b / \left( \rho^{(w)} b_f d_{50}^{(sb)} \sqrt{g(s-1)d_{50}^{(sb)}} \right) \), where \( G_b \) is the bedload discharge in mass per unit time, \( s = \rho^{(w)}/\rho^{(g)} \) is the specific gravity of the sediment particles and \( \rho^{(g)} = 2590 \) is the density of the bed material.

It was observed that the bed was fully active in all tests (finer sediment particles at the troughs were subjected to entrainment demonstrating vertical momentum fluxes down to \( Z_t \)). The flow depth was thus calculated as \( h = Z_s - Z_t \).

The friction velocity was calculated by three methods, employing definitions (D1) and (D2) (section 2). Methods (D2a) and (D1) of \( u_* \) employ the profile of the total shear stresses in the plane \( xz \). In both cases the profile of the Reynolds shear stresses was calculated from the instantaneous velocity data. Assuming zero pressure gradient in the \( x \) direction, a regression line fitted the data in the upper 85% of the flow depth above the plane of the crests. As seen in Figure 3, in the lower 15% of the flow above the plane of the crests, the vertical profiles of the ensemble-averaged Reynolds shear stresses exhibit some scatter, probably because form-induced stresses cease to be negligible. Should the regression line be termed \( \tau^{(lin)}(z) \), where
\( z = Z - Z_t \), the value of the bed shear stress is

\[
\tau_0^{(D2a)} \equiv \tau^{(\text{lim})}(z = (1 - \varphi_m)\delta),
\]

(6)

where \( \varphi_m = \int_0^\delta \varphi(z)dz \) is the depth-averaged void function, and

\[
\tau_0^{(D1)} \equiv \tau^{(\text{lim})}(z = \delta)
\]

(7)

for methods (D2a) and (D1), respectively. The density of point measurements of the bed microtopography was insufficient to obtain \( \varphi(z) \) and to guarantee an accurate estimate of \( \varphi_m \). Hence, the approximation \( \varphi_m = \delta/2 \) was assumed in all tests.

Method (D2b) in Table 1 corresponds also to definition (D2) but is calculated as the integral balance of momentum for uniform flows:

\[
\tau_0^{(D2b)} \equiv \rho_{(w)} g R_* S_0
\]

(8)

where \( R_* = b_f h_* / (b_f + 2h_*) \) and \( h_* = h - (1 - \varphi_m)\delta \) (a side-wall correction was applied, in accordance to the method proposed in [Chiew & Parker, 1994]). This estimation was essentially used for assessing the order of magnitude of \( u_* \) prior to the analysis of the velocity data.

The control parameters to design the tests of Table 1 were \( S_0, Q \), and the initial bed composition.

Tests of type E and T differ in the initial bed composition, a gravel-sand mixture in the former and a gravel mixture in the latter. The full grain-size distributions of the initial bed mixtures are shown in Figure 4.

Tests D1 to D3 were obtained from the respective tests of type E by subjecting the water-worked bed to an arming process. As tests of type D result from an arming process, it is legitimate to expect that the largest immobile particles \( (d_{100}^{(sr)}) \) in tests of type E and D are the same.
3.2. Instrumentation and Experimental Procedures

For all tests, the most relevant measurements comprised i) water depth, ii) bed microtopography, iii) instantaneous flow velocity (longitudinal and vertical components) and iv) composition of the bed-surface. The flow discharge was monitored during the tests and the bedload discharge was measured for a complete description of the flow.

Flow discharge was measured on a calibrated triangular weir placed in the downstream water tank. The water depth was measured with a 0.5 mm precision point-gage running along instrumentation rails. The detailed measurements of the bed topography were performed with a Keyence infra-red laser displacement sensor mounted on automated 3D traversing frame with 0.1 mm precision.

The instantaneous flow velocity was measured with Laser Doppler Anemometry. The system features a DANTEC 55X Modular Laser Doppler Anemometer (LDA) generating a 20 mW, monochromatic red (632.8 nm) He-Ne laser with a frequency shift of 40 MHz, imposed by a Bragg cell, thus capable of detecting positive and negative velocities. The optics are mounted in forward scatter mode with manual alignment of the receiving optics. The transmitting optics features three beams, placed at 120°, allow for the measurement of two orthogonal components of the instantaneous flow velocity. The length (in the direction of the mean flow) and width (in the direction normal to the walls) of the measurement volume are, approximately, 0.2 mm and 1.5 mm, respectively. The signal is processed in a DANTEC 55N20 Doppler Frequency Tracker and converted into a voltage output ready to be sampled on a personal computer. The data was re-sampled at even time intervals and stored in the memory. The sampling software enabled the storage of 12000 samples before writing to file. The LDA probes were placed at about 6.5 m from the inlet.
Uniform flow conditions were achieved and maintained by manually operating a vane weir at the flume outlet. In tests of type E and T, bedload samples were collected and water and bed elevations were monitored for no less than 8 hours to verify equilibrium sediment transport. Usable data was collected from this stage onwards.

In each test D the bed slope decreased and the water depth increased, as a consequence of the armoring process. Adjustments to the slope and to the downstream vanes were made to obtain a uniform flow in central part of the flume, encompassing the LDA section. Usable data was then collected.

Core samples of the bed were collected at three locations along the flume, including that of the LDA measurements. In tests of types E and T, the samples were taken after water-working the bed. In tests of type D, core samples were collected before and after the armoring process. The results of the in-situ core-sample analysis are shown in Table 1 in terms of the $d_{50}^{(sb)}$ and $d_{90}^{(sb)}$ of the substrate and in terms of the $d_{50}^{(srf)}$ and $d_{90}^{(srf)}$ of the bed surface.

Once uniform flow conditions were obtained in the reach were the LDA was placed, profiles of the instantaneous velocity were performed at no less than three separate occasions. The origin of the vertical axis was determined once the bed was laid: the LDA probe was placed so that the control volume was located at the elevation of the highest crests of the bed particles, whose elevation was then recorded.

At the end of the tests, after collecting velocity and bedload, four lines of 10 cm each were profiled at the centerline of the channel in the vicinity of the location of the velocity measurements. The spatial definition was 4 samples/cm. These measurements were used to estimate the thickness of the bed, $\delta$, defined as the ensemble average of the difference between the maximum and the minimum of each bed profile.
4. Results and Discussion

4.1. Methods of Calculation of the Parameters of the Log-law

Wall similarity, in the sense of [Townsend, 1976], is assumed valid. Considering that this does not require that the von Kármán constant is flow-independent and that there are different possible definitions of the parameters and scales of the log-law, three scenarios to interpret the laboratory data are proposed:

(s1) The boundary zero is set at the elevation of the lowest troughs \((z = Z - Z_t)\), the friction velocity is in accordance to definition (D2) (section 2), calculated as (D2a) in Table 1, and the von Kármán constant is considered flow-independent \((\kappa = 0.4)\). The roughness scale \(k_s\) and the constant \(B\) are subjected to a best fit procedure. Additionally, the roughness scale \(k_s\) is defined as the lowest height above the zero of the log-law for which the velocity profile is approximately logarithmic.

(s2) The boundary zero is set at the plane of the higher crests \((z = Z - Z_c)\), the friction velocity is in accordance with definition (D1) (section 2), calculated as (D1) in Table 1, and the von Kármán constant is considered flow-independent \((\kappa = 0.4)\). The constant \(B\) is 8.5 and the roughness scale \(k_s\) is calculated from a roughness function.

(s3) The boundary zero is set at the elevation of the lowest troughs \((z = Z - Z_t)\) and the friction velocity is calculated by definition (D2), calculated as (D2a) in Table ???. The von Kármán constant is assumed not universal but a fitting parameter. The roughness scale and constant \(B\) are calculated as in criterion (s1).

For each scenario, the longitudinal velocity data are shown in Figure 5.

In scenario (s1), the parameters of the log-law are calculated as follows:
the displacement height is first calculated from equation (3) with $\kappa = 0.4$. A fourth-order polynomial $p^{(4)}(z)$ is fitted to the longitudinal velocity data and differentiated, hence obtaining $p^{(3)}(z)$, a polynomial fitting to $\frac{d\{u\}}{dz}$. A line whose equation is $p(z) = 0.4z - 0.4\Delta$ is then fitted to $u_s \left(p^{(3)}\right)^{-1}$ in the region below $z/h = 0.35$. The actual regression bounds are restricted to maximize the correlation coefficient to a given tolerance. An example can be seen in Figure 6. The actual bounds are 0.0129 and 0.0259 m and the correlation coefficient is 0.992 for an minimum admissible of 0.990.

The value of the displacement height is finally obtained as $\Delta = -p(0)/0.4$.

* Having calculated $\Delta$, the log-law (1) is written as

$$\{u\} = \frac{u_s}{\kappa_{\text{try}}} \ln (z - \Delta) - \frac{u_s}{\kappa_{\text{try}}} \ln (k_s) + u_s B$$

which is in the form

$$Y = MX + A$$

with $Y = \{u\}$, $M = \frac{u_s}{\kappa_{\text{try}}}$, $X = \ln (z - \Delta)$ and $A = u_s \left( B - \frac{1}{\kappa_{\text{try}}} \ln (k_s) \right)$.

A linear regression to the longitudinal velocity data with $M$ and $A$ as fitting parameters is carried out. Parameter $\kappa_{\text{try}}$ retains the value of the von Kármán constant resulting from the linear regression of (9). The the bounds of the regression range are modified until $\kappa_{\text{try}} \approx 0.4$.

* Once suitable bounds are found and $M$ is such that $\kappa_{\text{try}} \approx 0.4$, the definition of the $k_s$ is retrieved from $k_s = z_{LB} - \Delta$, where $z_{LB}$ the lower bound of the regression range.

* Parameter $A$ is then employed to calculate constant $B$ as

$$B = \frac{A}{u_s} + \frac{1}{\kappa_{\text{try}}} \ln (k_s).$$

* At last, the roughness height $z_0$ is calculated from equation (4) with $\kappa = \kappa_{\text{try}}$, which concludes the procedure to calculate the parameters of the log-law in scenario (s1).
In scenario (s2), the procedure to calculate the parameters is as follows:

- the displacement height $\Delta$ is calculated first. The procedure is identical to that of scenario (s1), with the particularity that the vertical coordinate is now $z = Z - Z_c$.

- Equation (9) is re-written with $B = 8.5$. As in scenario (s1), equation (9) is fitted to the data with $M = \frac{u_*}{\kappa_{try}}$ and $A = u_s \left( 8.5 - \frac{1}{\kappa_{try}} \ln (k_s) \right)$ as fitting parameters. Again, the bounds of the regression are sought so that the slope $M$ renders $\kappa_{try} \approx 0.4$.

- The scale of the roughness elements is then calculated as $k_s = e^{\kappa_{try} \left( 8.5 - \frac{\Delta}{\kappa} \right)}$.

- Finally, equation (4) is used to calculate $z_0$.

Scenario (s3) differs from scenario (s1) in as much as the von Kármán constant is not considered universal. The parameters of the log-law are calculated as follows:

- The displacement height, $\Delta$, is calculated along with $\kappa$, from the smoothed version of equation (3), $p(z) = u_s \left( p^{(3)} \right)^{-1} = \kappa z - \kappa \Delta$, which is assumed to hold in some region within the limits considered in scenario (s1), and subjected to a linear regression. Again, the actual bounds are sought to maximize the correlation coefficient to 0.99. The value of $\kappa$ is obtained directly as the slope of the regression line and $\Delta$ is calculated as $-p(0)/\kappa$. It is emphasised that both slope and origin are allowed to vary in scenario (s3), unlike scenario (s1) where the slope was imposed.

- The data is then fitted to equation (9) hence obtaining $M$ and $A$. The trial value of the von Kármán constant is obtained from $u_*/M$. The range is selected so that the trial value of the von Kármán constant approaches the value calculated in the previous point.

- The scale of the roughness elements $k_s$ is defined as in scenario (s1) and constant $B$ is calculated from (10) with $\kappa_{try} = \kappa$.

- As in the previous scenarios, equation (4) is then used to calculate $z_0$. 
4.2. The Shear Rate in the Overlapping Layer between the Inner and Outer Regions as a Two-Phase Variable

In flows over mobile beds, the shear rate must be a variable of a two-phase phenomenon in the sense of [Yalin, 1977] and should be expressed as a function of the properties of the fluid, the properties of the sediment that composes the bedload, the variables that characterize the two-phase flow, the micro-topography of the bed and the gravity field.

Furthermore, in water-worked beds under equilibrium bedload transport, the composition of both the bed surface and of the bedload are unambiguously determined by the composition of the substrate for given flow, and fluid and sediment properties. The hypothesis of equal mobility ([Parker & Klingeman, 1982]), is an instance of this principle: given enough time the bed surface necessarily coarsens so that the bedload composition approaches that of the substrate.

The dimensional relation for the shear rate in flows over water-worked beds under equilibrium conditions is thus

$$\frac{d\{u\}}{dz} = F_m \left( z', h, d_k^{(sb)}(\sigma_j), \mu, \rho(w), \rho^{(g)}, \frac{\rho^{(g)} - \rho^{(w)}}{g} \right)$$

(11)

where $d_k^{(sb)}$ is a representative diameter of granular material of the substrate, $\sigma_j$ represents a sufficient set of higher-order centred moments of the grain-size distribution of the bed substrate, $w_s$ is the terminal fall velocity of the granular material, $\mu$ is the fluid viscosity and $\rho^{(g)}$ is the density of the particles that compose the granular material. Note that $u_*$ substituted $\tau_0$ for convenience of notation. Note also that the acceleration of gravity is not a relevant variable in the internal region of a shear flow only if the fluid is homogeneous and the flow is not associated to transport phenomena. In the case of the water-worked bed under equilibrium bedload, it is hypothesised that sediment transport may affect the shear rate both directly and through the bed texture. Given that bedload transport rates depend on the weight of the particles, the acceleration
of gravity must be included in equation (11). It is implied in this argument that the acceleration of gravity is important only inasmuch it determines, in conjunction with particle submerged density and form, the weight of the granular material. Hence, \( g \) was substituted by the group 
\[
R = g \left( \rho^{(a)} - \rho^{(w)} \right),
\]
the submerged specific weight of the sediment particles.

Considering \( z', u_*, \) and \( \rho^{(w)} \) as basic variables, applying Vaschy-Buckingham’s theorem, equation (11) becomes

\[
\frac{z'}{u_*} \frac{d\{u\}}{dz} = \Pi_m \left( \frac{h}{z'}, \frac{d_k^{(ab)}}{z'}, \frac{\sigma_j}{z'^j}, \frac{w_s}{u_*}, \frac{z'u_*}{\nu}, s, \frac{z'R}{\rho^{(w)} u_*^2} \right)
\]  

Equation (12) reveals that flows over water-worked beds under equilibrium bedload require more than one roughness scale. The classical \( k_s \) scale, essentially determined by bed amplitude and arrangement of large roughness elements, is insufficient and must be complemented by \( \sqrt{\sigma_j} \) and by \( \frac{(\rho^{(w)} u_*^2)}{R} \), expressing the fact that other roughness elements form out of particle clusters, assembled and destroyed as a function of the flow variables and conditioned by the grain-size distribution of the bed substrate.

The influence of the shape of the particles, expressed in \( w_s/u_* \), will not be discussed in this text. Furthermore, the sediment transport is considered \textit{en masse}, \textit{i.e.} no detailed analysis of the motion of individual grains is considered and, therefore, the inertial forces associated to individual particle motion are irrelevant ([Yalin, 1977]). In this case, the influence of the sediment density, \( s \), is indirect and restricted to parameter \( R \) in equation (12).

Combining parameter \( \frac{d_k^{(ab)}}{z} \) with parameters \( \left( \frac{\sigma_j}{z'} \right) \) and \( \frac{z'R}{\rho^{(w)} u_*^2} \) and assuming i) that the original Von Kármán hypothesis holds, \textit{i.e.} that viscosity plays no role in defining the shear rate away from the boundary and ii) that complete similarity to the inner rough scales and to the outer scale, then \( \lim_{\frac{w_s}{\rho^{(w)} u_*^2} \to +\infty} \frac{z}{u_*} \frac{d\{u\}}{dz} = \text{Constant} \), \( \lim_{\frac{d_k^{(ab)}}{z} \to 0} \frac{z}{u_*} \frac{d\{u\}}{dz} = \text{Constant} \) and
\[ \lim_{z \to +\infty} z \frac{d\{u\}}{dz} = \text{Constant.} \] Equation 12 becomes

\[ z \frac{d\{u\}}{dz} = \Pi_m \left( \left( \frac{\sigma_j}{d_k^{(sb)2}} \right), \frac{d_k R}{\rho^{(w)} u_*^2} \right) \] (13)

Equation (13) shows that the Shields parameter \( \theta_k^{(sb)} = \frac{d_k^{(w)} u_*^2}{d_k R} \) and the normalised moments of the grain size distribution of the bed substrate, \( m_j^{(sb)} = \left( \frac{\sigma_j}{d_k^{(sb)2}} \right) \), are relevant non-dimensional variables to interpret the variation of the parameters of the log-law with in flows over water-worked beds under equilibrium sediment transport. In the reminder of the text, the parameters of the log-law are discussed as potential functions of these non-dimensional parameters.

### 4.3. The Displacement Height and the von Kármán Constant

Figures 7 and 8 show the normalised values of \( \Delta \) as a function of the Shields parameter. In Figure 7, \( \Delta \) is normalised with the median diameter of the substrate, \( d_{50\text{sub}} \). Only scenario (sc1) is shown, the remaining are qualitatively similar.

In tests of series E and T, water-worked beds with negligible (or zero) sediment transport (tests E0, T0, T1, T4 and T2, to the left of the dashed line in Figure 7) seem to produce a constant \( \Delta/d_{50}^{(sb)} \). The constant is approximately 1.5 in scenarios (s1) and (s2) and 1.4 in scenario (s3).

There seems to be an increase of \( \Delta/d_{50}^{(sb)} \) with \( \theta_{50}^{(sb)} \) for the water-worked beds with generalised sediment transport (tests of types E and T to the right of dashed line in Figure 7). If that is the case, it can be argued that

\[ \frac{\Delta}{d_{50}^{(sb)}} = F \left( \theta_{50}^{(sb)}, m_2^{(sb)}, \ldots \right) \] (14)

which is compatible with equation (13): the shear rate is a function of flow and sediment parameters through its governing parameters.
In the case of the armored beds of tests D, a steady increase seems to be registered in scenario (s1). This may be a spurious result. Otherwise, it would mean that a memory of the pre-armored bed subsists.

To further discuss the argument expressed in equation (14), the bed thickness $\delta$ was employed to normalise the displacement height. The bed thickness and the bed texture are products of water-working a specific sediment mixture (expressed by $n_{12}^{(sb)}$) by a given flow in a gravity field and, hence, are also functions of the Shields parameter and the normalised moments of the grain size distribution. The ratio $\Delta/\delta$ is shown in Figure 8. Only scenarios (s1) and (s3) are shown. Scenario (s2) is qualitatively similar to (s1).

In scenarios (s1) and (s2), the ratio $\Delta/\delta$ for tests E and T seems to collapse into a constant plateau (0.87 for scenario s1 and 0.91 for scenario s2). This result expresses the fact that both $\Delta$ and $\delta$ are determined by the bed micro-topography which, in turn, is a product of water working, being thus highly correlated.

The ratio $\Delta/\delta$ seems also constant for the armored tests D (0.62, 0.71 and 0.61 for scenarios s1, s2 and s3, respectively). A consistent explanation is that the flow loses the memory of the pre-armored bed (and of the initial bed mixture, which indicates that the increase in $\Delta/d_{50}^{(sb)}$ registered in Figure 7s1 is spurious): after the armoring process is completed, there is no vertical exchange of sediment across the bed (cf. [Hirano, 1971]) and only the organization of the bed surface, namely its water-worked structure, depleted of the finer fractions, is relevant to determine the displacement height.

Figure 9 shows the variation of $\delta$ with the Shields parameter. There is a increase of the bed amplitude, relatively to the $d_{50}^{(sb)}$ of the substrate, with the Shields parameter, more conspicuous in tests with the sand mode. In the armored tests, the bed amplitude is larger than then in the...
mobile bed tests. Since the value of $\Delta/\delta$ is lower in the armored tests, it is concluded that the bed-surface structure in these tests is such that the average spacing of micro-structures is larger. This would explain why the zero of the log-law is further deeper below the crests of the most protruding elements in tests of type D.

A most relevant result is that the amplitude of the bed $\delta$ is not a universal scale of the displacement height, even in fully active beds.

The ratio $\Delta/\delta$ in scenario (s3) requires further discussion (Figure 8s3). It is apparent that this ratio is smaller in the tests of type E and T that feature very weak (but non-zero) sediment transport (values of $\theta_{s0}^{(sb)}$ between 0.040 and 0.055). For these near-threshold tests, the zero of the log-law is at a lower elevation, relatively to the plane of the crests, than in the remaining tests. This vertical shift is accompanied by a decrease of the value of the von Kármán constant, as seen in Figure 10.

Using [Nikora et al., 2002]’s interpretation of the displacement height (section 2), the reduction of the value of $\Delta$ represents a deeper penetration of the largest eddies in the bed. This suggests that a reduction in the value of $\kappa$ in weakly mobile beds may be associated to subtle changes in the integral scales in the overlapping layer, induced by or associated to a reorganisation of turbulence in the near-bed region, arguably due to the particle motion and bedload-bed surface interaction. The exact nature of this chain of phenomena should be addressed with theoretical and experimental work.

It should not be forgotten that the result $z=\Delta u_*/d_{(s)}$ is an asymptotic one ([Monin & Yaglom, 1971]); it is valid in a narrow region where the inner region, directly influenced by the geometry of the bed, overlaps with the outer region. This means that the range on which to perform the regression analysis to equation (3) could be very narrow. Indeed, in scenarios (s1)
and (s2), the adjustment of (3), with $\kappa = 0.4$, to the data was good only in a very limited set of points. Extending the regression range would undoubtedly bring about a better correlation coefficient but perhaps an erroneous one, since it may be forcing non-meaningful data into the analysis. This would be sufficient to explain the variable $\kappa$ in scenario (s3), without recurring to hydrodynamic considerations and salvaging a flow-independent $\kappa$.

A more complete portrait emerges from the joint appreciation of Figures 7, 8 and 10. Should $\kappa$ be universal, the zero of the log-law would be located very near the plane of the crests (in a narrow band between 0.8 and 1.0 of the bed thickness) in tests of type E and T, independently of bed mobility. Since the zero of the log-law tends to remain at a fixed location just below the plane of the crests, the increase of the ratio $\Delta / d_{50}^{(s)}$ with $\theta_{50}^{(s)}$ is thus a simple consequence of the increase of the bed thickness with bed mobility, which can be observed in Figure 9. For tests of type D, the zero of the log-law is deeper in the bed, at approximately 70% of the bed thickness. The difference between tests of type E and T and tests of type D expresses the influence of the bed surface micro-topography: a more “sparse” distribution of gravel micro-clusters in armored beds would be responsible for lowering the zero of the log-law, relatively to the plane of the most protruding elements. If $\kappa$ is flow-dependent, and susceptible to be calculated as in scenario (s3), the values of both $\Delta$ and $\kappa$ would be reduced at the onset of generalised sediment transport due to yet unknown hydrodynamic causes.

### 4.4. The Geometric Roughness Scale $k_s$

Because fluid properties and flow parameters are involved in determining the shear rate. Hence, $k_s$ should not be thought as a property of the bed or expressed as a bed statistic. In scenarios (s1) and (s3), it was intended that $B$ had the physical meaning of the normalised velocity in the lowermost edge of the logarithmic layer. Hence, $\Delta + k_s$ is the elevation above
the boundary zero that marks the lower bound of the logarithmic layer. The scale \( k_s \) is thus
the vertical distance between the zero of the log-law and the lower bound of the logarithmic
layer. Equivalently, \( \Delta + k_s \) can be thought as the thickness of the layer influenced exclusively
by inner variables (Figure 1). In scenario (s2), \( k_s \) is calculated by a roughness function. Con-
stant \( B = 8.5 \) may not correspond to an actual velocity observed in the flow and the scale of
the roughness elements \( k_s \) becomes a two-phase flow variable. In scenario (s2) \( k_s \) should not
be interpreted as a thickness of any “roughness” flow layer but only an indirect measure of the
influence of the bed roughness.

The value of \( k_s \) should be influenced by the grain-size distribution of the bed surface. Figure
11 shows the variation of \( k_s/d_{90}^{(srf)} \) with the Shileds parameter. It is apparent that the mobility
of the bed exerts little influence of the values of \( k_s \), except in scenario (s2), where \( k_s/d_{90}^{(srf)} \)
decreases in mobile tests of type E. In the other scenarios (Figures 11s1 and 11s3), \( k_s/d_{90}^{(srf)} \)
is approximately constant for all tests \( (k_s/d_{90}^{(srf)}) \approx 1.1 \) and \( k_s/d_{90}^{(srf)} \approx 1.2 \) for scenarios (s1)
and (s3), respectively). Judging from the results of these scenarios, it is argued that the layer
exclusively influenced by inner variables is mainly determined by the largest fractions in the
bed if no relevant bed-forms are present other than gravel micro-clusters. The influence of
bed mobility seems fully explained by the organization of the bed surface as a result of water-
working.

Computing the variables as in scenario (s2), a different picture emerges: the bed mobility
influences the values of \( k_s \) but only for tests of type, E, \textit{i.e.} with the moving sand mode (Figure
11s2). For tests of type T, the moving gravel seems to be irrelevant; \( k_s \) in both tests T and D
scales with the \( d_{90}^{(srf)} \) of the bed surface \( (k_s/d_{90}^{(srf)} \approx 1.0) \). It would thus be concluded that the
movement of smaller grain-sizes would contribute to reduce the roughness scale.
The influence of the composition and intensity of the bedload discharge over $k_s$ has been investigated by [Whiting & Dietrich, 1990] or [Smart, 1999], among others. It is generally assumed that (i) $k_s = m \cdot d_k$, where the constant $m$ is larger than 0.5 and $d_k$ is representative diameter of the substrate (the $d_{84}$, for instance) or of the bed surface (e.g. [Song et al., 1998]), or (ii) that $k_s$ increases with the thickness of the bedload layer ([Smart, 1999]) and, thus, with the Shields parameter. This later assumption results from considering that the ratio $z_0/k_s$ is constant and that, according to [Owen, 1964]’s hypothesis, $z_0$ increases with the thickness of the bedload layer.

As discussed above, when $k_s$ is interpreted as the thickness of the layer exclusively influenced by inner variables, the results shown in Figure 11 do not support the theoretical standpoint expressed in (ii), but are in agreement to the assumption expressed in (i). If $k_s$ is calculated from the roughness function with $B = 8.5$, assumption (i) for gravel-sand mixtures is not supported; $k_s$ appears affected by bed mobility but the effect is contrary to that assumed by [Bridge & Bennet, 1992], working on water-worked natural beds. A fundamental difference of criteria could account for this disagreement: in scenario (s2) of the present study, $k_s$ is calculated from a roughness function while the cited works assume that $k_s$ increases with the thickness of the bedload layer.

4.5. The Velocity at the Lower Bound of the Log-layer and the Roughness Height

The normalised velocity at lower bound of the logarithmic layer, $B$, can be seen in Figure 12 as a function of the Shields parameter. Only scenario (s1) is shown; scenario (s3) is similar, showing that the actual value of the von Kármán constant is qualitatively not important. It is apparent that $B$ increases with bed mobility only for tests of type E. It can thus be argued that the presence of moving fine sediment (sand sizes) smoothen the bed in the sense that it allows
for larger velocities near the crests of the protruding roughness elements. However, note that the location above the crests where $B$ is measured, $k_s + \Delta - \delta$, also increases, attenuating this effect. Still, a logarithmic profile of the longitudinal velocity can be found at a distance from the crests that is of the order of magnitude of the $d_{90}$ with a velocity, at that elevation, that increases with bed mobility, if the bed includes sand fractions.

A hydrodynamic consequence is that the profiles in the logarithmic reach will not be universal, they will depend on bed mobility expressed by the Shields parameter. This is well illustrated in Figures 5s1) and 5s3) where it is clear that the logarithmic reaches are not self-similar. On the contrary, by imposing $\kappa = 0.4$ and $B = 8.5$, in scenario (s2), the respective logarithmic reaches are self-similar.

Evidently, the depth-averaged longitudinal velocity is the same for all scenarios. Hence, any changes observed in $B$ are compensated by changes with contrary effect in $k_s$ (and possibly in $\Delta$) and, in the case of scenario (s3), in $\kappa$. For instance, flow phenomena determining the large values of $B$ in tests of type E of scenario (s1) (Figure 12) are expressed, in scenario (s2), by small values of $k_s$ since $B$ is considered constant (Figure 11). The comparison of results of different data sets should thus be performed with care and attending to the proposed definitions for the parameters of the log-law.

The roughness length, $z_0$, was calculated from the scale of the roughness elements $k_s$ and constant $B$ (equation 4). Its values, normalised by the $d_{50}^{(sub)}$ of the substrate, are shown in Figure 13. All scenarios are qualitatively similar; only scenario (s3) is shown.

Geometrically, $z_0$ is the height above the zero of the log-law for which the velocity, calculated from that logarithmic law, is zero (equation 1). Being a derived quantity, from $k_s$ and $B$, parameter $z_0$ is clearly a variable of a two-phase phenomenon. Hence, it is potentially sensitive to (i)
the dimension of the grains that compose the bed (equivalent uniform grain roughness), to (ii) the grain-scale structures (gravel micro-clusters, low amplitude bedload sheets) that the water-worked beds form for different equilibrium sediment transport rates (bed texture) and to (iii) the effect of particles undergoing near-bed movement. [Wiberg & Rubin, 1989], in the wake of [Owen, 1964], postulated that $z_0$ is a result of the linear superimposition of factors (i) and (iii) above. The latter incorporates [Owen, 1964]’s hypothesis, which states that the momentum sink that occurs as particles are accelerated from their resting positions in the bed is simply added to the momentum sink represented by the drag of immobile bed elements. The more particles in motion, the greater the energy expenditure through the work of drag and lift forces and the greater the roughness felt by the flow.

The present data does not support Owens’ hypothesis: for a given initial mixture (represented by $d_{50}^{(sb)}$), there is no obvious trend of variation of $z_0$ with the Shields parameter, irrespectively of the definitions of $k_s$ and $B$. The slight decrease of $z_0/d_{50}^{(sb)}$ in tests of type E is within experimental uncertainty.

The present data reveals that water-worked beds under equilibrium bedload represent a more complex case. The fact that $z_0/d_{50}^{(sb)}$ does not increase with bed mobility may be explained by the fact that the bed surface suffers changes that may reduce drag, counterbalancing the extra momentum sink represented by the moving particles. Also, as [Whiting & Dietrich, 1990] point out, in natural beds, the particles subjected to drag are the same, either resting or in movement.

Combining tests E and T, the value of $z_0/d_{50}^{(sb)}$ seems constant and about 0.06. In the armored bed tests D, the constant varies little among the scenarios, 0.13, 0.12 and 0.14 for (s1), (s2) and (s3) respectively. There is thus an increase of $z_0$ proportional to the $d_{50}^{(sb)}$ of the initial bed.
mixture. In tests of type D, the value of $z_0$ is determined by the surface arrangement which explains the fact that the $d_{50}^{(sb)}$ is not a universal scale.

The result that $z_0$ is proportional to $d_{50}^{(sb)}$ agrees with the analysis of [Whiting & Dietrich, 1990] but contradicts the observations of [Wiberg & Rubin, 1989] and of [Smart, 1999]. These authors argue that $z_0$ is not a function of a characteristic diameter of the bed mixture. [Smart, 1999] furthermore sustains that $z_0$ scales with $u_*^2$. Different definitions and methods of calculation may be at the root of the disagreement.

It is interesting to note that, in water-worked beds under equilibrium sediment transport, for a given initial mixture, increasing the drag force applied to the bed induces necessarily an adjustment of the thickness of the bed and a change in the composition of the bedload and of the surface composition. For the mixtures tested, the thickness of the bed increases with the increase of $u_*$ (Figure 9) and the bed surface becomes slightly coarser (slight increase of the $d_{50}^{srf}$) but this does not bring about an increase of $z_0$. This means that there is no link between the drag force exerted on a water-worked bed under equilibrium sediment transport and the roughness height. Hence, it is concluded that, in water-worked beds with equilibrium sediment transport, it is not useful to try to distinguish conceptually several modes of roughness height, each corresponding to a type of drag.

The ratio $z_0/k_*$ is related to constant $B$ through equation (4); they express exactly the same reality if the von Kármán constant is universal. In classic rough-wall fluid mechanics ([Schlichting & Gersten, 2000], [Townsend, 1976]), the normalised slip velocity $B$ is considered a function of the bed roughness, and of fluid and flow variables, i.e. $\frac{\delta}{k_*} = F\left(Re_*, m_2^{(sb)}\right)$, where $Re_* = k_*u_*/\nu^{(w)}$ is the boundary Reynolds number. For flows over water-worked beds under equilibrium sediment transport and for “k-type” beds ([Perry et al., 1969]), the dominant type
in water-worked gravel-sand beds, there is more than one roughness scale; the moments of the grain-size distribution, $\sqrt{\sigma_j}$ and $\frac{\bar{d}^{(s)}}{R}$, are also relevant scales. Hence, the more complete functional dependence

$$\frac{z_0}{k_s} = F\left(Re_*, \theta_{50}^{(sb)}, m_2^{(sb)}\right)$$

(15)

should hold.

If the boundary Reynolds number is high, similarity to this parameter is not sufficient to render $z_0/k_s$ constant, as predicted by classic rough-wall hydrodynamics ([Townsend, 1976]). Instead, $z_0/k_s$ can potentially vary as function of the Shields parameter and of the composition of the bed substrate. This poses a theoretical problem concerning the definition of hydraulically rough beds: for $B$ and $k_s$ defined as in scenarios (s1) and (s3), a value $B = 8.5$ is not sufficient to identify a hydraulically rough bed and, correspondingly, velocity profiles over hydraulically rough beds, defined as those exhibiting large values of $Re_*$, may have $B = 8.5$.

In Figure 14, the ratio $z_0/k_s$ is represented along with the classical value 0.0333 of [Nikuradse, 1933]. Note that scenario (s2) is not analysed because $\kappa = 0.4$ and $B = 8.5$, which means that the Nikuradse’s classical value is always retrieved.

The results seem to indicate that $z_0/k_s$ conforms to classical hydrodynamic description in the case of gravel beds (tests of type T) and coarse armored beds (tests of type D). In the case of gravel-sand mixtures, it appears that $z_0/k_s$ depends on the Shields parameter, which means that this ratio should be considered a two-phase flow variable, confirming the functional dependance expressed by equation 15.

In scenario (s1), $z_0/k_s$ seems to decrease with the increase of $\theta_{50}^{(sb)}$. This supports the hypothesis of bed smoothening already discussed apropos constant $B$ (Figure 12). In scenario (s3), $z_0/k_s$ seems constant. Since, for this scenario, $\kappa < 0.4$ (Figure 10), commonly associated
to drag reduction due to sediment transport (cf. [Gust & Southard, 1983]), the bed smoothening effect would be expressed by a decrease of $\kappa$ and not by a reduction of $z_0/k_s$. Again, the discussion of the variation of the parameters of the log-law should bear in mind their definitions.

4.6. Flow in the Pythmenic Region and Bed Characterization

The characteristic length scale of the flow in the overlapping layer between the pythmenic and the inner regions is $L_s$, given by equation (5). The bed was “fully active” in all tests since particles from finer grain sizes could be entrained from the lowest troughs, provided they were not hidden by coarsest protruding particles. The vertical length scale $L_s$ was thus be larger than the bed thickness, as seen in1.

The friction velocity, defined as the vertical momentum flux at the plane of the crests (definition D1, section 2) seems the most appropriate kinematic scale for equation (2). Herein $u_p \equiv u_s(D1)$. Definition (D2) seems less appropriate since it involves phenomena, such as momentum fluxes between the pythmenic and hyporheic regions (the case of fully active beds), that should not influence the velocity profile in the vicinity of the plane of the crests.

The normalisation length scale, $L_p$, in equation (2) was calculated from the $\langle \bar{u} \rangle$ data in the vicinity of the plane of the crests. In agreement with the definition (D1) of $u_s$, the boundary zero was set at the plane of the crests, as in scenario (s2) for the discussion of the log-law.

The best correlation between $L_p$ and other geometric parameters susceptible to calculated with the current database was attained with the displacement height for the log-law, calculated as in scenario (s2). Figure 15 shows that the ratio $L_p/\Delta$ is approximately 0.356 and seems independent of bed mobility.

The velocity defect $\langle \bar{u} \rangle - \langle \bar{u} \rangle(Z_c)$ normalised by $u_s(D1)$ seems to collapse on a single straight line $z/(0.356\Delta)$ at the vicinity of the plane of the crests (Figure 16). The universality of this
result should be tested for other bed geometries. Should it hold, it may indicate that vertical
momentum exchanges at the plane of the crests are the relevant phenomenon that influence the
location of the zero of the log-law.

The value of the longitudinal velocity at the plane of the crests can be seen in Figure 17. A
slight increase of $\langle \bar{u} \rangle (Z_c) / u_*$ with the Shields parameter is registered. It appears that the effect
of bed mobility is to stretch the mean shear rate at the bed, increasing the value of $\langle \bar{u} \rangle$ at the
plane of the crests.

4.7. Friction Coefficient

The friction factor is, for simplicity, defined as $C_f = (u_* / U)^2$, in this text. Should the
bed shear stress be understood as the drag on the bed per unit plan area, the partition $C_f =
C_{f_{i;ii}} + C_{f_{iii}}$, where $C_{f_{i;ii}}$ accounts for the drag on the immobile bed elements and where $C_{f_{iii}}$
accounts for drag on the particles travelling by rolling, sliding or saltation in the near-bed re-

gion, is meaningful since drag is defined by a (time-averaged) integral of pressure over the entire
particle-fluid contact surface ([Nikora et al., 2007a]).

To calculate the parcel attributable to particle movement, [Rákózsi, 1967] chemically “froze”
the bed after water-working and introduced the same discharge over the thus-obtained immo-
bile bed that preserved the bed micro-topography and, according to the author, the porosity.
The friction velocity was then measured and compared with the friction velocity of the mo-
bile bed. In general, [Rákózsi, 1967] found that the $u_*^{(2)} = u_*^{(2)_{i;ii}} + u_*^{(2)_{iii}}$ of the mobile
bed was larger than the $u_*^{(1)} = u_*^{(1)_{i;ii}}$ of the immobile bed. To actually calculate the parcel
$C_{f_{iii}}^{(2)} \equiv \left( \frac{u_*^{(2)_{iii}}}{U^{(2)}} \right)^2$ from the difference $\left( \frac{u_*^{(2)_{i;ii}}}{U^{(2)}} \right)^2 + \left( \frac{u_*^{(2)_{iii}}}{U^{(2)}} \right)^2 - \left( \frac{u_*^{(1)_{i;ii}}}{U^{(1)}} \right)^2$
it is necessary that $U^{(2)} = U^{(1)}$, i.e. that the flow over the roughness elements is the same
with and without sediment transport. This is theoretically impossible to achieve exactly, since
the near-bed particle motion introduces changes in the flow structure ([Ferreira et al., 2002], [Campbell et al., 2005]). One other difficulty concerning the immobilisation of a previously mobile bed is that the immobile bed is smoother than the mobile bed because all the previously moving sediment particles are deposited in the bed, filling voids and diminishing the bed amplitude (cf. [Whiting & Dietrich, 1990], in particular Figure 1).

Hence, separating the parcels of the friction coefficient was not attempted in this study. The study was focused on assessing the influence of the mobile bed on the flow friction factor, expressed by its evolution with a Froude number. The results, for scenarios (s1) and (s2), are shown in Figure 18. The friction coefficient was calculated as $C_f = \left(\frac{u_*}{U}\right)^2$, where $U$ is the depth-averaged flow velocity in the channel centreline. The mean flow Froude number $Fr = U/\sqrt{gh}$ is favoured to substitute the Shields parameter so to prevent the friction factor to appear both in the horizontal and the vertical axis of the plot.

Figure 18 reveals that the actual definition of $u_*$ (different in scenarios s1 and s2, see Table 1 and section 4.1) is not relevant, arguably because the relative submersion is sufficiently high. It is also observed that the friction factor is insensitive to the increase of the Froude number (and thus the bedload discharge), in the range of the present laboratory work. The relevant influence seems to be the composition of the bed. For tests of type E, $C_f$ appears constant and equal to 0.054 and 0.052, for scenarios (s1) and (s2), respectively. For tests of type T and for the armored tests D, $C_f$ is 0.066 and 0.063, for scenarios (s1) and (s2), respectively. The $d_{100}$ of both mixtures is the same (Figure 4). Since the values of the $C_f$ of tests T and E are different, it can be concluded that the bed micro-topography is more relevant than the largest particles in the bed to determine the friction factor. This micro-topography can be substantially different.
if a sand mode is present in the bed. If that is not the case, it appears that the bed texture of a mobile gravel bed is similar to that of an armored bed with the same $d_{90}^{(srf)}$ of the bed surface.

These results are in agreement with those of [Whiting & Dietrich, 1990] who, having performed measurements on a natural water-worked bed, found no appreciable differences between mobile and immobile (sub-threshold) beds. Thus, the reported increase of the friction factor found by [Song et al., 1998] (imposing a sediment feed on a sub-threshold bed) and by [Carbonneau & Bergeron, 2000] or [Habibzadeh & Omid, 2009] (imposing an upstream sediment feed on a fixed bed), among others, is not representative of water-worked beds under equilibrium bedload. The contribution of near-bed particle motion is, in these studies, effectively superimposed to the contribution of the fixed bed. That is not the case of water-worked beds were the bed micro-topography adapts, contributing to keep the value of $C_f$ essentially independent of the amount of bedload.

5. Conclusion

The effect of bed mobility and bedload transport on the vertical profile of the longitudinal velocity and on flow resistance was assessed in this work through the analysis of laboratory data relative to uniform open-channel flows with porous mobile rough beds composed of poorly sorted gravel-sand mixtures undergoing equilibrium sediment transport.

The database was explored under three scenarios: definition (D2) of $u_*$ (section 2, item 6), variable $B$ and $\kappa = 0.4$ (scenario s1); definition (D1) of $u_*$ (section 2, item 6), $B = 8.5$ and $\kappa = 0.4$ (scenario s2); and definition (D2) of $u_*$, variable $B$ and variable $\kappa$ (scenario s3).

It was proposed that the bed can be characterized as “fully active”, i.e. vertical momentum fluxes penetrate down to the elevation of the lowest troughs, or as “deep”, depending on the
relative magnitude of its amplitude, $\delta/L_s$ (Table 1). The current database was obtained in a “fully active” bed, for which the ratio $\delta/L_s$ is less than 1.

A theoretical analysis revealed that, in rough water-worked beds under equilibrium bedload transport, there is more than one relevant roughness scale. The classical $k_s$ scale must be complemented with $\sqrt{\sigma_f}$ and $\frac{\sigma^{(w)}_{u^2}}{R}$, accounting for the influence of the grain-size distribution and of the intensity of the bedload transport. A fundamental theoretical result is that the variables of the laws that describe velocity profiles in these flows may vary with the substrate composition, represented by the moments of its grain-size distribution, and with Shields’ parameter.

Having discussed the vertical profile of the mean longitudinal velocity as a function of those parameters, the following main conclusions were obtained.

- The displacement height, $\Delta$, increases relatively to the $d_{50}^{(sb)}$ of the substrate with the increase of bed mobility (expressed by the increase of the Shields parameter). As bedload discharge increases, the zero-plane displacement for the log-law must be placed further above the lowest bed troughs in order to ensure a good fitting with $\kappa = 0.4$.

- The amplitude of the bed also increases with the increase of the bed mobility; both are determined by the bed micro-topography. If the von Kármán constant is considered universal, it is thus found that the bed amplitude is sufficient to determine $\Delta$ for a given bed composition ($\Delta/\delta = \text{const}$). However, the amplitude of the bed is not a universal scale of the displacement height: the constant changes for each particular bed mixture and for armored beds.

- If the von Kármán constant is flow dependent, $\Delta$ seems to decrease at the onset of generalised bedload transport. This coincides with a pronounced decrease of the von Kármán constant, suggesting a reorganisation of turbulence with a subtle increase of the integral scale in the
near bed region. If indeed the von Kármán constant is flow dependent, the characterisation of
these phenomena should be addressed in future studies.

- If it is imposed $B = 8.5$, $k_s$ is calculated by a roughness function. In this case, $k_s$ appears
to decrease with the increase of bed mobility, expressing a bed smoothening effect. Constant
$B$ increases with the Shields parameter only if the bed contains a sand mode. The moving
fine sediment (sand sizes) apparently smoothen the bed in the sense that it allows for larger
velocities near the crests of the protruding roughness elements.

- There seems to be no effect of bed mobility on the values of the roughness height $z_0$.
Apparently, in water-worked beds under equilibrium sediment transport, any effect of increasing
$z_0$ due to particle movement is compensated by the rearrangement of bed micro-topography. In
this case, it is thus not useful to try to distinguish conceptually several modes of roughness
height, each independently parameterized to the Shields parameter.

- The ratio $z_0/k_s$ decreases with the increase of $\theta_{50}^{(sb)}$. This supports the hypothesis of bed
smoothening alluded in the discussion of constant $B$. The decreasing trend is less pronounced
if the von Kármán constant is flow-dependent and smaller than 0.4. This brings new light to
the findings of drag-reduction studies: the bed smoothening effect can be expressed by both
variables $z_0/k_s$ and $\kappa$. If one of them is held constant, the other should exhibit a stronger
variation.

- The normalized velocity defect in the vicinity of the plane of the crests seems to collapse
in a universal profile $z/\Delta$ provided that the velocity scale is derived from the momentum flux
at that elevation.

- The actual definition of $u_*$ is not relevant for the discussion of the friction factor $C_f$, ar-
guably because the relative submersion is sufficiently high. For the range of Froude numbers
discussed in this work, the friction factor is insensitive to the increase of the bed mobility. The relevant influence seems to be the composition of the bed. If there are no sand sizes, the bed texture of a mobile gravel bed is similar to that of an armored bed with the same $d_{90}^{(srf)}$, and so is the corresponding friction factor.

Considering three different scenarios for data treatment allowed for the clarification of issues that were obscured by ambiguity in the definitions of roughness-related parameters. The two most important are the following:

- The value of roughness scale $k_s$ appears to be determined by the largest fractions in the bed mixture and independent of the bed mobility only if it is defined as the thickness of the layer exclusively influenced by inner variables (scenarios s1 and s3, section 4.1). If $k_s$ is calculated from a roughness function (scenario s2), its value appears affected by the sand content and its mobility. Evidently, none of the views is more correct than the other; both express the same vertical profile of the longitudinal velocity and must be discussed in conjunction with the value of $B$. In scenarios (s1) and (s3) a constant $k_s/d_{90}^{(srf)}$ is associated to an increase of $B$; in scenario (s2), a decreasing $k_s/d_{90}^{(srf)}$ is associated to a constant $B$. In both cases, the underlying phenomena seems to be a bed smoothening in the presence of moving sand fractions.

- The nature of the von Kármán constant, universal or flow-dependent, influences the debate on the nature of roughness in water-worked beds under equilibrium bedload transport, namely the discussion of the values of the roughness height $z_0$ and of $z_0/k_s$. The bed smoothening effect can be detected and discussed solely with the ratio $z_0/k_s$. However, if there are hydrodynamic reasons to justify the variability of $\kappa$, then a bed smoothening effect can be legitimately expressed by a decrease of $\kappa$. This issue should be addressed with more detailed investigations on the structure of near-bed turbulence in flows over water-worked mobile beds.
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References


Figure 1. Idealized physical system. The boundary zero is set at the plane of the troughs. The involved variables are identified in the text.
Figure 2. Reynolds stresses in a vertical plane at the centerline of a rough bed. The amplitude of the bed (distance between troughs and crest) was $\delta = 0.065$ m, the mean-flow Reynolds number was $Re = 9.1 \times 10^4$, and the Froude number was $Fr = 0.62$ (laboratory tests performed at IST, TU Lisbon, partially shown in Ferreira et al. 2010).
Figure 3. Shear-stress for tests E (⋆), D (♢) and T (□).
Figure 4. Grain-size distribution of the initial gravel mixtures of tests T (○) and of the initial gravel-sand mixtures of tests E and D (◇).
Figure 5. Vertical profile of the longitudinal velocity. The same data is normalised differently according to scenario (s1), scenario (s2) and scenario (s3). Profiles of tests E represented by filled diamonds (●), tests D represented by open diamonds (◇) and tests T represented by open circles (○). Dotted line stands for equation (1) with $\kappa = 0.4$ and $z_0/k_s = 0.0333$ ($B = 8.5$).
Figure 6. Regression line for the calculation of the displacement height ($\text{___}$). Circles ($\bigcirc$) stand for the normalised polynomial $u_r \left( \frac{p^{(3)}}{u^*} \right)^{-1}$. Data of test $E_1$. 
Figure 7. Variation of the displacement height, normalised with $d_{50}^{(s)}$, with the Shields parameter (scenario s1). Data of tests of type E represented by filled diamonds (●), type D represented by open diamonds (◇) and type T represented by open circles (○). Dashed line marks the value of the Shields parameter of test T₂.
Figure 8. Variation of the displacement height, normalised with the bed thickness, with the Shields parameter (scenarios s1 and s3). Data identified as in Figure 7.
Figure 9. Variation of the bed thickness, normalised by the median diameter of the substrate with the Shields parameter calculated for scenario (s1). Data identified as in Figure 7.
Figure 10. Von Kármán constant, $\kappa$, scenario (s3), as a function of the Shields parameter. Data identified as in Figure 7.
Figure 11. Variation of the roughness scale $k_s$, normalised by $d_{90}^{(srf)}$, with the Shields parameter. Data identified as in Figure 7.
Figure 12. Variation of the constant $B$ (defined as the normalised velocity at the top of the layer directly influenced by roughness). Data identified as in Figure 7.
Figure 13. Variation of the roughness height normalised by $d_{s50}^{(sb)}$, as a function of the Shields parameter. Data identified as in Figure 7.
Figure 14. Variation of the roughness height normalised by the roughness scale $k_s$, as a function of the Shields parameter. Data identified as in Figure 7.
Figure 15. Scale parameter $L_p$ normalised by the displacement height ($L_p/\Delta$). Data identified as in Figure 7.
Figure 16. Normalized velocity defect in the vicinity of the plane of the crests, $(\langle \bar{u} \rangle - \langle \bar{u} \rangle_{Z_c}) / u^*_c$. Data identified as in Figure 7, except tests of type T: subthreshold tests $T_0$, $T_1$ and $T_2$ represented by open circles ($\bigcirc$); low-bedload tests $T_3$, $T_4$ and $T_7$ represented by gray circles ($\bigcirc$); high-bedload tests $T_5$, $T_6$, $T_8$ and $T_9$ identified with black circles ($\bigcirc$).
Figure 17. Normalized longitudinal mean velocity at the plane of the crests of the roughness elements \( \left( \langle \bar{u} \rangle \left( Z_c \right) / u_* \right) \). Data identified as in Figure 7.
Figure 18. Friction coefficient $C_f$ as a function of the mean-flow Froude number for scenarios (s1) and (s2). Data identified as in Figure 7.

![Graphs of friction coefficient $C_f$ vs. Froude number for scenarios (s1) and (s2).](image)

Table 1. Summary of the characteristics of the experimental tests.

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<th>Name</th>
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