Flow over rough mobile beds: friction factor and vertical distribution of the longitudinal mean velocity

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Abstract. The main objective of the present study is to identify the impacts of bed mobility on the vertical profile of the mean longitudinal velocity and on resistance in flows over water-worked beds of poorly sorted mixtures of sand and gravel. Water-worked beds with sediment transport are explicitly distinguished from immobile beds with imposed sediment feed. Flows with different equilibrium sediment transport rates are generated in a laboratory flume. The initial bed mixtures featured combinations of sand and gravel modes. Data collection included instantaneous velocities measured with Laser Doppler Anemometry. Wall similarity, in the sense of [Townsend, 1976], is assumed. The parameters of the formulae are discussed within three scenarios comprising different definitions of $u_*$ and $k_s$ combined with different conceptions of the Von Kármán constant ($\kappa$ flow-independent or flow-dependent). It is shown that the parameters of the formulae that express the velocity profile vary with the Shields number and with the initial bed composition. The variation is independent of the adopted scenario, except in what concerns the formulation of hydraulic smoothening in the presence of sand sizes, which depends on the definition of $k_s$. 
1. Introduction

The empirical characterisation of the mean flow velocity and of flow resistance is a classical theme in open-channel rough-wall hydrodynamics (reviews in [Schlichting & Gersten, 2000], [Castro, 2007], among others). A crucial idealisation, inherited from smooth-wall hydrodynamics, is the partition of the flow column into overlapping inner and outer regions, valid if (i) gradients in the longitudinal direction, notably the pressure gradient, are small (gradually varied flow in the longitudinal direction), (ii) the aspect ratio is high (time-averaged flow far from the wall is essentially two-dimensional in the vertical plane) and (iii) the relative submersion is high. Under these conditions, the vertical distribution of the mean longitudinal velocity in the overlapping layer can be fitted to the logarithmic law of the wall

\[
\frac{\{u\}(z')}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z'}{z_0} \right),
\]

(1)

where \( \{u\} \) is an ensemble-averaged longitudinal velocity (generally time-averaged), \( z' = Z - Z_0 \) is the vertical coordinate above \( Z_0 \), the zero of the log-law, \( Z \) is the vertical coordinate above and arbitrary datum, \( z_0 \) is the bed’s characteristic roughness height, \( \kappa \) is the von Kármán constant and \( u_* = \sqrt{\tau_0/\rho^{(w)}} \) defines the kinematic scale for both inner and outer flow variables, where \( \tau_0 \) is the wall shear stress and \( \rho^{(w)} \) is the fluid density.

A basic premise of this text is that, in flows over hydraulically rough beds with bedload, the characterisation of the vertical profile of \( \{u\} \) can be performed with the same formal apparatus of flow over immobile boundaries. In fact, this is in accordance with the current research practice (cf. [Song et al., 1998], [Carbonneau & Bergeron, 2000], [Smart, 1999], [Ferreira et al., 2002], [Campbell et al., 2005] and [Dey & Raikar, 2007], among others).
Especially important in this context is the distinction between (a) porous beds composed of cohesionless poorly-sorted sediment, water-worked under equilibrium bedload transport, and (b) sub-threshold or fixed rough beds with upstream-imposed sediment transport. In the case of (a), under uniform flow and for the same bed mixture, the same slope and the same discharge, there is no degree of freedom between the composition of the bed surface and the quantity and composition of the bedload. In fixed beds, on the contrary, the bedload rate and composition may be arbitrarily defined, with no relation with the composition of the bed surface (cf. [Carbonneau & Bergeron, 2000] and [Habibzadeh & Omid, 2009]).

This paper will be mostly concerned with water-worked beds composed of poorly-sorted sediment mixtures. In this case, the strong feedback between the bed surface and the bedload poses specific challenges when it comes to quantify the parameters of equation (1).

First of all, in porous beds, there is no “natural” zero. In fact, there is an irredeemable arbitrariness in the definition of the boundary zero. As a result, the quantification of the log-law parameters, namely the roughness height $z_0$ and $\kappa$, becomes relative to the adopted definition of the boundary zero and of the zero of the log-law.

A second difficulty concerns the calculation of the bed shear stress. While in smooth beds the bed shear stress is unambiguously defined as the momentum transmitted to the boundary, in rough porous beds there are several possible definitions (cf. [Manes et al., 2007]). The description of the flow in the pythmenic layer ([Ferreira et al., 2010]) has benefited from the application of double (time and space) averaging methodologies ([Nikora et al., 2001], review in [Nikora et al., 2007a]), allowing for formally sound definitions of $\tau_0$. However, the quantification of the parameters of log-law will still be relative to the adopted definition.
Bearing these issues in mind, the main objective of this study is the discussion of the influence of bed mobility and bedload transport on the vertical profile of the longitudinal velocity and on the friction factor, in flows over water-worked beds of poorly sorted mixtures of sand and gravel. For that purpose, a new laboratory database is employed, featuring Laser Doppler Anemometry (LDA) velocity measurements in various flows conditions.

Addressing the issue of the non-uniqueness of the definitions of the parameters that arise in rough-wall hydrodynamics, a secondary objective is to describe the way different definitions of $u_*$, $\kappa$ and of the geometric roughness scale, $k_s$, may lead to different flow descriptions. Out of the many possible combinations, three scenarios are proposed in this text. Scenario (s1) comprises given definitions of $u_*$ and of $k_s$ and considers $\kappa = 0.4$. Scenario (s2) proposes different definitions of $u_*$ and of $k_s$ maintaining $\kappa = 0.4$. Scenario (s3) retakes the definitions of $u_*$ and of $k_s$ proposed in scenario (s1) but considers a flow-dependent $\kappa$.

The paper formulates answers to questions such as: what are the non-dimensional variables that should be employed to discuss the influence of bed mobility on the parameters of equation (1)? what is the pattern of variation of these parameters? how sensitive is the flow description to the particular definitions of parameters such as $u_*$ or $k_s$? in mobile beds, is it necessary to change the value of the Von Kármán constant to fit $\{u\}$ data to a log-law?

2. The Physical System

Consider a wide channel with a steady open-channel flow, uniform in the longitudinal direction. The turbulent boundary layer is fully developed over an irregular, porous, mobile bed composed of a poorly sorted mixture of cohesionless particles in the sand-gravel range. The relative submersion of the largest particles is about or larger than 10. The total transport rate of contact load – the bedload discharge – and the fractional bedload discharges are time-invariant.
in a domain whose longitudinal scale is several times the flow depth and no significant bed
forms develop. Under these conditions, the time average of the elevation of the bed is constant,
at any location. Furthermore, after enough time is elapsed, complete mixing is achieved and the
space-averaged bed texture becomes time-invariant and uniform in the longitudinal direction.

Figure 1 shows a short longitudinal reach of this idealised stream. The flow is divided in four
regions, the outer region (A), the inner region (B), the pythmenic region (C) and the hyporheic
region (D). There may be overlapping between every two adjacent regions as the phenomena
that characterise each region does not cease to exist abruptly.

In region (A) the turbulent flow is influenced by the free-surface and scales with outer vari-
ables $h$, the flow depth, and $u_\ast$. The flow in the inner region (B) is affected by the characteristics
of the rough wall, directly in the lowermost layers and indirectly, through $u_\ast$, in the uppermost
layers. There may be more than one relevant length scale, expressing the arrangement of the bed
particles, the amplitude of the fluctuation of the bed micro-topography and the bedload transport
rate and composition. The dominant characteristic length scale in region (B) is $k_s$. In order to
ensure compatibility with smooth bed analysis, the lower boundary of the inner region is set at
the elevation of the zero of the log-law, $Z_0$, located $|\Delta|$ above (or below) the boundary zero. $\Delta$
is known as the displacement height and can be negative (Figure 1). Accepting wall similarity,
in the sense of [Townsend, 1976], in the overlapping layer between the inner and outer regions
the vertical profile of the longitudinal velocity is logarithmic and expressed by equation (1).

In the pythmenic region (C), the flow is three-dimensional and mainly determined by the
bed micro-topography. In the overlapping layer between the pythmenic and inner regions, near
the crests of the roughness elements, there are strong vertical momentum fluxes resulting from
the interaction of separated regions with flow diverted at roughness elements ([Giménez-Curto
A shear layer is formed (Figure 2) that can be characterized with [Raupach et al., 1996]’s idealisation for canopy/free-stream momentum exchange ([Katul et al., 2002], [Nikora et al., 2007a]). If the amplitude of the bed, $\delta$, is large, it is reasonable to believe that vertical transfer of fluid momentum by the eddies generated at the shear layer is attenuated and will vanish below a certain elevation. Let $Z_b$ be this elevation (Figure 1). If the amplitude of the bed, $\delta$, is larger than $Z_c - Z_b$, it may be called “deep”. Otherwise, it may be called “fully active”, in the sense that vertical momentum fluxes penetrate down to the elevation of the lowest troughs.

Below $Z_b$ and, in general, in the hyporheic region (D), the flow should be determined by the permeability (or the void function), the hydraulic gradient, the fluid viscosity and density and the acceleration of gravity.

The definitions of the parameters of the log-law require the following remarks:

1. The zero of the vertical scale (the boundary zero) is a matter of convention. In this text, it is proposed that should coincide with the elevation $Z_b$. Hence, the flow depth is $h = Z_s - Z_b$, where $Z_s$ is the elevation of the free surface (Figure 1).

2. [Monin & Yaglom, 1971] suggested that the boundary zero could be inferred by fitting the data to a logarithmic profile and forcing the slope to be $1/\kappa \approx 1/0.40$. [Jackson, 1981], in the wake of [Thom, 1971], postulated (note that he did not prove it, analytically or otherwise) that whatever the origin of the vertical axis, the displacement height adjusts the zero-plane for the log-law so that it coincides with the plane that contains the net drag force over the roughness elements. [Nikora et al., 2002] proposed that the zero plane of the log-law marks the limit of penetration of large eddies in the rough bed. Using the image of eddy cascade, the linear distribution of Prandtl’s mixing length is $\ell = \kappa(z - \Delta)$, where $z = Z - Z_b$, from which one
obtains, in the overlapping layer

\[ u_\ast \left( \frac{d\{u\}}{dz} \right)^{-1} = \kappa (z - \Delta) \]  

(2)

The value of \( \Delta \) is retrieved once the shear-rate and the friction velocity are calculated independently from experimental data. Evidently, equation (2) can be obtained from the derivative of equation (1) with no reference to the distribution of the mixing length. If \( \kappa = 0.4 \), this is equivalent to Monin and Yaglom’s remark.

In this text, no physical explanation of \( \Delta \) is endorsed. Equation (2) is employed as a non-ambiguous method of calculation that allows for a discussion of its variation with bed mobility.

3. The assumption that \( \kappa \) is universal has long been an object of dispute. One set of arguments ([Krogstad et al., 1992], [Oncley et al., 1996]) is based on the conviction that wall similarity may not hold for all types of rough beds, even if relative submergence is high, rendering the von Kármán constant a function of the roughness Reynolds number \( Re_0 \equiv u_\ast z_0 / \nu \) (\( \nu \) is the fluid kinematic viscosity). Other arguments, compatible with wall similarity, are drawn from the fact that \( \kappa = 0.4 \) is an observational result, not a theoretical one ([Dittrich & Koll, 1997]). In this case, equation (2) would provide a straightforward way to calculate \( \kappa \) from the shear rate data in the overlapping layer ([Dey & Raikar, 2007]), or to calculate \( \kappa \) and \( \Delta \) simultaneously ([Nikora et al., 2002], [Koll, 2006] or [Franca et al., 2008]).

The estimates of \( \kappa \) are generally incommensurable across studies due to different criteria for setting the location of the zero of the log-law and for defining \( u_\ast \). In this text, accepting wall similarity, two scenarios for \( \kappa \) will be adopted: constant, and equal to 0.4, and flow-dependent, calculated in accordance to equation (2).

4. In spite of research efforts (notably [Jackson, 1981]), there is not a general dynamic model for the roughness height. It is a measure of the boundary roughness and is related to the geo-
metric roughness scale $k_s$ by

$$\frac{z_0}{k_s} = e^{-\kappa B}$$

(3)

where $B$ is the normalised flow velocity ($\{u\}/u_s$) at the elevation $k_s$ above the zero of the log-law. For simple roughness geometries, including sand-like roughness, and for high roughness Reynolds numbers ($Re_0 > 2.5$), $z_0/k_s = c_r$ where $c_r \approx 0.0333$ (or $c_r \approx 1/32.5$ according to [Schlichting & Gersten, 2000]) and $B = 8.5$. In this case, the bed is called hydraulically rough.

The discussion about the effect of bed mobility and bedload transport has been dominated by the idealisation of [Owen, 1964], which postulated that $z_0$ increases with saltation height as it includes the momentum sink due to particle movement (cf. [Bridge & Bennet, 1992], [Wiberg & Rubin, 1989], among others). In this text, this hypothesis is investigated and discussed.

5. The scale $k_s$ is, in this text, not a geometrical property of the bed or a statistic of the bed micro-topography. Instead, it is considered a function of the latter and of fluid and flow parameters. Presumably it is mainly influenced by the effective bed amplitude $Z_c - Z_b$, understood as the “active” part of the bed, where vertical momentum exchanges take place.

6. In fully developed turbulent flows over smooth beds, the friction velocity $u_s$ is unambiguously defined as the viscous shear stress at the elevation of the smooth boundary, expressed in kinematic terms. If the bed is rough, $u_s$ is a matter of definition (see [Manes et al., 2007] for a discussion of different definitions). The void function, defined as $\varphi(z) = A_v(z)/A_T(z)$, where $A_T$ is the area of a sampling region inscribed in a plane parallel to the bed and $A_v$ is the parcel of that region occupied by voids, is a relevant variable for this discussion. Being $g$ the acceleration of gravity and $S_0$ the bed slope, two definitions of $u_s$ are employed:
\( u^* = \sqrt{gS_0(Z_s - Z_c)} \), representing the momentum flux at the elevation of the crests of the roughness elements ([Jackson, 1981]), [Manes et al., 2007] among others) and;

\( u^* = \sqrt{\int_{Z_b}^{Z_s} \varphi gS_0 \, dz} \), representing the total momentum transmitted by gravity to the fluid from the free-surface down to \( Z_b \). The distinction between “fully active” and ”deep” is relevant in this context. If the bed is fully active, \( Z_b = Z_t \) and \( u^* = \sqrt{gS_0 (h - (1 - \varphi_m)\delta)} \), where \( \varphi_m = \frac{1}{\delta} \int_{Z_b}^{Z_s} \varphi(z) \, dz \) is the depth-averaged void function. If the bed is deep, the active part of the pythmenic layer must be previously determined.

As a kinematic scale, the role of \( u^* \), for any valid definition, should be to express the bulk effect of the bed roughness on flow variables as it normalises the velocity profile in the overlapping layer between inner and outer regions. Both will depend on the bed micro-topography but only (D2) is explicitly defined in terms of the void function.

3. Laboratory Work

3.1. Description of the Laboratory Tests

Data collection took place in a 11 m long and 40 cm wide prismatic recirculating tilting flume of the Laboratory of Fluid Mechanics of the University of Aberdeen. The side-walls were made of glass, enabling visualisation and laser measurements. For each test, the flume floor was covered with a 6 cm deep sediment layer.

A total of 17 subcritical and nearly-uniform flow experimental tests were carried out. Their main characteristics are summarised in Table 1. The non-previously defined variables and parameters in this table are the (steady) flow discharge, \( Q \), the depth-averaged mean longitudinal velocity, \( U \), the aspect ratio, \( b_f/h \) (\( b_f \) is the channel width), the Froude number, \( Fr = U/\sqrt{gh} \), the width of the mesh that retains 50% and 10%, respectively, in weight, of a sediment sample of the bed substrate (below the lowest bed troughs) \( d_{50}^{(ab)} \) and \( d_{90}^{(ab)} \), the same...
relatively to the bed surface, $d_{50}^{(sr,f)}$ and $d_{90}^{(sr,f)}$, the Shields number, $\theta_{50}^{(sb)} = u_{*}^2 / (g(s-1)d_{50}^{(sb)})$, the boundary Reynolds number, $Re_* \equiv k_{sb}u_{*}/\nu$ and the non-dimensional bedload discharge, $\phi_{sb} = G_{b}/(\rho (g) b f d_{50}^{(sb)} \sqrt{g(s-1)d_{50}^{(sb)}})$, where $G_{b}$ is the bedload discharge in mass per unit time, $s = \rho^{(g)}/\rho^{(w)}$ is the specific gravity of the sediment particles and $\rho^{(g)} = 2590$ is the density of the bed material.

It was observed that the bed was fully active in all tests (finer sediment particles at the troughs were subjected to entrainment demonstrating vertical momentum fluxes down to $Z_t$). Hence, $Z_b = Z_t$ and $h = Z_s - Z_t$.

The friction velocity was calculated by three methods, employing definitions (D1) and (D2) (section 2). Methods (D2a) and (D1) of $u_{*}$ employ the profile of the Reynolds shear stresses $\rho^{(w)} (\overline{u'w'})$, where $u'$ and $w'$ are fluctuating longitudinal and vertical velocities, respectively. $\tau$ stands for time-average and $\langle \cdot \rangle$ for space average). In both cases, the profile of the Reynolds shear stresses was calculated from the instantaneous velocity data. Assuming zero pressure gradient in the longitudinal ($x$) direction, a regression line fitted the data in the upper 85% of the flow depth above the plane of the crests. As seen in Figure 3, in the lower 15% of the flow above the plane of the crests, the vertical profiles of the ensemble-averaged Reynolds shear stresses exhibit some scatter, probably because form-induced stresses cease to be negligible. Should the regression line be termed $\tau^{(lim)}(z)$, where $z = Z - Z_t$, the value of the bed shear stress is

$$\tau_0^{(D2a)}(z) = \tau^{(lim)}(z = (1 - \varphi_m)\delta)$$ \hspace{1cm} (4)$$

$$\tau_0^{(D1)}(z) = \tau^{(lim)}(z = \delta)$$ \hspace{1cm} (5)
for methods (D2a) and (D1), respectively. The density of point measurements of the bed micro-
topography was insufficient to obtain \( \varphi(z) \) and to guarantee an accurate estimate of \( \varphi_m \). Hence, the approximation \( \varphi_m = 1/2 \) was assumed in all tests.

Method (D2b) in Table 1 corresponds also to definition (D2) but is calculated as the integral balance of momentum for uniform flows:

\[
\tau_{0}^{(D2b)} \equiv \rho^{(w)} g R_* S_0
\]  

where \( R_* = b_f h_*/(b_f + 2h_*) \) and \( h_* = h - (1 - \varphi_m)\delta \) (a side-wall correction was applied, in accordance to the method proposed in [Chiew & Parker, 1994]). This estimation was essentially used to assess the order of magnitude of \( u_* \) prior to the analysis of the velocity data.

The control parameters to design the tests of Table 1 were \( S_0, Q \), and the initial bed compo-
sition.

Tests of type E and T differ in the initial bed composition, a gravel-sand mixture in the former and a gravel mixture in the latter. The full grain-size distributions of the initial bed mixtures are shown in Figure 4. The geometric standard deviation is 3.6 and 1.7 for tests of type E and T, respectively.

Tests D1 to D3 were obtained from the respective tests of type E by subjecting the water-
worked bed to an armoring process. As tests of type D result from an armoring process, it is legitimate to expect that the largest immobile particles \( d_{100}^{(sr)} \) in tests of type E and D are the same. The geometric standard deviation of the bed surface is 2.8, 3.1 and 2.6 for tests D1, D2 and D3, respectively. The average over all tests is 2.8.

### 3.2. Instrumentation and Experimental Procedures
For all tests, the most relevant measurements comprised i) water depth, ii) bed micro-
topography, iii) instantaneous flow velocity (longitudinal and vertical components) and iv) com-
position of the bed-surface. The flow discharge was monitored during the tests and the bedload
discharge was measured for a complete description of the flow.

Flow discharge was measured on a calibrated triangular weir placed in the downstream water
tank. The water depth was measured with a 0.5 mm precision point-gage running along in-
strumentation rails. The detailed measurements of the bed topography were performed with a
Keyence infra-red laser displacement sensor mounted on automated 3D traversing frame with
0.1 mm precision.

The instantaneous flow velocity was measured with Laser Doppler Anemometry. The system
features a DANTEC 55X Modular LDA system generating a 20 mW, monochromatic red (632.8
nm) He-Ne laser with a frequency shift of 40 MHz, imposed by a Bragg cell, thus capable of
detecting positive and negative velocities. The optics are mounted in forward scatter mode with
manual alignment of the receiving optics. The transmitting optics features three beams, placed
at 120°, allowing for the measurement of two orthogonal components of the instantaneous flow
velocity. The length (in the direction of the mean flow) and width (in the direction normal to
the walls) of the measurement volume are, approximately, 0.2 mm and 1.5 mm, respectively.
The signal is processed in a DANTEC 55N20 Doppler Frequency Tracker and converted into a
voltage output ready to be sampled on a personal computer. The data was re-sampled at even
time intervals and stored in the memory. The sampling software enabled the storage of 12000
samples before writing to file. The LDA probes were placed at about 6.5 m from the inlet.

Uniform flow conditions were achieved and maintained by manually operating a vane weir
at the flume outlet. In tests of type E and T, bedload samples were collected and water and
bed elevations were monitored for no less than 8 hours to verify equilibrium sediment transport. Usable data was collected from this stage onwards.

In each test D, the bed slope decreased and the water depth increased, as a consequence of the armoring process. Adjustments to the slope and to the downstream vanes were made to obtain a flow with \( \frac{dh}{dx} = 0 \) in the central part of the flume, encompassing the LDA section. Usable data was then collected.

Core samples of the bed were collected at three locations along the flume, including that of the LDA measurements. In tests of types E and T, the samples were taken after water-working the bed. In tests of type D, core samples were collected before and after the armoring process. The results of the in-situ core-sample analysis are shown in Table 1 in terms of the \( d_{s50}^{(sb)} \) and \( d_{s90}^{(sb)} \) of the substrate and in terms of the \( d_{s50}^{(srf)} \) and \( d_{s90}^{(srf)} \) of the bed surface.

Once uniform flow conditions were obtained in the reach where the LDA was placed, profiles of the instantaneous velocity were performed at no less than three separate occasions. The origin of the vertical axis was determined once the bed was laid: the LDA probe was placed so that the measuring volume was located at the elevation of the highest crests of the bed particles.

At the end of the tests, after collecting velocity and bedload, four lines of 10 cm each were profiled at the centerline of the channel in the vicinity of the location of the velocity measurements. The spatial definition was 4 samples/cm. These measurements were used to estimate the thickness of the bed, \( \delta \), defined as the ensemble average of the difference between the maximum and the minimum of each bed profile.

4. Results and Discussion

4.1. Methods of Calculation of the Parameters of the Log-law
Wall similarity, in the sense of [Townsend, 1976], is assumed valid. Considering that this does not require that the von Kármán constant is flow-independent and that there are different possible definitions of the parameters and scales of the log-law, three scenarios to interpret the laboratory data are proposed:

(s1) The boundary zero is set at the elevation of the lowest troughs \((z = Z - Z_t)\), the friction velocity is in accordance to definition (D2) (section 2), calculated as (D2a) in Table 1, and the von Kármán constant is considered flow-independent \((\kappa = 0.4)\). The geometric roughness scale \(k_s\) and the constant \(B\) are subjected to a best fit procedure. The roughness scale \(k_s\) is defined as the lowest height above the zero of the log-law for which the velocity profile is logarithmic.

(s2) The boundary zero is set at the plane of the higher crests \((z = Z - Z_c)\), the friction velocity is in accordance with definition (D1) (section 2), calculated as (D1) in Table 1, and the von Kármán constant is considered flow-independent \((\kappa = 0.4)\). The constant \(B\) is 8.5 and the roughness scale \(k_s\) is calculated from a roughness function.

(s3) The boundary zero is set at the elevation of the lowest troughs \((z = Z - Z_t)\) and the friction velocity is calculated by definition (D2), calculated as (D2a) in Table 1. The von Kármán constant is assumed not universal but a fitting parameter. The values of \(k_s\) and of \(B\) are calculated as in criterion (s1).

For each scenario, the longitudinal velocity data are shown in Figure 5.

In scenario (s1), the parameters of the log-law are calculated as follows:

- the displacement height is first calculated from equation (2) with \(\kappa = 0.4\). A fourth-order polynomial \(p^{(4)}(z)\) is fitted to the longitudinal velocity data and differentiated, hence obtaining \(p^{(3)}(z)\), a polynomial fitting to \(\frac{d\{u\}}{dz}\). A line whose equation is \(p(z) = 0.4z - 0.4\Delta\) is then fitted to \(u_\ast \left(\frac{1}{p^{(3)}}\right)^{-1}\) in the region below \(z/h = 0.35\). The actual regression bounds are restricted to...
maximize the correlation coefficient to a given tolerance. An example can be seen in Figure 6. The actual bounds are 0.0129 and 0.0259 m and the correlation coefficient is 0.992 for an minimum admissible of 0.990.

The value of the displacement height is finally obtained as $\Delta = -p(0)/0.4$.

- Having calculated $\Delta$, the log-law (1) is written as

$$\{u\} = \frac{u_s}{\kappa_{try}} \ln (z - \Delta) - \frac{u_s}{\kappa_{try}} \ln (k_s) + u_s B \tag{7}$$

which is in the form

$$Y = MX + A$$

with $Y = \{u\}$, $M = \frac{u_s}{\kappa_{try}}$, $X = \ln (z - \Delta)$ and $A = u_s \left( B - \frac{1}{\kappa_{try}} \ln (k_s) \right)$.

A linear regression to the longitudinal velocity data with $M$ and $A$ as fitting parameters is carried out. Parameter $\kappa_{try}$ retains the value of the von Kármán constant resulting from the linear regression of (7). The the bounds of the regression range are modified until $\kappa_{try} \approx 0.4$.

- Once suitable bounds are found and $M$ is such that $u_s/M = \kappa_{try} \approx 0.4$, the definition of $k_s$ is retrieved from $k_s = z_{LB} - \Delta$, where $z_{LB}$ is the lower bound of the regression range.

- Parameter $A$ is then employed to calculate constant $B$ as

$$B = \frac{A}{u_s} + \frac{1}{\kappa_{try}} \ln (k_s). \tag{8}$$

- At last, the roughness height $z_0$ is calculated from equation (3) with $\kappa = \kappa_{try}$, which concludes the procedure to calculate the parameters of the log-law in scenario (s1).

In scenario (s2), the procedure to calculate the parameters is as follows:

- the displacement height $\Delta$ is calculated first. The procedure is identical to that of scenario (s1), with the particularity that the vertical coordinate is now $z = Z - Z_c$.
• Equation (7) is re-written with $B = 8.5$. As in scenario (s1), equation (7) is fitted to the
data with $M = \frac{u_s}{\kappa_{try}}$ and $A = u_s \left( 8.5 - \frac{1}{\kappa_{try}} \ln (k_s) \right)$ as fitting parameters. As in the previous
scenario, the bounds of the regression are sought so that the slope $M$ renders $\kappa_{try} \approx 0.4$.

• The scale of the roughness elements is then calculated from $\ln (k_s) = \kappa_{try} \left( 8.5 - \frac{A}{u_s} \right)$.

• Finally, equation (3) is used to calculate $z_0$.

Scenario (s3) differs from scenario (s1) in as much as the von Kármán constant is not consid-
ered universal. The parameters of the log-law are calculated as follows:

• The displacement height, $\Delta$, is calculated along with $\kappa$, from the smoothed version of
equation (2), $p(z) = u_s \left( p^{(3)} \right)^{-1} = \kappa z - \kappa \Delta$, which is assumed to hold in some region within
the limits considered in scenario (s1), and subjected to a linear regression. Again, the actual
bounds are sought to maximize the correlation coefficient to 0.99. The value of $\kappa$ is obtained
directly as the slope of the regression line and $\Delta$ is calculated as $-p(0)/\kappa$.

• The data is then fitted to equation (7) hence obtaining $M$ and $A$. The trial value of the von
Kármán constant is obtained from $u_s/M$. The range is selected so that the trial value of the von
Kármán constant approaches the value calculated in the previous point.

• The geometric roughness scale $k_s$ is defined as in scenario (s1) and constant $B$ is calculated
from (8) with $\kappa_{try} = \kappa$.

• As in the previous scenarios, equation (3) is then used to calculate $z_0$.

4.2. The Shear Rate in the Overlapping Layer between the Inner and Outer Regions as a
Two-Phase Variable

In flows over mobile beds, the shear rate must be a variable of a two-phase phenomenon in
the sense of [Yalin, 1977]. It should be expressed as a function of the properties of the fluid,
the properties of the sediment that composes the bedload, the variables that characterize the two-phase flow, the micro-topography of the bed and the gravity field.

Furthermore, in water-worked beds under equilibrium bedload transport, the composition of both the bed surface and of the bedload are unambiguously determined by the composition of the substrate for given flow, and fluid and sediment properties. The hypothesis of equal mobility ([Parker & Klingeman, 1982]), is an instance of this principle: given enough time the bed surface necessarily coarsens so that the bedload composition approaches that of the substrate.

Neglecting the effects of the shape of the particles, the dimensional relation for the shear rate in flows over water-worked beds under equilibrium conditions is thus

\[
\frac{d\{u\}}{dz} = F_m \left( z', h, d_{k}^{(sb)}, (\sigma_j), u_s, \mu, \rho^{(w)}, \rho^{(g)}, g \left( \rho^{(g)} - \rho^{(w)} \right) \right) \tag{9}
\]

where \(d_{k}^{(sb)}\) is a representative diameter of granular material of the substrate, \((\sigma_j)\) represents a sufficient set of higher-order centred moments of the grain-size distribution of the bed substrate and \(\mu\) is the fluid viscosity. Note that \(u_s\) substituted \(\tau_0\) for convenience of notation. Note also that, in the case of the water-worked bed under equilibrium bedload, it is hypothesised that sediment transport may affect the shear rate both directly and through the bed texture. Given that bedload transport rates depend on the weight of the particles, the acceleration of gravity must be included in equation (9). It is implied in this argument that the acceleration of gravity is important only in as much as it determines, in conjunction with particle submerged density and form, the weight of the granular material. Hence, \(g\) was substituted by the group \(R = g \left( \rho^{(g)} - \rho^{(w)} \right)\), the submerged specific weight of the sediment particles.
Considering $z', u_s$ and $\rho^{(w)}$ as basic variables, applying Vaschy-Buckingham’s theorem, equation (9) becomes

$$\frac{z' \, d\{u\}}{u_s \, dz} = \Pi_m \left( \frac{h}{z'}, \frac{d_k^{(sb)}}{z'}, \frac{\sigma_j}{z' \nu}, s, \frac{z'R}{\rho^{(w)} u_s^2} \right)$$

(10)

Equation (10) reveals that flows over water-worked beds under equilibrium bedload require more than one roughness scale. The classical geometric roughness scale, $k_s$, essentially determined by bed amplitude and arrangement of large roughness elements, is insufficient and must be complemented by $\sqrt{\sigma_j}$ and by $\frac{\rho^{(w)} u_s^2}{R}$, expressing the fact that other roughness elements form out of particle clusters, assembled and destroyed as a function of the flow variables and conditioned by the grain-size distribution of the bed substrate.

The sediment transport is considered en masse, i.e. no detailed analysis of the motion of individual grains is considered and, therefore, the inertial forces associated to individual particle motion are irrelevant ([Yalin, 1977]). In this case, the influence of the sediment density, $s$, is indirect and restricted to variable $R$ in equation (10).

Combining $\frac{d_k^{(sb)}}{z'}$ with $\left( \frac{\sigma_j}{z' \nu} \right)$ and $\frac{z'R}{\rho^{(w)} u_s^2}$ and assuming i) that the original Von Kármán hypothesis holds, i.e. that viscosity plays no role in defining the shear rate away from the boundary, and ii) that complete similarity to the inner rough scales and to the outer scale, then

$$\lim_{\frac{z'u_s}{\rho^{(w)} u_s^2} \to +\infty} \frac{z' \, d\{u\}}{u_s \, dz} = \text{const}, \quad \lim_{\frac{d_k^{(sb)}}{z'} \to 0} \frac{z' \, d\{u\}}{u_s \, dz} = \text{const} \quad \text{and} \quad \lim_{\frac{h}{z'} \to +\infty} \frac{z' \, d\{u\}}{u_s \, dz} = \text{const}.$$  

Equation 10 becomes

$$\frac{z' \, d\{u\}}{u_s \, dz} = \Pi_m \left( \frac{\sigma_j}{d_k^{(sb)}}, \frac{d_k^{(sb)} R}{\rho^{(w)} u_s^2} \right)$$

(11)

Equation (11) shows that the Shields number $\theta_k^{(sb)} = \frac{\rho^{(w)} u_s^2}{d_k^{(sb)} R}$ and the normalised moments of the grain size distribution of the bed substrate, $m_j^{(sb)} = \left( \frac{\sigma_j}{d_k^{(sb)}} \right)$, are relevant non-dimensional...
variables to interpret the variation of the parameters of the log-law with in flows over water-worked beds under equilibrium sediment transport.

In the reminder of the text, it will be assumed that $m_2^{(sb)}$ is sufficient to characterize a given mixture and that it may be quantified as the geometric standard deviation. The parameters of the log-law are herein discussed as functions of $\theta_{50}^{(sb)}$ and $m_2^{(sb)}$. The latter is substituted by $m_2^{(srf)}$ in the case of tests of type D.

### 4.3. The Displacement Height and the von Kármán Constant

Figures 7 and 9 show the normalised values of $\Delta$ as a function of the Shields number. In Figure 7, $\Delta$ is normalised with the median diameter of the substrate, $d_{50}^{(sb)}$. Only scenario (s1) is shown, the remaining are qualitatively similar.

In tests of series E and T, water-worked beds with negligible (or zero) sediment transport (tests E$_0$, T$_0$, T$_1$, T$_4$ and T$_2$, to the left of the dashed line in Figure 7) seem to produce a constant $\Delta/d_{50}^{(sb)} \approx 1.5$.

There seems to be an increase of $\Delta/d_{50}^{(sb)}$ with $\theta_{50}^{(sb)}$ for the water-worked beds with generalised sediment transport (tests of types E and T to the right of dashed line in Figure 7). If that is the case, it can be argued that

$$\frac{\Delta}{d_{50}^{(sb)}} = F\left(\theta_{50}^{(sb)}, m_2^{(sb)}, \ldots\right) \quad (12)$$

which is compatible with equations (2) and (11): the shear rate is a function of flow and sediment parameters through its governing parameters.

In the case of the armored beds of tests D, a steady increase seems to be registered in scenario (s1). This may be a spurious result. Otherwise, it would mean that a memory of the pre-armored bed subsists.
To further discuss the argument expressed in equation (12), the bed thickness $\delta$ was employed to normalise the displacement height. The bed thickness and the bed texture are products of water-working a specific sediment mixture (expressed by $n_{t_2}^{(sb)}$) by a given flow in a gravity field and, hence, are also functions of the Shields number and the normalised moments of the grain size distribution, as seen in Figure 8. There is an increase of the bed amplitude, relatively to the $d_{50}^{(sb)}$, with the Shields number, more conspicuous in tests with the sand mode. In the armored tests, the bed amplitude is larger than then in the mobile bed tests.

The ratio $\Delta/\delta$ is shown in Figure 9. Only scenarios (s1) and (s3) are shown. Scenario (s2) is qualitatively similar to (s1). In scenario (s1), the ratio $\Delta/\delta$ for tests E and T seems to collapse into the constant plateau 0.87. This result expresses the fact that both $\Delta$ and $\delta$ are determined by the bed micro-topography which, in turn, is a product of water-working, being thus highly correlated.

The ratio $\Delta/\delta$ seems also constant for the armored tests D (0.62 and 0.61 for scenarios s1 and s3, respectively). A consistent explanation is that the flow loses the memory of the initial bed mixture: after the armoring process is completed, there is no vertical exchange of sediment across the bed ([Hirano, 1971]) and only the organization of the bed surface, namely its water-worked structure, depleted of the finer fractions, is relevant to determine the displacement height. The lower value of $\Delta/\delta$ in the armored tests indicates that the average spacing of bed micro-clusters is larger, relatively to the recirculation tests. This would explain why the zero of the log-law is deeper below the crests of the most protruding elements in tests of type D.

A most relevant result is that the amplitude of the bed $\delta$ is not a universal scale of the displacement height, even in fully active beds.
The ratio $\Delta/\delta$ in scenario (s3) requires further discussion (Figure 9s3). It is apparent that this ratio is smaller in tests that feature very weak (but non-zero) sediment transport (values of $\theta_{50}^{(sb)}$ between 0.040 and 0.055). For these near-threshold tests, the zero of the log-law is at a lower elevation, relatively to the plane of the crests, relatively to the remaining tests. This vertical shift is accompanied by a decrease of the value of the von Kármán constant, as seen in Figure 10.

Using [Nikora et al., 2002]'s interpretation of the displacement height (section 2), the reduction of the value of $\Delta$ represents a deeper penetration of the largest eddies in the bed. This suggests that a reduction in the value of $\kappa$ in weakly mobile beds may be associated to subtle changes in the integral scales in the overlapping layer. This may be induced by, or associated to, a reorganisation of turbulence in the near-bed region, arguably due to the particle motion and bedload-bed surface interaction. The exact nature of this chain of phenomena should be addressed with theoretical and experimental work.

It should not be forgotten that the result $\frac{z-\Delta}{u^*} \frac{d[u]}{dz} = \text{const}$ is an asymptotic one ([Monin & Yaglom, 1971]); it is valid in a narrow region where the inner region, directly influenced by the geometry of the bed, overlaps with the outer region. This means that the range on which to perform the regression analysis to equation (2) could be very narrow. Extending the regression range could bring about a better correlation coefficient but may be forcing non-meaningful data into the analysis. This would be sufficient to explain the variable $\kappa$ in scenario (s3), without recurring to hydrodynamic considerations and salvaging a flow-independent $\kappa$.

A more complete portrait emerges from the joint appreciation of Figures 7, 9 and 10. Should $\kappa$ be universal, the zero of the log-law would be located very near the plane of the crests (in a narrow band between 0.8 and 1.0 of the bed thickness) in tests of type E and T, independently
of bed mobility. Since the zero of the log-law tends to remain at a fixed location just below the plane of the crests, the increase of the ratio $\Delta/d_{50}^{(sb)}$ with $\theta_{50}^{(sb)}$ is thus a simple consequence of the increase of the bed thickness with bed mobility, which can be observed in Figure 8. For tests of type D, the zero of the log-law is deeper in the bed, at approximately 70% of the bed thickness.

The difference between tests of type E and T and tests of type D expresses the influence of the bed surface micro-topography: a more “sparse” distribution of gravel micro-clusters in armored beds would be responsible for lowering the zero of the log-law, relatively to the plane of the most protruding elements.

If $\kappa$ is flow-dependent, and susceptible to be calculated as in scenario (s3), the values of both $\Delta$ and $\kappa$ are reduced at the onset of generalised sediment transport due to yet unknown hydrodynamic causes. Thus, the value of $\kappa$ does not have to be decreased to guarantee that the mean longitudinal velocity data can be fitted to a log-law provided that the value of $\Delta$ is understood as a variable of the bed texture.

### 4.4. The Geometric Roughness Scale $k_s$

Because fluid properties and flow parameters are involved in determining the shear rate, $k_s$ should not be thought as a property of the bed or expressed as a bed statistic. In scenarios (s1) and (s3), it was intended that $B$ had the physical meaning of the normalised velocity in the lowermost edge of the logarithmic layer. Hence, $\Delta + k_s$ can be thought as the thickness of the layer influenced exclusively by inner variables (Figure 1). In scenario (s2), $k_s$ is calculated by a roughness function. Constant $B = 8.5$ may not correspond to an actual normalised velocity observed in the flow and the geometric roughness scale $k_s$ should not be interpreted as a thickness of any “roughness” flow layer but only an indirect measure of the influence of the bed roughness.
For tests of type T, $k_s$ does not vary with $\theta_{50}^{(ab)}$. In the case of tests of type E, the definition of $k_s$ is relevant to discuss its values. Considering scenario (s1), $k_s$ slightly increases (11%) with $\theta_{50}^{(ab)}$ (not shown). As for scenario (s2), $k_s$ seems independent of $\theta_{50}^{(ab)}$ (not shown). Since the surface of a water-worked bed is coarser than the initial mixture, the relevant issue is to know whether $k_s$ varies relatively to the $d_{90}$ of the bed surface.

Figure 11 shows the variation of $k_s/d_{90}^{(sr)}$ with the Shileds parameter (scenario s3 is not shown as it is similar to s1). It is apparent that the mobility of the bed exerts little influence in scenario (s1) since $k_s/d_{90}^{(sr)} \approx 1.1$ for all tests (Figure 11s1). This indicates that the layer exclusively influenced by inner variables is mainly determined by the largest fractions in the bed if no relevant bed-forms are present, other than gravel micro-clusters. The influence of bed mobility seems fully explained by the organization of the bed surface as a result of water-working.

Computing the variables as in scenario (s2), a different picture emerges: $k_s/d_{90}^{(sr)}$ decreases in mobile tests of type E, indicating that the bed mobility influences the values of $k_s$ but only for tests with a moving sand mode (Figure 11s2). For tests of type T, the moving gravel seems to be irrelevant; $k_s$ in both tests T and D scales with the $d_{90}^{(sr)}$ of the bed surface ($k_s/d_{90}^{(sr)} \approx 1.0$). In scenario (s2), a hydraulic smoothening hypothesis can formulated in the sense that the movement of smaller grain-sizes contributes to reduce the roughness scale.

The influence of the composition and intensity of the bedload discharge over $k_s$ has been investigated by [Whiting & Dietrich, 1990] or [Smart, 1999], among others. It is generally assumed that (i) $k_s = c_s d_k$, where the constant $c_s$ is larger than 0.5 and $d_k$ is representative diameter of the substrate or of the bed surface (the $d_{84}$, for instance) (e.g. [Song et al., 1998]), or (ii) that $k_s$ increases with the Shields number ([Smart, 1999]), as a result of considering that
the ratio $z_0/k_s$ is constant and that, according to [Owen, 1964]'s hypothesis, $z_0$ increases with bed mobility.

When $k_s$ is interpreted as the thickness of the layer exclusively influenced by inner variables (scenario s1), the present database (see Table 1 and Figure 11) show that, for the range of investigated $\theta_{sb}^{(sb)}$, both theoretical standpoints are compatible. The values of $k_s$ may increase with the Shields parameter but only in as much as the $d_{(50)}^{(srf)}$ increases, as a result of bed surface coarsening. The ratio $k_s/d_{(50)}^{(srf)}$ remains constant.

If $k_s$ is calculated from the roughness function with $B = 8.5$ (scenario s2), assumption (i) for gravel-sand mixtures is not supported; $k_s$ appears affected by bed mobility although the effect is contrary to that assumed by [Bridge & Bennet, 1992], working on water-worked natural beds. This is a case where the definition of the log-law parameters matters when it comes to interpreting its variation with bed mobility.

### 4.5. The Roughness Height and the Velocity at the Lower Bound of the Log-layer

The roughness height, $z_0$, was calculated from the geometric roughness scale $k_s$, constant $B$ and $\kappa$ (equation 3). Its values, normalised by $d_{(50)}^{(srf)}$, are shown in Figure 12. All scenarios are qualitatively similar; only scenario (s3) is shown.

Geometrically, $z_0$ is the height above $Z_0$ for which the velocity, calculated from the log-law, is zero (equation 1). Being a derived quantity, parameter $z_0$ is clearly a variable of a two-phase phenomenon. Hence, it is potentially sensitive to (i) the dimension of the grains that compose the bed (equivalent uniform grain roughness), to (ii) the grain-scale structures (gravel micro-clusters, low amplitude bedload sheets) that the water-worked beds form for different equilibrium sediment transport rates (bed texture) and to (iii) the effect of particles undergoing near-bed movement. [Wiberg & Rubin, 1989], in the wake of [Owen, 1964], postulated that $z_0$ is
a result of the linear superimposition of factors (i) and (iii) above. The latter incorporates [Owen, 1964]’s hypothesis, which states that the momentum sink that occurs as particles are accelerated from their resting positions in the bed is simply added to the momentum sink represented by the drag of immobile bed elements. The more particles in motion, the greater the energy expenditure through the work of drag and lift forces and the greater the roughness felt by the flow.

The present data does not support Owens’ hypothesis: for a given initial mixture (represented by $d_{50}^{(sb)}$), there is no obvious trend of variation of $z_0$ with the Shields number, as seen in Figure 12, irrespectively of the definitions of $k_s$, $B$ and $\kappa$. Combining tests E and T, the value of $z_0/d_{50}^{(sb)}$ seems constant and about 0.06 which contradicts the observations of [Wiberg & Rubin, 1989] and of [Smart, 1999]. The slight decrease of $z_0/d_{50}^{(sb)}$ in tests of type E is within experimental uncertainty.

The fact that $z_0/d_{50}^{(sb)}$ does not increase with bed mobility cannot be fully explained with the present database. However, a line of thought can be advanced: for higher bed shear stress gravel micro-clusters are less stable ([Strom et al., 2004]) hence introducing less form drag associated to fixed elements. This effect would counterbalance the extra momentum sink represented by the moving particles. Equivalently, it can be said, following [Whiting & Dietrich, 1990], that, in natural beds, the particles subjected to drag are the same, either resting or in movement.

In the armored bed tests D, $z_0/d_{50}^{(sb)}$ is 0.13, 0.12 and 0.14 for (s1), (s2) and (s3) respectively, which configures an increase of $z_0$ relatively to the mobile bed experiments. It must be concluded that this increase is related to bed surface coarsening and that the value of $z_0$ is thus determined by the surface grain-size distribution. This implies that $d_{50}^{(sb)}$ is not a universal scale.

The ratio $z_0/k_s$ is related to constant $B$ through equation (3); they express exactly the same reality if the von Kármán constant is universal. In classic rough-wall fluid mechanics ([Schlichting
& Gersten, 2000], [Townsend, 1976]), the normalised slip velocity $B$ is considered a function of the bed roughness, and of fluid and flow variables, $i.e.$ \( \frac{z_0}{k_s} = F \left( Re_s, m_2^{(sb)} \right) \). For flows over water-worked beds under equilibrium sediment transport and for “k-type” beds ([Perry et al., 1969]), the dominant type in water-worked gravel-sand beds, there is more than one roughness scale; the moments of the grain-size distribution, \( \sqrt{\sigma_j} \) and \( \frac{\mu^2}{R} \), are also relevant scales. Hence, the more complete functional dependence

\[
\frac{z_0}{k_s} = F \left( Re_s, \theta_{50}^{(sb)}, m_2^{(sb)} \right) \tag{13}
\]

should be held.

If the boundary Reynolds number is high, similarity to this parameter is not sufficient to render \( \frac{z_0}{k_s} \) constant, as predicted by classic rough-wall hydrodynamics ([Townsend, 1976]). Instead, \( \frac{z_0}{k_s} \) can potentially vary as function of the Shields number and of the composition of the bed substrate.

In Figure 13, the ratio \( \frac{z_0}{k_s} \) is represented along with the classical value 0.033 of [Nikuradse, 1933]. Note that scenario (s2) is not shown because \( \kappa = 0.4 \) and \( B = 8.5 \), which means that the Nikuradse’s classical value is always retrieved.

The results seem to indicate that \( \frac{z_0}{k_s} \) conforms to classical hydrodynamic description in the case of gravel beds (tests of type T) and coarse armored beds (tests of type D). In the case of gravel-sand mixtures (tests E), it appears that \( \frac{z_0}{k_s} \) depends on \( \theta_{50}^{(sb)} \), which means that this ratio should be considered a two-phase flow variable, confirming the functional dependence expressed by equation 13. Indeed, in both scenarios (s1) and (s3), \( \frac{z_0}{k_s} \) is larger for test E0.

This is compatible with a hydraulic smoothening hypothesis: water-working at high values of \( \theta_{50}^{(sb)} \) results in a coarser bed surface and a larger \( k_s \); however, the value of \( z_0 \) does not increase; eventually, the presence of moving fine sediment (sand sizes) precludes the formation
of perennial micro-clusters, preventing the increase of $z_0$. This is thus the meaning of such hydraulic smoothening: relatively to the (coarser) bed surface, moving sand sizes contribute to decrease the importance of the roughness height.

It is interesting to note that, in scenario (s2), hydraulic smoothening can not be discussed since, by definition, $z_0/k_s = 0.033$. In the case of (s2), the hydraulic smoothening hypothesis was formulated in terms of $k_s/d_{90}^{(sr)}$. Conversely, it is recalled that, in scenarios (s1) and (s3), hydraulic smoothening was not observed in terms of $k_s/d_{90}^{(sr)}$. This shows that the same phenomenon may be expressed in terms of different parameters, depending on their definition.

### 4.6. Friction Coefficient

In this text, the friction factor is, for simplicity, defined as $C_f = (u_\ast / U)^2$. Let the bed shear stress be understood as the drag on the bed per unit plan area. In that case, for a given flow with a given depth-averaged velocity $U$, $C_f = C_{f0} + C_{fs}$, where $C_{f0}$ accounts for the drag on the immobile bed elements and where $C_{fs}$ accounts for drag on the particles travelling by rolling, sliding or saltation in the near-bed region. The linear superimposition is possible because drag (and, hence $u_\ast$) is defined by a (time-averaged) integral of pressure over the entire particle-fluid contact surface ([Nikora et al., 2007a]).

In an early but well accomplished laboratory research effort, [Rákózsi, 1967] attempted to calculate $u_{s\ast}$ attributable to particle movement. He chemically “froze” the bed after water-working and introduced the same discharge over the thus-obtained immobile bed that, according to the author, preserved the bed micro-topography and the porosity. The slope was adjusted to obtain a new uniform flow. The friction velocity was then measured and compared with the friction velocity of the mobile bed. In general, [Rákózsi, 1967] found that the $u_{s\ast}^{(2)} = u_{s\ast}^{(2)} = u_{s\ast}^{(1)} = u_{s\ast}^{(1)}$ of the mobile bed was larger than the $u_{s\ast}^{(1)} = u_{s\ast}^{(1)}$ of the immobile bed. Hence, $u_{s\ast}^{(2)} =$
$u^*_s^{(2)} - u^*_s^{(1)}_0$ are larger for the mobile bed experiments. These experiments show that in flows with, essentially, the same micro-topography, overall drag (for which the friction velocity is a proxy) is higher if, additionally, sediment is being transported. Since the reduction of the depth-averaged velocity was smaller than the increase of the friction velocity, [Rákózsi, 1967] found that the mobile bed $Cf_s$ was larger than that of the “frozen” bed.

This result, along with those of, among others, [Song et al., 1998] (imposing a sediment feed on a sub-threshold bed) and by [Carbonneau & Bergeron, 2000] or [Habibzadeh & Omid, 2009] (imposing an upstream sediment feed on a fixed bed) has been used as an argument for the increase of the friction factor with bed mobility and sediment transport.

Yet, it is often overlooked that these results are not directly applicable to water-worked beds under equilibrium bedload transport. The sediment discharge rate and the bed micro-topography are a result of the same physical processes and can not be separated. The issue, in this case, is to find out if the increase in $U$ is greater or smaller than the increase in $u_s$ associated to a higher bedload rate. The results of the present database are used to clarify this issue.

The variation of the friction factor with $Fr$ is shown in Figure 14, only for scenario (s1) since (s2) is qualitatively similar. The mean flow Froude number is favoured to substitute the Shields number so to prevent the friction factor to appear both in the horizontal and the vertical axis of the plot.

It is observed that the friction factor is insensitive to the increase of $Fr$ (and thus of the bedload discharge), in the range of the present laboratory work. Increasing $U$ (for instance increasing $Q$ at constant slope) is associated to a concomitant increase of $u_s$, rendering, for one given initial mixture, a constant $Cf$. 
The relevant influence seems to be the composition of the bed. For tests of type E, \( C_f \approx 0.054 \).

For tests of type T and for the armored tests D, \( C_f \) is 0.066. The \( d_{100} \) of both mixtures is the same (Figure 4). Since the values of the \( C_f \) of tests T and E are different, it can be concluded that the bed micro-topography is more relevant than the largest particles in the bed to determine the friction factor. This micro-topography can be substantially different if a sand mode is present in the bed. If that is not the case, it appears that the bed texture of a mobile gravel bed is similar to that of an armored bed with the same \( d_{90}^{(sr)} \) of the bed surface.

These results are in agreement with those of [Whiting & Dietrich, 1990] who, having performed measurements on a natural water-worked bed, found no appreciable differences between mobile and immobile (sub-threshold) beds. Thus, the reported increase of the friction factor is probably a consequence of the laboratory methodology, namely feeding sediment on a fixed bed. In such studies, the effect of sediment transport, namely the associated momentum sink, is actually superimposed to the contribution of the fixed bed. That is not the case of water-worked beds were the bed micro-topography adapts, contributing to keep the value of \( C_f \) essentially independent of the amount of bedload.

5. Conclusion

The effect of bed mobility and bedload transport on the vertical profile of \( \{u\} \) and on \( C_f \) was assessed in this work through the analysis of laboratory data. The tests featured uniform open-channel flows with porous mobile rough beds composed of poorly sorted gravel-sand mixtures undergoing equilibrium sediment transport.

The database was explored under three scenarios: (s1) featuring definition (D2) of \( u_* \) (section 2, item 6), \( k_s \) interpreted as the thickness of the layer, above the zero of the log-law, influenced by the bed micro-topography and \( \kappa = 0.4 \); (s2) featuring definition (D1) of \( u_* \) (section 2, item
6), $k_s$ calculated from a roughness function and $\kappa = 0.4$; and (s3), the same as (s1) in what concerns $u_*$ and $k_s$ but with a flow-dependent $\kappa$.

A theoretical analysis revealed that, in rough water-worked beds under equilibrium bedload transport, there is more than one relevant roughness scale. The classical $k_s$ geometric roughness scale must be complemented with $\sqrt{\sigma_j}$ and $\frac{(w)u^2}{R}$, accounting for the influence of the grain-size distribution and of the intensity of the bedload transport. These scales are indispensable to interpret flows over water-worked mobile beds. A fundamental theoretical result is that the parameters of the laws that describe longitudinal velocity profiles may vary with the substrate composition, represented by $m_j^{(sb)}$, the normalised moments of its grain-size distribution, and with $\theta_{50}^{(sb)}$, the Shields number.

Having discussed the vertical profile of the mean longitudinal velocity as a function of those non-dimensional variables, the following main conclusions were obtained.

The parameters of the log-law exhibit, in general, a variation with $\theta_{50}^{(sb)}$ and with the composition of the bed (represented by $m_j^{(sb)}$). The variation, however is not independent of their definition. The two most relevant cases are summarized next.

- The value of $k_s/d_{90}^{(sr)}$, is independent of $\theta_{50}^{(sb)}$ only if the geometric roughness scale is defined as the thickness of the layer exclusively influenced by inner variables (scenarios s1 and s3, section 4.1). If $k_s$ is calculated from a roughness function (scenario s2), the value of $k_s/d_{90}^{(sr)}$ decreases, affected by the sand content and its mobility.

- The ratio $z_0/k_s$ is constant, by definition, if $k_s$ is calculated from a roughness function with $B = 8.5$ (scenario s2). In scenarios (s1) and (s3), the presence of a mobile sand mode is associated to a decrease of the values of $z_0/k_s$ (and, thus an increase of $B$).
Evidently, none of the above views is more correct than the other. In fact, it can be argued that both are consistent with hydraulic smoothening but only if a mobile sand mode is present. Here, hydraulic smoothening means the existence of phenomena that preclude the increase of $z_0$ in the presence of a coarsening bed. For tests of type E (with a mobile sand mode), the bed amplitude increases with $\theta_{90}^{(sb)}$ and the bed surface coarsens. In this case, the following can be concluded.

- The thickness of the near-bed layer where the log-law is not valid increases with the increase of bed mobility. Yet the roughness height $z_0$ remains constant. Apparently, in water-worked beds under equilibrium sediment transport, any effect of increasing $z_0$ due to particle movement is compensated by the rearrangement of bed micro-topography, for instance destruction of gravel micro-clusters responsible for important form drag. In scenarios (s1) and (s3), this hydraulic smoothening is expressed as a decrease of $z_0/k_s$. Note that, in these scenarios, the increase in the values of $B$ is nothing but a consequence of the fact that the log-law is valid only further up in the water column.

- In scenario (s2), $k_s$ does not express the thickness of a layer. It is directly a measure of the bed roughness. Hydraulic smoothening is thus directly expressed by the decrease of $k_s/d_{90}^{(surf)}$ with the increase of $\theta_{90}^{(sb)}$.

- For all scenarios, the values of $k_s/d_{90}^{(surf)}$ and $z_0/k_s$ are constant and independent of bed mobility in the gravel bed tests (type T), even if gravel sizes are moving, and in the armored bed tests (type D). No hydraulic smoothening is envisaged. The flow can be described as if the bed was a rough immobile boundary.
The ability of a water-worked bed under equilibrium bedload transport to adjust itself to stronger drag, inherent to hydraulic smoothening, distinguishes it from fixed beds with imposed sediment feed. The following consequences are worth noticing.

- It is not useful to try to distinguish conceptually several modes of roughness height, each independently parameterized to the Shields number, since \( z_0 \) does not necessarily increase with \( \theta_{90}^{(ab)} \).

- For the range of Froude numbers discussed in this work, the friction factor is insensitive to the increase of the bed mobility. While in fixed beds the momentum sink associated to sediment transport is actually superimposed to the contribution of the fixed bed, in water-worked beds the bed micro-topography adapts, keeping the value of \( C_f \) essentially independent of the amount of bedload. The relevant influence for \( C_f \) seems to be the composition of the bed.

The discussion of nature of the von Kármán constant, universal or flow-dependent, arises frequently when describing turbulent flows over mobile beds. In particular, it is often observed that a reduction of \( \kappa \) allows for a better fitting of \( \{u\} \) to the log-law.

Without discussing the nature of \( \kappa \), the present data shows that a perfectly consistent flow description can be achieved with \( \kappa = 0.4 \). The relevant discussion becomes the behaviour of the displacement height. If \( \kappa \) is universal, the values of \( \Delta \) increase relatively to \( \delta_{50}^{(ab)} \) with the increase of \( \theta_{50}^{(ab)} \). As bedload discharge increases, the zero-plane displacement for the log-law must be placed further above the lowest bed troughs in order to ensure a good fitting with \( \kappa = 0.4 \). The amplitude of the bed, \( \delta \), also increases with \( \theta_{50}^{(ab)} \); both \( \Delta \) and \( \delta \) are determined by the bed micro-topography. For \( \kappa = 0.4 \), the bed amplitude is sufficient to determine \( \Delta \) for a given bed composition (\( \Delta/\delta = \text{const} \)). However, \( \delta \) is not a universal scale of the displacement height: the constant changes for each particular bed mixture and for armored beds.
If the von Kármán constant is flow-dependent, $\Delta$ seems to decrease at the onset of generalised bedload transport. This coincides with a pronounced decrease of the von Kármán constant, suggesting a reorganisation of turbulence with a subtle increase of the integral scale in the near bed region. This issue should be addressed with more detailed investigations on the structure of near-bed turbulence in flows over water-worked mobile beds.

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References


Figure 1. Idealized physical system. $Z_s$ is the elevation of the free-surface, $Z_c$ and $Z_t$ are the space-averaged elevations of the planes of the crests and of the lowest bed troughs, respectively, $Z_b$. All elevations are relative to an arbitrary datum. The remaining variables are identified in the text.
Figure 2. Reynolds stresses in a vertical plane at the centerline of a rough bed showing the development of a shear layer. The amplitude of the bed (distance between troughs and crest) was $\delta = 0.065$ m, the mean-flow Reynolds number was $Re = 9.1 \times 10^4$, and the Froude number was $Fr = 0.62$ (laboratory tests performed at IST, TU Lisbon, partially shown in Ferreira et al. 2010).
Figure 3. Normalized Reynolds shear-stress for tests E (●), D (◇) and T (◇).
Figure 4. Grain-size distribution of the initial gravel mixtures of tests T (○) and of the initial gravel-sand mixtures of tests E and D (♦).
Figure 5. Vertical profile of the longitudinal velocity. The same data is normalised differently according to scenario (s1), scenario (s2) and scenario (s3). Profiles of tests E represented by filled diamonds (●), tests D represented by open diamonds (◇) and tests T represented by open circles (○). Dotted line stands for equation (1) with $\kappa = 0.4$ and $z_0/k_s = 0.0333 (B = 8.5)$. 
Figure 6. Regression line for the calculation of the displacement height (\( \_ \)). Circles (\(\circ\)) stand for the normalised polynomial \( u_* (p^{(3)})^{-1} \). Data of test E1.
Figure 7. Variation of the displacement height, normalised with $d_{50}^{(sb)}$, with the Shields number (scenario s1). Data of tests of type E ($m_{2}^{(sb)} = 3.8$) represented by filled diamonds ($\bullet$), type D ($m_{2}^{(srf)} = 2.8$) represented by open diamonds ($\bigdiamond$) and type T ($m_{2}^{(sb)} = 1.7$) represented by open circles ($\bigcirc$). Dashed line marks the value of the Shields number of test T2.
Figure 8. Variation of the bed thickness, normalised by the median diameter of the substrate with the Shields number calculated for scenario (s1). Data identified as in Figure 7.
Figure 9. Variation of the displacement height, normalised with the bed thickness, with the Shields number (scenarios s1 and s3). Data identified as in Figure 7.
Figure 10. Von Kármán constant, \( \kappa \), scenario (s3), as a function of the Shields number. Data identified as in Figure 7.
Figure 11. Variation of the geometric roughness scale $k_s$, normalised by $d_{srf}^{(s)}$, with the Shields number. Data identified as in Figure 7.
Figure 12. Variation of the roughness height normalised by \(d_{50}^{(a/b)}\), as a function of the Shields number. Data identified as in Figure 7.
Figure 13. Variation of the roughness height normalised by the geometric roughness scale $k_s$, as a function of the Shields number. Data identified as in Figure 7.
Figure 14. Friction coefficient $C_f$ as a function of the mean-flow Froude number for scenarios (s1). Data identified as in Figure 7.
### Table 1. Summary of the characteristics of the experimental tests.

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<th>Name</th>
<th>$Q$</th>
<th>$S_0$</th>
<th>$h$</th>
<th>$U$</th>
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<th>$u^*_{(D1)}$</th>
<th>$b_d/h$</th>
<th>$F_r$</th>
<th>$d_{50}^{(A)}$</th>
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A: outer region
B: inner region
C: pythmenic region
D: hyporeic region

overlapping (logarithmic) layer

$h$

$Z_i$

$Z_b$

$Z_0$

$Z_s$