Protecting Vertical-Wall Abutments with Riprap Mattresses
António H. Cardoso and Cristina M. S. Fael, Ph.D.

Abstract: This study addresses the design of riprap mattresses as a scour countermeasure near vertical-wall bridge abutments under clear water flow conditions. It specifically deals with the diameter of riprap, \( D_{50} \), the lateral extent of mattresses, \( w \), and their thickness, \( t \). Experiments were performed in a rectangular, sand-bed open channel using different abutment lengths, three riprap stone sizes, and two different sands. The minimum size of stable stones as well as the mattress dimensions depend on the ratio between the abutment length and the flow depth. New equations for the evaluation of \( D_{50} \) and \( w \) are suggested. The geometric properties of the scour holes which develop at the edge of riprap mattresses are similar to those reported in the literature for spill-through abutments. Although it is not possible to fully arrest scour by winnowing, the corresponding scour depth is negligible when the mattress layer thickness is at least \( 6D_{50} \).

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CE Database subject headings: Riprap; Bridge abutments; Experimentation; Scour.

Introduction

Local scour around bridge abutments is widely recognized as one of the major causes of the failure of bridges during floods. Due to the complexity of the local scour phenomenon, however, there is a need for further development of design guidelines for countermeasures to mitigate scour. Those found in the literature can be classified into two groups: the first group refers to armouring countermeasures, including riprap mattresses; the second refers to flow-altering devices, which are not within the scope of this paper.

The riprap matting around abutments creates a physical barrier intended to resist the eroding capacity of the flow and is the most widespread countermeasure. Some guidance for the use of riprap at abutments is included, for example in Ministry of Works and Development (1979), Austroads (1994), or Richardson and Davis (1995). According to Melville et al. (2007), the theoretical and empirical basis of the existing equations for the sizing of riprap blocks is limited.

Recently, there has been a significant research effort focused on the design of riprap aprons to protect abutments. Most of these studies are described in Melville et al. (2006a,b, 2007). They cover spill-through as well as wing-wall abutments for both clear water and live bed flow conditions. In spite of these recent efforts, studies concerning countermeasures for vertical-wall abutments are not abundant, and engineering practice is still based on limited evidence. Ignoring that protection itself is a factor influencing scour, engineers frequently assume, for instance, that mattresses should cover the predicted scour area of the riverbed around bridge piers or abutments.

In this context, the further characterization of stable stone sizes, plan dimensions of riprap mattresses, and their corresponding thickness are the particular aspects considered in this study focused on vertical-wall abutments; the thickness of mattresses will be addressed for the particular case of no underlying filters or geotextiles. Vertical-wall abutments were selected since they typically generate the most severe vortex system in their surrounding and, thus, they can also be assumed to be governed by the most severe design criteria for riprap.

The study adopted an experimental approach. Laboratory tests were carried out under clear-water flow conditions, i.e., conditions in which the mean velocity of the undisturbed approach flow is below or at the threshold velocity for the beginning of motion of the bed sediment. This choice corresponds to the common situation encountered in floodplains where abutments are most frequently built.

Literature Review

The correct characterization of design variables such as the stone size, layer thickness, and mattress plan dimensions requires an understanding of mattress failure mechanisms. According to Eve and Melville (2000), the failure mechanisms of riprap are essentially the same for bridge abutments as for piers. For clear-water conditions, Parola (1993), Chiew (1995), and Lauchlan (1999) identified three failure mechanisms: shear failure, winnowing failure, and edge failure. Shear failure is clearly linked with the riprap size; winnowing depends on the thickness of mats and on the gradation coefficient of the riprap stones; and edge failure and plan dimensions are strongly interrelated.

To determine the stable riprap stone size, several authors have suggested empirical formulas that can be expressed as

\[
\frac{D_{50}}{d} = \frac{C}{(s - 1)^m}F^n
\]

where \( D_{50} \) = median riprap stone diameter; \( d \) = flow depth; \( s \) = specific gravity of blocks; \( F = \frac{U}{\sqrt{gd}} \) = flow Froude number; \( U \) = approach flow velocity or contracted section velocity, depending
on the author; $g =$acceleration of gravity; and $C$, $m$, and $n$ =empirical coefficients. Pagán-Ortiz (1991), Atayee et al. (1993), Austroads (1994), Richardson and Davis (1995), and Lagasse et al. (2001) suggested values for these coefficients. For design purposes of wing-wall abutments, Melville et al. (2007) suggested the use of either Pagán-Ortiz (1991) or Lagasse et al. (2001) sets of coefficients, with appropriate factors of safety. The coefficients of Lagasse et al. (2001) for wing-wall abutments, as corroborated by Melville et al. (2007), for the following design parameters are given in Table 1. The materials used can be considered as being uniform, since $\sigma_D < 1.5$ in all cases. The specific gravity of bed and riprap materials was verified to be $s \simeq 2.65$ in all cases.

Experimental Setup and Procedure

Experiments were carried out in a 28.0 m long, 4.0 m wide, and 1.0 m high concrete flume. The right lateral wall of the flume is made of glass panels, permitting observation of the flow in the reach where countermeasures were tested. This reach includes a 3.0 m long, 4.0 m wide, and 0.6 m deep bed recess, starting at 13.9 m from the flume entrance.

Vertical wall abutments were placed on the base of the recess, at its mid cross section (at 16.4 m from the flume entrance), protruding at right angles to the glass wall (see Figs. 1 and 2). For a given test, the recess was almost filled with natural quartz sand; one specific type of riprap was placed around each abutment, on top or embedded in the quartz sand. Three types of riprap stones and two types of sand were used in the study.

The characteristics of the quartz sands and of the riprap stones, including the gradation coefficient, $\sigma_D = (D_{84.1}/D_{50} + D_{50}/D_{15.9})/2$, are given in Table 1. The materials used can be considered as being uniform, since $\sigma_D < 1.5$ in all cases. The specific gravity of bed and riprap materials was verified to be $s \simeq 2.65$ in all cases.

The abutments were simulated by 140 mm wide, parallelepiped Perspex boxes with vertical walls. The tops of the Perspex boxes were kept open, allowing for the manipulation of a video camera to record images from inside the boxes. Abutment lengths equal to 0.30, 0.51, 0.72, 0.93, and 1.13 m were used. This way, $L/d$ varied between 2.46 and 9.42, while $L/B$ ($B =$flume width) ranged from 0.075 to 0.283.

The study was carried out with a practically constant flow depth ($d \simeq 0.12$ m). During the experiments, the water was pumped from a large reservoir; the maximum available discharge was 0.18 m$^3$/s. Just upstream of the flume entrance, a diffuser pipe was installed to ensure a uniform flow distribution across the flume width. At the downstream end of the flume, a hand-
operated tailgate was used to regulate the water level. Downstream of the tailgate the water fell into the large reservoir, closing the water circuit.

The bed of the flume was horizontal, which implies that the flow was not exactly uniform. The variation of the flow depth within the observation reach ranged between 0.5 and 2 mm and the universal log-law was verified to hold at the entrance and in the middle cross section of the observation reach, indicating that the effect of the velocity gradient on the flow structure is negligible. Velocity data and their analysis can be found in Fael (2007).

Three sets of experiments were carried out. Prior to these three sets of experiments, some preliminary tests were run so as to visually determine the critical flow velocity, $U_c$, for the beginning of motion of quartz sand grains as well as of the riprap stones.

The first set of experiments aimed at evaluating the critical flow velocity for scour inception, i.e., the approach flow velocity, $U_s$, above which scour develops close to the abutment. This corresponds to critical shear failure. Fifteen short duration tests were carried out. For each type of riprap, sand # 1 was placed in the recess box, entirely covered with a filter fabric and a layer of riprap whose plan dimension coincided with the recess box. The thickness of this riprap layer, whose top was leveled with the concrete flume bottom, was $3D_{50}$ [see Fig. 2(b)]. The upper $\approx 1D_{50}$ layer around the abutment’s nose was composed of painted stones. Tests started with a very low flow velocity. The velocity was successively increased while the flow depth was maintained constant by increasing the discharge and adjusting the downstream tailgate. The procedure was repeated until the riprap stones began to move close to the abutment. The number of trials and associated durations varied widely from test to test, but near failure, the same discharge and flow depth were maintained for at least 1 h or until the upper painted layer was disrupted and stones moved downstream. No visible waves were generated by the change of bed roughness from concrete to riprap.

The second set of experiments was designed to address the thickness of the mattresses directly placed on sand. Forty two tests were performed: 21 with sand #1 and 21 with sand #2; only riprap # 2 and riprap # 3 were used in this set of experiments. As for the first set, the sand recess box was covered by a riprap layer leveled with the adjacent concrete flume bed. No filter fabric was used [see Fig. 2(c)]. Several riprap layer thicknesses, $t$, were tested: for sand #1, $t$ varied between $D_{50}$ and $3D_{50}$; for riprap #2 on sand #2, $t$ was made equal to $2D_{50}$ or $3D_{50}$; for riprap #3 on sand #2, the layer thickness has varied between $2D_{50}$ and $20D_{50}$. The approach flow velocities were kept equal to 90% of those inducing shear failure at the abutments’ nose, as defined in the first set of experiments. A ruler was fixed to the transparent wall of the abutments and scouring was monitored with the video camera until effective equilibrium had been achieved.

The last set of experiments was intended to evaluate the minimum plan dimensions of mattresses. In this case, the recess box was filled with sand. Mattresses of different plan sizes, made of riprap #3 and set on filter fabric with a pore space of 0.07 mm, were embedded in the sand around the abutments noses, their tops being leveled with the sand and the adjacent concrete bed (see Fig. 1). Tests were carried out for $U \approx U_s$, $U_c$ being the critical approach velocity for the beginning of sand motion. The layer thickness was, in all cases, $t = 2D_{50} \approx 31$ mm. The dimensions of the filters used in these tests were set equal to 75% of the plan dimensions of the riprap layers, according to Melville and Coleman (2000).

Table 1. Sand and Riprap Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>$D_{15.9}$ (mm)</th>
<th>$D_{50}$ (mm)</th>
<th>$D_{94.1}$ (mm)</th>
<th>$\sigma_D$ (−)</th>
<th>Neil (1967)</th>
<th>Garde (1970)</th>
<th>Shields</th>
<th>Tested Lower</th>
<th>Tested Upper</th>
<th>Adopted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand #1</td>
<td>0.87</td>
<td>1.28</td>
<td>1.87</td>
<td>1.46</td>
<td>0.34</td>
<td>0.37</td>
<td>0.48</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>Sand #2</td>
<td>0.64</td>
<td>0.86</td>
<td>1.17</td>
<td>1.35</td>
<td>0.30</td>
<td>0.31</td>
<td>0.37</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Riprap #1</td>
<td>2.65</td>
<td>3.59</td>
<td>5.79</td>
<td>1.48</td>
<td>0.54</td>
<td>0.58</td>
<td>0.82</td>
<td>0.50</td>
<td>0.57</td>
<td>0.53</td>
</tr>
<tr>
<td>Riprap #2</td>
<td>5.28</td>
<td>7.48</td>
<td>10.91</td>
<td>1.44</td>
<td>0.73</td>
<td>0.78</td>
<td>1.11</td>
<td>0.67</td>
<td>0.77</td>
<td>0.72</td>
</tr>
<tr>
<td>Riprap #3</td>
<td>13.43</td>
<td>15.69</td>
<td>18.66</td>
<td>1.18</td>
<td>0.98</td>
<td>1.04</td>
<td>1.38</td>
<td>1.01</td>
<td>1.10</td>
<td>1.05</td>
</tr>
</tbody>
</table>

The approach flow velocities were kept equal to 90% of those inducing shear failure at the abutments’ nose, as defined in the first set of experiments. A ruler was fixed to the transparent wall of the abutments and scouring was monitored with the video camera until effective equilibrium had been achieved.

The last set of experiments was intended to evaluate the minimum plan dimensions of mattresses. In this case, the recess box was filled with sand. Mattresses of different plan sizes, made of riprap #3 and set on filter fabric with a pore space of 0.07 mm, were embedded in the sand around the abutments noses, their tops being leveled with the sand and the adjacent concrete bed (see Fig. 1). Tests were carried out for $U \approx U_s$, $U_c$ being the critical approach velocity for the beginning of sand motion. The layer thickness was, in all cases, $t = 2D_{50} \approx 31$ mm. The dimensions of the filters used in these tests were set equal to 75% of the plan dimensions of the riprap layers, according to Melville and Coleman (2000).
Shear failure at abutments. The inability for Eq. (1) to predict the critical value of $D_{50}$ for a given approach flow (or $U_c$ for a given $D_{50}$) may also arise from the fact that $C$, $m$, and $n$ do not take into account variations in the value of $L/d$.

An alternative approach to assess the critical value of $D_{50}$ is to define the critical value, $I_c$, of the approach flow intensity, $I = U/ U_c$, below which shear failure does not occur, for a given value of $L/d$; $I_c$ is defined as $I_c = U/ U_c$. When no obstacle exists in the flow, $I_c = 1$, and the beginning of sediment motion can be predicted through standard formulations for the initiation of sediment motion. Scour around an obstacle may start under clear water flow conditions, i.e., when there is no general sediment movement elsewhere, which means that, for $I < 1$, scour may occur close to an obstacle. Some controversy exists on the value of $I_c$. Some authors assume, after Hanco (1971), that no scour occurs as long as $I < I_c = 0.5$. Cardoso et al. (2002) corroborated this conclusion for comparatively short abutments ($L/d < 7$). Chiew (1995) suggests $I_c = 0.3$ for cylindrical piers. Melville and Coleman (2000) give an approximate value of $I_c = 0.35$. Hager and Oliveto (2002) have proposed an equation to calculate $I_c$ as a function of the flow contraction, $L/B$. For vertical-wall abutments in wide sand-bed open channels, where scour is not significantly influenced by flow contraction, Fael et al. (2006) suggested

$$I_c = 1 - \frac{3}{8} \left( \frac{L}{d} \right)^{2/9}$$

for $2.01 < L/d < 29.17$ (5)

Since Eq. (5) was established for sand-bed material, there is no guarantee that it applies to the design of the size of riprap; Unger and Hager (2006) have shown that the criterion for the inception

**Results and Discussion**

**Preliminary Tests on Beginning of Motion**

The beginning of sediment motion is difficult to identify in the laboratory because of its random character; its visual evaluation is, therefore, somewhat subjective. Hence, two approximate values of $U_c$, critical velocity for the beginning of motion—are determined: a lower limit, where sediment motion is about to take place but has not yet been observed and an upper one, corresponding to very weak sediment motion (incipient motion). Table 1 includes the observed values of $U_c$, together with the results obtained with the equations of Neil (1967) and Garde (1970), as well as with the results from Shields’ diagram, associated with the log law of rough walls.

Next, the following values of $U_c$ will be adopted: sand $U_c = 0.36$ ms$^{-1}$; sand $U_c = 0.33$ ms$^{-1}$; riprap $U_c = 0.53$ ms$^{-1}$; riprap $U_c = 0.72$ ms$^{-1}$; riprap $U_c = 1.05$ ms$^{-1}$.

**Size of Riprap Stones, $D_{50}$**

As for the beginning of motion, the determination of the approach flow velocity corresponding to shear failure or scour inception at the abutment, $U_c$, is somewhat subjective; it also led to the definition of intervals. The values reported below correspond to the center of such intervals.

The results of the 15 tests carried out on this topic are summarized in Table 2. Bearing in mind the structure of Eq. (1), the values of $D_{50}/d$ are plotted against the Froude numbers at shear failure in Figs. 3 and 4. In Fig. 3, the approach flow failure velocity is used in defining $F_{1c}$, while in Fig. 4 the contracted section failure velocity is used in $F_{2c}$. Fig. 4 also includes the results derived from Eq. (1) for the constants $C$, $m$, and $n$ suggested by Pagán-Ortiz (1991) and Richardson and Davis (1995). The proposal of Pagán-Ortiz (1991) largely overpredicts $D_{50}$, resulting in safe design. The proposal of Richardson and Davis (1995) leads to less accentuated overprediction and one unsafe situation was identified. Apart from the tendency of checked proposals for overprediction, Figs. 3 and 4 indicate that $F$ alone—irrespective of the cross section where it is calculated—does not completely describe

**Table 2. Velocity Corresponding to Scour Inception or Beginning of Shear Failure**

<table>
<thead>
<tr>
<th>Test</th>
<th>$D_{50}$ (mm)</th>
<th>$L$ (m)</th>
<th>$U_c$ (m/s)</th>
<th>$L/d$</th>
<th>$d/D_{50}$</th>
<th>$I_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>3.59</td>
<td>1.13</td>
<td>0.24</td>
<td>9.42</td>
<td>33.43</td>
<td>0.45</td>
</tr>
<tr>
<td>S2</td>
<td>3.59</td>
<td>0.93</td>
<td>0.24</td>
<td>7.75</td>
<td>33.43</td>
<td>0.45</td>
</tr>
<tr>
<td>S3</td>
<td>3.59</td>
<td>0.72</td>
<td>0.25</td>
<td>6.00</td>
<td>33.43</td>
<td>0.47</td>
</tr>
<tr>
<td>S4</td>
<td>3.59</td>
<td>0.51</td>
<td>0.26</td>
<td>4.25</td>
<td>33.43</td>
<td>0.49</td>
</tr>
<tr>
<td>S5</td>
<td>3.59</td>
<td>0.30</td>
<td>0.28</td>
<td>2.46</td>
<td>33.43</td>
<td>0.53</td>
</tr>
<tr>
<td>S6</td>
<td>7.48</td>
<td>1.13</td>
<td>0.31</td>
<td>9.42</td>
<td>16.04</td>
<td>0.43</td>
</tr>
<tr>
<td>S7</td>
<td>7.48</td>
<td>0.93</td>
<td>0.31</td>
<td>7.75</td>
<td>16.04</td>
<td>0.44</td>
</tr>
<tr>
<td>S8</td>
<td>7.48</td>
<td>0.72</td>
<td>0.35</td>
<td>6.00</td>
<td>16.04</td>
<td>0.49</td>
</tr>
<tr>
<td>S9</td>
<td>7.48</td>
<td>0.51</td>
<td>0.35</td>
<td>4.25</td>
<td>16.04</td>
<td>0.49</td>
</tr>
<tr>
<td>S10</td>
<td>7.48</td>
<td>0.30</td>
<td>0.42</td>
<td>2.46</td>
<td>16.04</td>
<td>0.59</td>
</tr>
<tr>
<td>S11</td>
<td>15.69</td>
<td>1.13</td>
<td>0.34</td>
<td>9.42</td>
<td>7.65</td>
<td>0.33</td>
</tr>
<tr>
<td>S12</td>
<td>15.69</td>
<td>0.93</td>
<td>0.39</td>
<td>7.75</td>
<td>7.65</td>
<td>0.37</td>
</tr>
<tr>
<td>S13</td>
<td>15.69</td>
<td>0.72</td>
<td>0.43</td>
<td>6.00</td>
<td>7.65</td>
<td>0.41</td>
</tr>
<tr>
<td>S14</td>
<td>15.69</td>
<td>0.51</td>
<td>0.48</td>
<td>4.25</td>
<td>7.65</td>
<td>0.46</td>
</tr>
<tr>
<td>S15</td>
<td>15.69</td>
<td>0.30</td>
<td>0.54</td>
<td>2.46</td>
<td>7.65</td>
<td>0.51</td>
</tr>
</tbody>
</table>
of scour for uniform sediment at circular bridge piers does not directly apply to riprap on finer bed material. Fig. 5 compares the results of Eq. (5) with the “observed” values of \( I_c \) (Table 2). It also includes the equation of Hager and Oliveto (2002) for scour inception. It is clear that \( I_c \) depends on \( L/d \). As \( L/d \) increases, there is an increasing susceptibility of scouring. Eq. (5) fits the data better than the equation by Hager and Oliveto (2002), but the observed values of \( I_c \) can be slightly smaller than those derived from Eq. (5), particularly for coarser riprap. For \( L/d = 9.42 \), \( I_c \) may be as low as \( \approx 0.3 \). The widespread criterion of Hancock (1971)—\( I_c = 0.5 \)—leads to unstable riprap stones if \( L/d > 3 – 4 \).

A close examination at Fig. 5 seems to indicate that \( I_c \) increases with \( d/D_{50} \). For the data obtained in this study, the dependence of \( I_c \) on both \( L/d \) and \( d/D_{50} \) is given by the following regression equation (\( R^2 = 0.803 \)):

\[
I_c = 0.195 \left( \frac{L}{d} \right)^{-0.222} \left( \frac{d}{D_{50}} \right)^{0.106}
\]

Eq. (6) is not recommended for design purposes since it was obtained on the basis of a narrow range of \( d/D_{50} \) values. The lower envelope curve (see Fig. 5)

\[
I_c = 1 - \frac{2}{5} \left( \frac{L}{d} \right)^{1/4}
\]

is suggested instead, but it should be noted that it only applies for \( L/d < 9.42 \).

Riprap Layer Thickness, \( t \)

The main objective of this section is to assess the minimum mattress thickness, \( t \), needed to avoid failure due to winnowing. Since the experiments were run for \( I_c = 0.9 \), shear failure was never observed; as the recess box was entirely covered with riprap, edge failure was also not possible. Thus, only failure due to winnowing could be expected.

It is important to check whether the riprap layers act as granular filters themselves. Several criteria can be found in the literature to verify this hypothesis. The most stringent seems to be the criterion of Terzaghi–Vicksburg, according to which riprap acts as a filter if the following conditions are simultaneously satisfied:

\[
D_{15}/D_{50} < 5; \quad 5 < D_{15}/D_{15} < 20; \quad D_{50}/D_{50} < 40
\]

The results of the experiments on the thickness of mattresses set directly onto the sand bed are presented in Figs. 6 and 7, for riprap #2 and riprap #3, respectively. Results are expressed as \( d_{se}/d \) versus \( L/d \) and \( N \); for more details, particularly on the duration of tests, \( T \) (30 h \( \leqslant T \leqslant 216 \) h), refer to Fael (2007). In those figures, \( d_{se} \) represents the equilibrium scour depth generated by winnowing, as measured at the end of each test and \( N \) is the number of equivalent layers, such that \( t = N D_{50} \).

From Fig. 6, scour is nonexistent for riprap #2 on sand #1, quasi-regardless of the layer thickness. The exception occurs for \( L/d = 9.42 \) and \( N = 1 \), where a 1.5 cm deep scour hole was observed. Scour is also absent for riprap #2 on sand #2 provided \( N \geq 3 \). These results seem to corroborate the criterion of Terzaghi–Vicksburg according to which riprap #2 acts as a filter relative to the underlying sands.

According to Fig. 7, scour is practically absent when riprap #3 is placed on sand #1 as soon as \( N \geq 3 \). When it is placed on the finer sand #2, winnowing seems to occur for thicknesses as large as \( t = 20 D_{50} \). However, for \( N \geq 6 \), the equilibrium scour depth remains practically constant, irrespective of \( N \). The order of magnitude of the scour depth is then observed to be around \( D_{50} \). In practice, this means that although it is not possible to fully arrest scour by winnowing, the corresponding scour depth is negligible as soon as \( N \geq 6 \). This result must be confirmed in future studies for different values of the ratios \( D_{50}/D_{50} \) and \( L/d \). It encourages the present practice of using \( t = 2 D_{50} \) for riprap placed on an adequate filter.

Fig. 7 depicts a rather surprising result for one single layer of riprap #3 placed on sand #1. The scour depth decreases as \( L \)
Table 4. Results of Tests on Layout of Riprap Mattresses

<table>
<thead>
<tr>
<th>Test</th>
<th>$L$ (m)</th>
<th>$w$ (m)</th>
<th>$T$ (h)</th>
<th>Failure</th>
<th>$d_{e}$ (m)</th>
<th>$d_{i}$ (m)</th>
<th>$d_{y}$ (m)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.72</td>
<td>0.75</td>
<td>289.0</td>
<td>No</td>
<td>0.34</td>
<td>1.45</td>
<td>0.93</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>0.72</td>
<td>0.15</td>
<td>137.8</td>
<td>Yes</td>
<td>0.33</td>
<td>0.85</td>
<td>0.52</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>0.72</td>
<td>0.17</td>
<td>143.9</td>
<td>No</td>
<td>0.34</td>
<td>0.93</td>
<td>0.51</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>0.93</td>
<td>0.45</td>
<td>264.0</td>
<td>No</td>
<td>0.21</td>
<td>0.71</td>
<td>0.91</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>0.93</td>
<td>0.20</td>
<td>139.0</td>
<td>No</td>
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increases. This trend seems to be corroborated for the shortest abutment ($L/d=2.46$), and also in the case of thicker mattresses ($N=2$ and $N=3$). This is possibly related to the structure of the separated flow generated at the upstream vertical edge of the abutment, but this hypothesis needs confirmation through detailed three-dimensional (3D) measurements of the local flow structure.

**Mattress Plan Dimensions**

Experiments on edge failure and minimum mattress plan dimensions were carried out for 31 mm thick layers ($=2D_{o}$) of riprap # 3, laying on filter fabric, to avoid winnowing failure. Since the approach flow velocity was approximately equal to $U_r$ of the surrounding sand, riprap shear failure was also avoided.

At the beginning of the third set of experiments, it was assumed that a reasonable layout for the mattresses would correspond to the configuration of the scour holes previously observed by Fael et al. (2006) at the undisturbed sand-bed level of similar unprotected abutments. For this reason, $w$ (see Fig. 1) was predicted as $w=d_{e}/\tan \theta$, where $\theta$ is angle of repose of sand. In this way, $w$ was set equal to 0.75 m for Test 1 (see Table 4).

Experiments have shown that values of $w$ of this order of magnitude do not lead to mattresses that completely avoid scour at their edges, particularly downstream. It was also observed that increasing the dimensions of the mattresses does not completely inhibit scour but, mostly, moves the scour hole further from the abutment. Stones tend to roll into the scour hole that develops close to the mattress. This behavior is largely illustrated by the experiments described by Melville et al. (2006a,b). Thus, a decision was made to reassess edge failure, assuming it to occur only when riprap stones are removed from the immediate vicinity (closest row) of the abutment toe, by rolling into the scour hole. This criterion is basically the same as that suggested by Unger and Hager (2006). For $L=0.70$ m, the required mattress evolved from a comparatively large mattress (Test 1, Table 4) to the one that strictly fulfilled the reassessed failure criterion (Test 2).

Table 4 summarizes the results of 13 experiments. It includes the values of $L$ as well as those of $w$=characteristic length of the tested mattress (see Fig. 1), $T$=test duration, $d_{e}$=equilibrium depth of the observed scour hole, $d_{i}$ and $d_{y}$=distances of the deepest point of the scour hole to the upstream vertical edge of the abutment; and $\alpha$=angle defining the position of the deepest point (see Fig. 1). It also indicates the results of the edge failure assessment. Ten tests (two per abutment length) define the boundary of edge failure.

A decision was made to evaluate the maximum observed scour depth by comparing the values issued from the protected-bed tests with those of similar tests carried out for the unprotected bed by Fael et al. (2006) as well as with the predictions of Eq. (3). A comparison is presented in Fig. 8; scour depths of Tests 1, 4, and 11 were not included because they do not correspond to conditions close to the threshold of edge failure. The protection reduces the equilibrium depth of the associated scour hole. Predictions of Eq. (3) are $\approx 2.2$ times bigger than observations for $L/d=9.42$; the overprediction reduces to $\approx 1.65$ for $L/d=2.46$.

For practical engineering applications it may be important to address the position of the deepest point of the scour hole, for instance, to determine if the scour hole remains in the flood plain or reaches the main channel. Consequently, $d_{i}$, $d_{y}$, and $\alpha$ were also measured (see Fig. 1). Fig. 9 presents the variation of $\alpha$ with $L/d$. The figure does not make it possible to identify any trend of $\alpha$ with respect to $L/d$; $\alpha=30.6 \pm 2.9^\circ$ (average $\pm$ SD). This is in excellent agreement with the findings of Melville et al. (2006a) in spite of the use of different abutment shapes. Fig. 10 shows the variations of $d_{i}/d$ and $d_{y}/d$ with $L/d$. There is a small increasing trend of both $d_{i}$ and $d_{y}$ with $L$ but, for the reported experimental range, it may be assumed that $d_{i} \approx 7.5d$ and $d_{y} \approx 4.5d$ (consistent with $\alpha$). Predicted values of $R$ as given by Eq. (4) are, on average,  

![Fig. 8. Effect of mattresses on equilibrium scour depth](image-url)
approx. 2.5% smaller than the observed ones; the maximum deviation is approx. 10%. Again this is in rather good agreement with the findings of Melville et al. (2006a).

The most important results reported in Table 4 refer to the values of \( w \) corresponding to the threshold of edge failure. These values are plotted against \( L/d \) in Fig. 11. This figure also includes the predictions by Richardson and Davis (1995)——\( w=2d \)—and Melville et al. (2006a)—Eq. (2). This equation was used with both the measured scour depth values and those given by Eq. (3). Eq. (2) overpredicts \( w \) for the vertical-wall tests reported in this study if Eq. (3) is used to calculate \( d_{sw} \), but the same equation produces rather more reasonable predictions when the observed values of \( d_{sw} \) are used instead. The measured values of \( w \) increase with \( L/d \); according to the data of the present study, edge failure at vertical-wall abutments can be avoided provided

\[
\frac{w}{d} > \frac{1}{2} \left( \frac{L}{d} \right)^{3/5}
\]  

Eq. (8) is valid for \( d/D_{50} \approx 7.5 \) and \( L/d < 9.42 \), in the presence of a filter fabric underneath the riprap. The influence of \( d/D_{50} \) cannot be addressed since it was kept practically constant. This indicates the need to vary \( d/D_{50} \) in future research studies. It is also clear that the proposal of Richardson and Davis (1995)—see Fig. 11—leads to safe predictions of \( w \), within the experimental range of this study.

From the experiments reported in this section, it seems that an adequate layout of a riprap mattress for vertical-wall abutments would be as sketched in Fig. 12. Although this was not discussed above, it became evident during the experimental study that \( w_d \) can be very small or even zero. The criterion \( w_d=3D_{50} \) was observed to avoid edge failure along the mattress boundary facing the channel wall. For the time being, no evidence was produced on the value of \( w_d \).

**Conclusions**

The design of riprap protection for vertical-wall abutments under clear-water flow conditions was addressed in this study. Abutments protruded at a right angle to the wall of a rectangular, sand-bed channel. For the experimental range of this study \( (2.46 \leq L/d \leq 9.42) \), the following conclusions can be drawn:

1. Regarding the design of stone sizes for stable riprap, characteristic stable diameters depend on the Froude number as well as on \( L/d \). The influence of \( L/d \) is not taken into consideration by several formulations identified in the literature;
2. The concept of critical flow intensity, \( I_c \), for scour inception applies to the design stone sizes. Eq. (7) is suggested, which leads to increasing stone sizes as \( L/d \) increases;
3. Although it is not possible to fully prevent scour by winnowing, the corresponding scour depth is negligible when the layer thickness is such that \( N \approx 6 \);
4. For reasonable extents of mattresses, some scouring at its boundaries seems to be unavoidable, inducing the rolling of riprap stones into the scour hole;
5. Scour holes develop downstream of the abutment; their depth seems to be less than for an unprotected bed. The location of the deepest scour is at \(d_{\theta} = 7.5d\) for \(\theta = 30^\circ\) (see Fig. 1); and
6. Assuming edge failure to occur when stones are removed from the immediate vicinity of the abutment, the lateral extent, \(w\), of mattresses required to prevent edge failure increases with \(L/d\), according to Eq. (8), for \(L/d < 9.42\), \(d/D_{50} \approx 7.5\), and riprap layers placed on filter fabric. The proposal of Richardson and Davis (1995) seems to predict safe values of \(w\).

The study reported in this paper encourages the adoption of the layout presented in Fig. 12 but further research is needed to extend the conclusions and proposals to different abutment shapes, a wider range of \(L/d\) values, and different values of \(d/D_{50}\) and \(D_{50}/D_{50}\).

Acknowledgments

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Notation

The following symbols are used in this paper:

- \(B\) = channel width;
- \(C, m, n\) = empirical coefficients;
- \(D_{n}\) = sand particle sieving diameter for which \(n\%\) are finer by weight;
- \(D_{m}\) = riprap stone sieving diameter for which \(n\%\) are finer by weight;
- \(d\) = approach flow depth;
- \(d_{e}\) = equilibrium scour depth;
- \(d_{c}; d_{l}\) = distances of deepest point of scour hole to upstream vertical edge of abutment;
- \(F\) = flow Froude number;
- \(F_{lv}\) = Froude number of approach flow;
- \(F_{rc}\) = Froude number of contracted section;
- \(I = U/U_{c}\) = flow intensity parameter defined with approach flow velocity;
- \(I_{c} = U_{c}/U_{c}\) = value of \(I\) corresponding to onset of shear failure;
- \(K\) = abutment shape factor;
- \(L\) = abutment length;
- \(N\) = number of equivalent layers, such that \(t = ND_{50}\);
- \(R\) = position of deepest point of scour;
- \(s\) = specific gravity of riprap blocks or bed material;
- \(T\) = test duration;
- \(t\) = thickness of riprap layer;
- \(U\) = approach flow velocity or contracted-section velocity;
- \(U_{c}\) = critical velocity for beginning of motion of riprap blocks or bed material;
- \(U_{e}\) = approach flow velocity corresponding to onset of shear failure;
- \(w, w_{a}, w_{d}, w_{f}\) = characteristic apron lengths;
- \(\alpha\) = angle defining position of deepest point of scour hole relative to upstream vertical edge of abutment;
- \(\theta\) = angle of repose of sand; and
- \(\sigma_{D}\) = gradation coefficient of riprap blocks or bed material.

References


