STABILITY ANALYSIS OF A SHAKE TABLE HYBRID SIMULATION FOR LINEAR AND NON-LINEAR SDOF SYSTEMS

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OUTLINE

- Virtual hybrid simulation (VHS)
- Mechanical and complete system identification of LNEC’s uniaxial shake table (ST1D)
- ST1D under payload and uncertainty of its parameters
- Stability analysis of a VHS using Routh-Hurwitz stability test
- Stability test on linear and non-linear SDOFs
- VHS using OpenSees & OpenFresco (OSOF-VHS)
- Modelling experimental errors in OSOF-VHS
- Future directions
Virtual Hybrid Simulation (VHS)

- Virtual hybrid simulation as a pre-hybrid testing for:
  - Checking bugs in the testing software
  - Selecting appropriate controllers
  - Addressing experimental errors and stability test

- Requirements of VHS:
  - Complete model of the servo-hydraulic actuator or the shake table
  - Model of the experimental element
1. Mechanical system identification
Includes identification of:

- Total moving mass
  - Experimental sub-structure mass
  - Platen mass
  - Actuator mass
  - Other moving parts

- Dissipative force
  - Experimental sub-structure damping
  - Resistance in actuator chamber
  - Resistance in platen bearings

- Elastic force
  - Experimental sub-structure
  - Other sources

2. Complete system identification
Includes:

- Mechanical system identification
- Servo-hydraulic actuator control (SHAC) identification
Mechanical system identification

- Mathematical models assumed for fitting:
  - $F_I = Mu \downarrow x$, where $M=M_{\text{Platen}} + M_{\text{actuator}}$
  - $F_E = Ku \downarrow x$
  - $F_D = [F_\mu + C/u \downarrow x \uparrow \alpha \text{sign}(u \downarrow x)]$, where $0 \leq \alpha \leq 1$

- The experimental method uses:
  - Sinusoidal and triangular displacement signals
  - Uses periodicity of measured signals:
    - Triangular test:
      - Sinusoidal test:
        - $u_1 \approx 0$, $u_2 \approx 0$
        - $F_I(u_1, u_2) = \frac{1}{2} [F_A(t_1) + F_A(t_2)] = M \downarrow x(t_1)$
        - $u_1 \approx -u_2$

Gidewon G. Tekeste / Stability analysis of a shake table hybrid simulation for linear and non-linear SDOF systems
Mechanical system identification continued…

**Elastic force estimation:** estimated from triangular signals
- A low-pass Fourier filter with a $f_c$ 2-4 times the $f_{cmd}$
- Uses a synthesized velocity signal (from accelerometer and LVDT measurements)
- Signals used for estimation:

<table>
<thead>
<tr>
<th>Test</th>
<th>Freq [Hz]</th>
<th>Amplitude [cm]</th>
<th>$V_{max}$ [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T5</td>
<td>0.4</td>
<td>1.0</td>
<td>1.6</td>
</tr>
<tr>
<td>T6</td>
<td>0.5</td>
<td>1.0</td>
<td>2</td>
</tr>
</tbody>
</table>

- Null or near zero elastic force
- Spurious estimates due to small SNR of the load cell force

same applies to mass and dissipative force estimation
Effective horizontal mass estimation: estimated from sinusoidal signals

- Excludes velocities below $|\pm 5/mm/s$
- Minimizing velocity sum at $t_1$ and $[T/2-t_1-Tol, T/2-t_1+Tol]$

Test signals and estimates:

<table>
<thead>
<tr>
<th>Test</th>
<th>Freq [Hz]</th>
<th>Amplitude [cm]</th>
<th>$V_{max}$ [cm/s]</th>
<th>$A_{max}$ [%g]</th>
<th>Mass estimates [ton]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.2</td>
<td>4.0</td>
<td>5.0265</td>
<td>0.6435</td>
<td>1.98</td>
</tr>
<tr>
<td>S2</td>
<td>0.4</td>
<td>4.0</td>
<td>10.1300</td>
<td>2.5760</td>
<td>2.11</td>
</tr>
<tr>
<td>S3</td>
<td>0.5</td>
<td>4.0</td>
<td>12.5660</td>
<td>4.0240</td>
<td>2.06</td>
</tr>
</tbody>
</table>

Weighting estimated through a simplified approach:

$$wli = fli / \sum^n_{i=1} fli, \quad m\overline{avg} = \sum^n_{i=1} mli(wli \times mli),$$

where $fli$ and $mli$ are frequency and mass estimates of each test respectively.

(a deduction from Lanese et al., 2012)
Dissipative force estimation: estimated from triangular signals

- Straightforward coupling of samples (no displacement sum minimization)
- Propagation of disturbance is pronounced after inversion of motion
- Each triangular test yields positive and negative estimates of dissipative force
- The Coulomb friction force governs the dissipative force
**Complete system identification**

- Involves modelling:
  - Servo-controller - proportional gain only controller
  - Servo-valve - first order transfer function (good in 0-50Hz range)
  - Hydraulic actuator - linearized flow equation and oil-column frequency
  - Estimated effective mass of the platen (elastic force was set to zero)
  - Dissipative force (simplified to viscous damping only)
  - Payload (specimen) dynamic properties - no payload condition to produce a generalized system model; Model subject to changes under any experimental specimen

\[ G \downarrow x \downarrow p \downarrow \downarrow c = A_k \downarrow p \downarrow k \downarrow sv \downarrow k \downarrow q / k \downarrow pl / (A \uparrow^2 / k \downarrow pl K \downarrow h \downarrow m \downarrow T \uparrow^* \downarrow \tau \downarrow sv) \]

\[ s \uparrow^4 + \{m \downarrow T \uparrow^* \downarrow \tau \downarrow sv + A \uparrow^2 / k \downarrow pl \} s \uparrow^3 + \{\tau \downarrow sv (c \downarrow t + A \uparrow^2 / k \downarrow pl)) + m \downarrow T \uparrow^* + c \downarrow t A \uparrow^2 / k \downarrow pl \} s \uparrow^2 + \{c \downarrow t + A \uparrow^2 / k \downarrow pl\} s + A_k \downarrow p k \downarrow sv k \downarrow q / k \downarrow pl \]

Where:

\[ k \downarrow pl = K \downarrow c + C \downarrow l, \quad K \downarrow h = 4 \beta \downarrow e A \uparrow^2 / V \downarrow t \]

and

\[ m \downarrow T \uparrow* = m \downarrow p \] under no payload condition, while, under payload \( m \downarrow sp \) and \( H \downarrow sp(s) \) are non-zero terms
Complete system identification continued…

- Test signal characteristics:
  - Band Limited White Noise (BLWN) ranging 0-50Hz
  - RMS value of 0.345 cm

- Parametric identification of the ST1D model in SIMULINK using:
  - Transfer system equation developed
  - Command signal, measured signal, and the gain error identified
  - Constrained non-linear least square solution

- Validation of estimated shake table parameters

\[ H_{ST1D} = 1.9485e+7/s^4 + 92.29s^3 + 2.5132e + 4s^2 + 9.3478e+5s + 1.8746e+7 \]

(4 poles without zeros)

<table>
<thead>
<tr>
<th>Op. freq</th>
<th>Model</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
<td>Magnitude (dB)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-60</td>
<td>-60</td>
</tr>
<tr>
<td>5</td>
<td>-40</td>
<td>-40</td>
</tr>
<tr>
<td>10</td>
<td>-20</td>
<td>-20</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>35</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>40</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>45</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>50</td>
<td>140</td>
<td>140</td>
</tr>
</tbody>
</table>

| Frequency [Hz] | Phase (deg) | |
| --- | --- | |
| 0 | 0 | 0 |
| 5 | -30 | -30 |
| 10 | -60 | -60 |
| 15 | -90 | -90 |
| 20 | -120 | -120 |
| 25 | -150 | -150 |
| 30 | -180 | -180 |
| 35 | -210 | -210 |
| 40 | -240 | -240 |
| 45 | -270 | -270 |
| 50 | -300 | -300 |

| Time [sec] | Displacement [mm] | |
| --- | --- | |
| 0 | 0 | 0 |
| 0.5 | 10 | 10 |
| 1 | 20 | 20 |
| 1.5 | 30 | 30 |
| 2 | 40 | 40 |

\[ \beta \text{le} \] (4 poles without zeros)

<table>
<thead>
<tr>
<th>Servo-valve time constant (TV)</th>
<th>Valve pressure gain</th>
<th>Valve leakage factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil bulk modulus (BSV)</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>
**ST1D model under payload & uncertainty of its parameters**

**SDOF payload**: Flexible and rigid
- Flexible SDOF comes with Control-structure interaction (CSI)
  - Behaves similarly to a no-payload condition, except it introduces:
    - A notch and a peak in the magnitude plot
    - A notch in the phase plot
- Rigid SDOF results in a stiffer response and a significant shift in the oil-column resonance

**Uncertainty of ST1D parameters**: No payload
- Undershooting mass results in a spongy response
- Overshooting apparently makes system stiffer
- Increase in $K_p$: stiffer but smaller margins of stability

$K_{p,cr}$ = 10.529

$f_{SDOF} = 5 \text{ Hz}$, $f_{n} = 24.08 \text{ Hz}$
Stability analysis of a VHS using Routh-Hurwitz test

Why and how stability test:
- Delay in HS is interpreted as a ‘negative damping’
- Stability under a linear transfer system is dictated by:
  - Experimental stiffness
  - Experimental mass and
  - Experimental damping
- Shortcomings of Mercan & Ricles (2007) stability study:
  - A ‘pure delay’ assumption in HS
- System delay is a function of frequency, hence it requires the system model
- Stability test using Routh-Hurwitz stability:
  - Sufficient for stability of LTI control systems
  - Allows parametric study under a linear response

Routh-Hurwitz method:
- Finds roots of the characteristic polynomial equation (D(s)) that fall in right-half S-plane
- Procedure:
  - Develop the Hurwitz matrix (upper triangular matrix)
  - Any sign change in 1st column indicates unstable test

\[
G_{\downarrow VHS}(s) = X_{\downarrow t}(s)/P(s) = \frac{N(s)}{D(s)}
\]

\[
D(s) = b\downarrow m \ s^{\uparrow m} + b\downarrow m-1 \ s^{\uparrow m-1} + \ldots = 0
\]

\[
G_{\downarrow VHS}(s) = X_{\downarrow t}(s)/P(s) = \frac{N(s)}{D(s)}
\]

\[
D(s) = b\downarrow m \ s^{\uparrow m} + b\downarrow m-1 \ s^{\uparrow m-1} + \ldots = 0
\]

\[
G_{\downarrow VHS}(s) = X_{\downarrow t}(s)/P(s) = \frac{N(s)}{D(s)}
\]

\[
D(s) = b\downarrow m \ s^{\uparrow m} + b\downarrow m-1 \ s^{\uparrow m-1} + \ldots = 0
\]
Stability analysis of a linear SDOF

Analysis and results:

- A SDOF properties: $\omega=1\text{Hz}$, $m=2t$ and $\zeta=2\%$
- Generating stability surface by constantly changing the fractions of experimental and numerical sub-structures, e.g., $M_{\text{exp}} = M_{\text{exp}}/m$
- A percentage step of 5% was adopted for each parameter
- 2-parameter study, namely:
  
  i. $K_{\text{exp}}$ versus $C_{\text{exp}}$ at:
     - $M_{\text{exp}} = 0$, 20%, 40%, 60%, 80%, 100%  
  ii. $C_{\text{exp}}$ versus $M_{\text{exp}}$ at:
     - $K_{\text{exp}} = 0$, 20%, 40%, 60%, 80%, 100%  
  iii. $K_{\text{exp}}$ versus $M_{\text{exp}}$ at:
     - $C_{\text{exp}} = 0$, 20%, 40%, 60%, 80%, 100%  
- 2-parameter study at a fixed third parameter, but at varying values of stiffness and viscous damping of the SDOF (for cases i and ii)
  - $m=2t$ (constant), $\omega=1,2,5\text{ Hz}$, and $\zeta=2\%, 5\%, 10\%$
Stability analysis of a linear SDOF

\( K_{\text{exp}} \) versus \( C_{\text{exp}} \) at constant \( M_{\text{exp}} \):

- As \( M_{\text{exp}} \) increases (0-80%), \( C_{\text{exp}} \) falls
- As \( M_{\text{exp}} \) increases (0-40%), \( K_{\text{exp}} \) rises, but falls when \( M_{\text{exp}} \) increases further
- At full experimental inertial mass, system attains improved stability
- At 20% of \( M_{\text{exp}} \), as \( \omega \) and \( \zeta \) of the SDOF increase (constant mass), \( K_{\text{exp}} \) ↓, while \( C_{\text{exp}} \) ↑
- Validation of stability test at a transition zone, using El Centro at 0.3g PGA and a free-vibration tail, at \( M_{\text{exp}} \)=20%
Stability analysis of a linear SDOF continued...

$C_{\text{exp}}$ versus $M_{\text{exp}}$ at constant $K_{\text{exp}}$:

- Increasing $K_{\text{exp}}$, $C_{\text{exp}}$ falls and the minimum required $M_{\text{exp}}$ rises
- $M_{\text{exp}}$ needs to be bounded for stability
- At full experimental mass, stability is attained if the minimum $C_{\text{exp}}$ is increased
- When $\omega$ and $\zeta$ of the SDOF increase (constant mass), system exhibits a complex pattern
- Validation of stability test at $K_{\text{exp}} = 20\%$
Stability analysis of a linear SDOF continued…

\( K_{\text{exp}} \) versus \( M_{\text{exp}} \) at varying \( C_{\text{exp}} \):

- Regardless of the \( C_{\text{exp}} \), stability contour is identical, except at \( C_{\text{exp}} = 100\%
- A positive linear relationship exists between \( M_{\text{exp}} \) and \( K_{\text{exp}} \) until 30\% of \( M_{\text{exp}} \)
- Stable at right top corner of contour plot at \( C_{\text{exp}} = 100\% \), i.e., shake table test
- Small deviations of the stable zone under varying frequencies and damping, but the linear relationship still prevails

\[\begin{array}{c}
\text{Mass ratio} \\
\text{Stiffness ratio} \\
0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\
\end{array}\]

Tekeste, G.G., Correia, A.A., Costa, A. G., [2017], “Stability analysis of a real-time shake table hybrid simulation for linear and non-linear SDOF systems”, 7th international conference on Advances in Experimental Structural Engineering, EUCENTRE Foundation, Pavia, Italy
Stability analysis of a nonlinear SDOF

Why?
- To address the nonlinear range of response
- Study the relation between degree of nonlinearity and stability

Methodology:
- Simulink model of VHS
  - Bouc-Wen model of the experimental part
  - Linear model of numerical part

\[ f_{\text{exp}}(t) = a kl int \exp x_{\text{exp}}(t) + (1-a)kl int \exp \delta y Z(t) + c^{\text{exp}} x_{\text{exp}}(t) \]

\[ Z(t) = x_{\text{exp}}(t) [A + \beta \cdot \text{sign}(Z(t) x_{\text{exp}}(t))] + \gamma |Z(t)| n / \delta y \]

- The Simulink model calls a MATLAB .m function of Bouc-Wen at every step

Comparison:
- Validation at unstable coordinates of the linear case for comparison (\( M_{\text{exp}} = 20\% \), \( C_{\text{exp}} = 20\% \) and \( K_{\text{exp}} = 35\% \))
- Comments:
  - Improved stability under a non-linear response and/or a material with high non-linear behaviour
  - A larger margin from instability with increasing degree of non-linearity, defined by

\[ \rho = \max |f_{\text{lin}}(t)| / f_{\text{linmax}} = |f_{\text{lin}}(t)| / f_{\text{lin}} [A/(\beta + \gamma)] \]
Virtual hybrid simulation using OpenSees and OpenFresco

- Virtual hybrid tests using OS as the computational driver and OF as the middleware
- Restoring forces returned to OS are simulated using OS uniaxial material (restoring forces in actual hybrid tests are measured in laboratory)

**Experimental errors modelling in OF:**
- Introducing experimental errors through ExpSignalFilter control object using:
  - ErrorSimUndershoot
  - ErrorSimOvershoot
  - ErrorSimRandomGauss (models random process in nature)

Modelling experimental errors in OSOF-VHS

- Offline estimation of error parameters using the ST1D model in Simulink

```plaintext
# expSignalFilter ErrorSimRandomGauss $tag $avg $std
expSignalFilter ErrorSimRandomGauss 1 -0.0003475 0.2051730
```

- One-bay frame with a truss element and two non-linear columns
- Columns are “experimental” substructures
- Truss element and masses are the “numerical” substructures
Modelling experimental errors in OSOF-VHS continued…

- **OpenFresco definition:**
  
  ```
  expControl SimUniaxialMaterials 1 1 -ctrlFilters 1 0 0 0 0
  expSetup OneActuator 1 -control 1 2 -sizeTrialOut 3 3 (only 1 ctrl and 1 out were used)
  expSite LocalSite 1 1
  expElement BeamColumn 1 1 3 1 -site 1 -initStif . . .
  ```

- Since “real” experimental errors cannot be directly feedback to OS, a number of stochastic realizations were necessary to model the errors using WGN.
- Expected response is found by averaging over 50 realizations.
- The sub-space synchronization plot (SSP) shows both gain and time-lag errors.

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![Trial displacements without error vs with WGN (averaged)](image1)

![Coupling force returned to OS](image2)
Conclusions:
- Experimental errors result in a coupled gain and delay (introduced by ST1D dynamics)
- Reduction in energy dissipation capacity is prevalent

Solutions sought:
- Higher frequency of AD/DA conversion
- Tuning the controller gains for minimal settling time and overshoot
- Implementation of adaptive model based compensator in the outer loop

Column hysteresis of VHS with & without Exp. errors

Feedforward compensation of ST1D
Future directions

- Completion of a LabVIEW based testing software that works in conjunction with OF and OS
- Conducting hybrid simulations on a steel frame using the ST1D only
- Development of advanced control strategies for the RTHS framework
  - Shake table control
  - Force control of actuators
  - Model based and adaptive compensation
THANK YOU FOR YOUR ATTENTION!

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