STABILITY ANALYSIS OF A SHAKE TABLE HYBRID SIMULATION FOR LINEAR AND NON-LINEAR SDOF SYSTEMS

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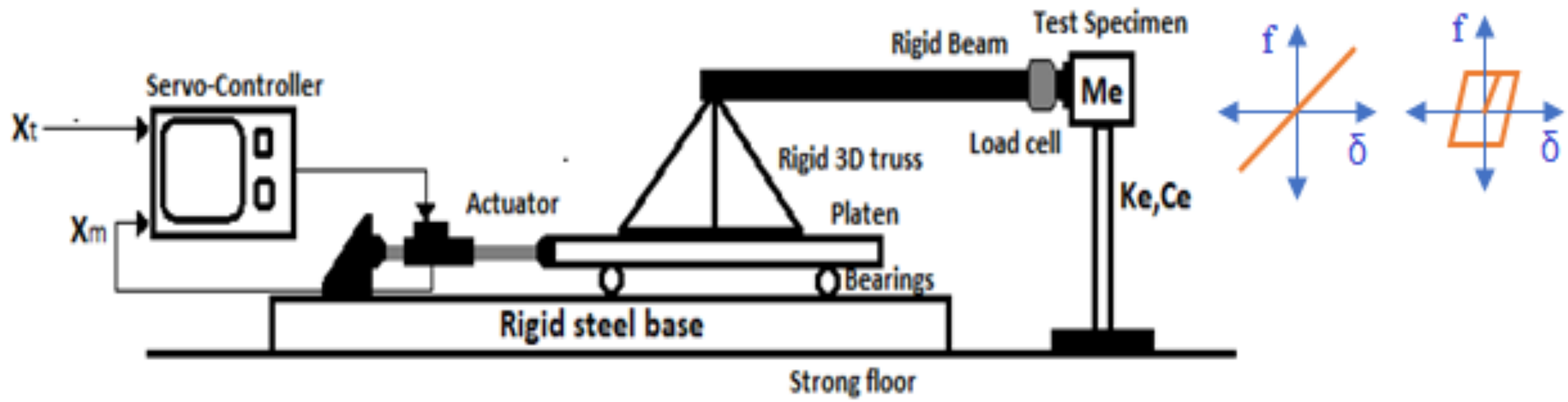
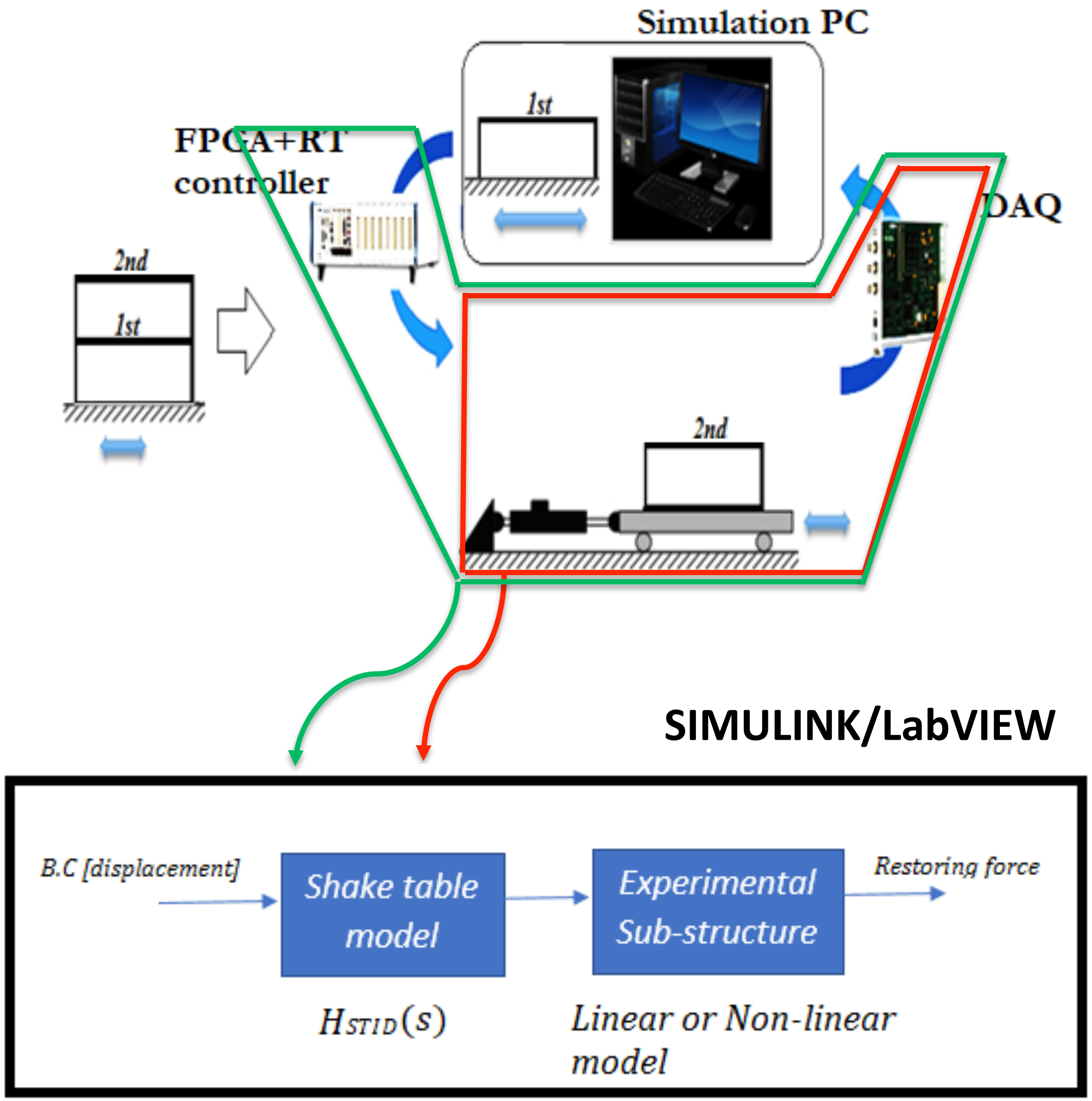


OUTLINE

- Virtual hybrid simulation (VHS)
- Mechanical and complete system identification of LNEC's uniaxial shake table (ST1D)
- ST1D under payload and uncertainty of its parameters
- Stability analysis of a VHS using Routh-Hurwitz stability test
- Stability test on linear and non-linear SDOFs
- VHS using OpenSees & OpenFresco (OSOF-VHS)
- Modelling experimental errors in OSOF-VHS
- Future directions

Virtual Hybrid Simulation (VHS)



- ❖ Virtual hybrid simulation as a pre-hybrid testing for:
  - ✓ Checking bugs in the testing software
  - ✓ Selecting appropriate controllers
  - ✓ Addressing experimental errors and stability test
- ❖ Requirements of VHS:
  - ✓ Complete model of the servo-hydraulic actuator or the shake table
  - ✓ Model of the experimental element

Uniaxial shake table system identification

1. Mechanical system identification

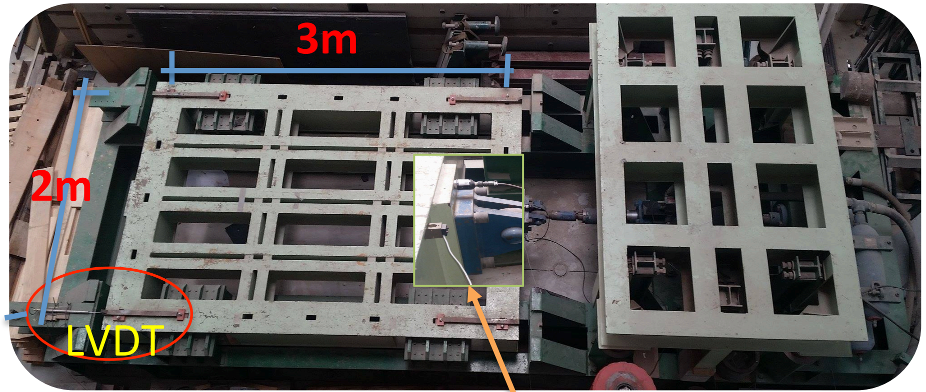
Includes identification of:

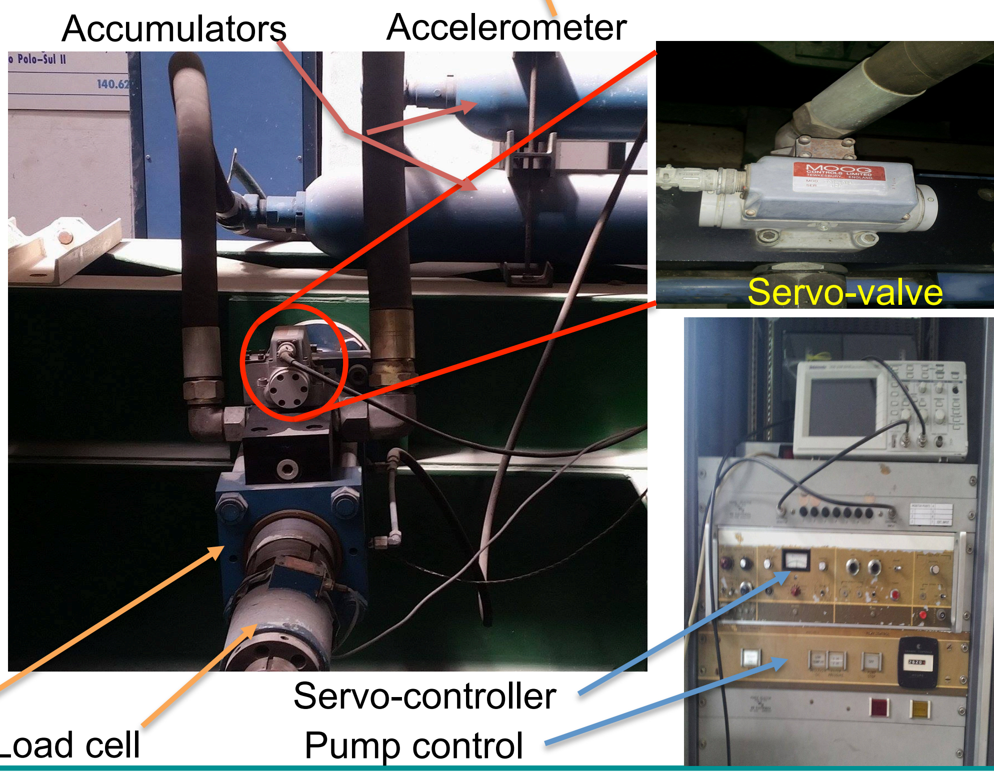
- ✓ Total moving mass
  - Experimental sub-structure mass
  - Platen mass
  - Actuator mass
  - Other moving parts
- ✓ Dissipative force
  - Experimental sub-structure damping
  - Resistance in actuator chamber
  - Resistance in platen bearings
- ✓ Elastic force
  - Experimental sub-structure
  - Other sources

2. Complete system identification

Includes:

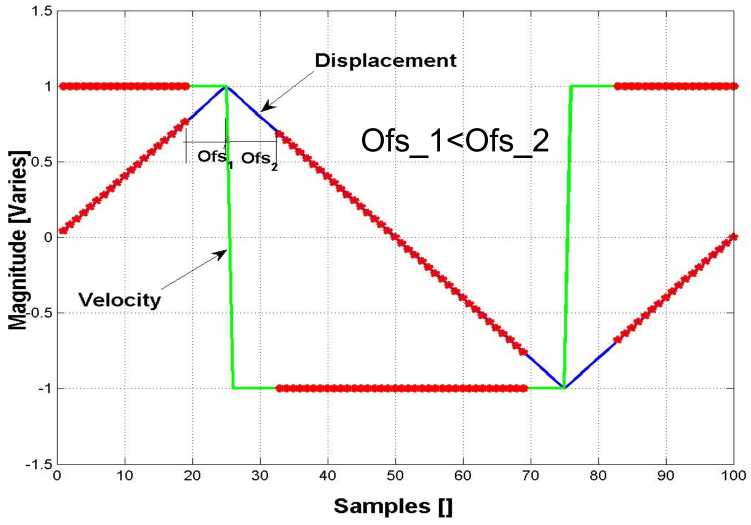
- ✓ Mechanical system identification
- ✓ Servo-hydraulic actuator control (SHAC) identification



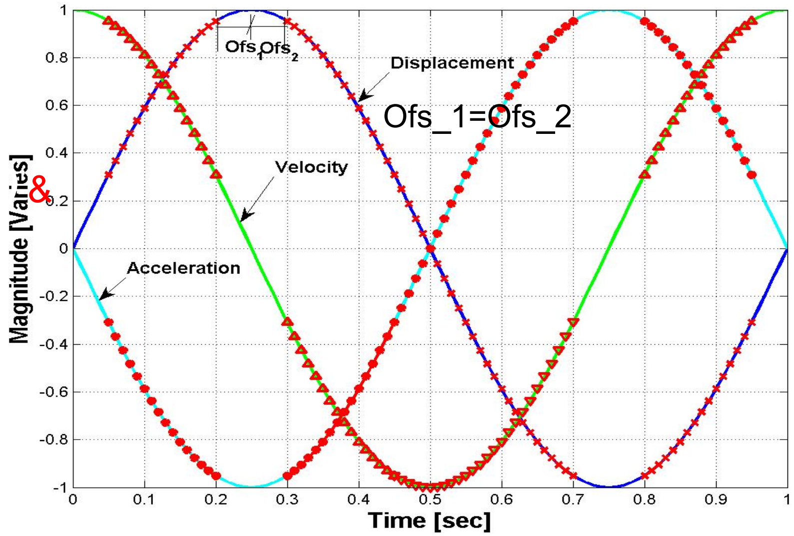


Mechanical system identification

- Mathematical models assumed for fitting:
  - $F_I = M u_x$, where $M = M_{Platen} + M_{ac}$
  - $F_E = K u_x$
  - $F_D = [F_\mu + C/u_x/\uparrow\alpha]\,\mathrm{sign}(u_x)$, where
- The experimental method uses:
  - Sinusoidal and triangular displacement signals
- Uses periodicity of measured signals:
  - Triangular test:



  - Sinusoidal test:



Mechanical system identification continued…

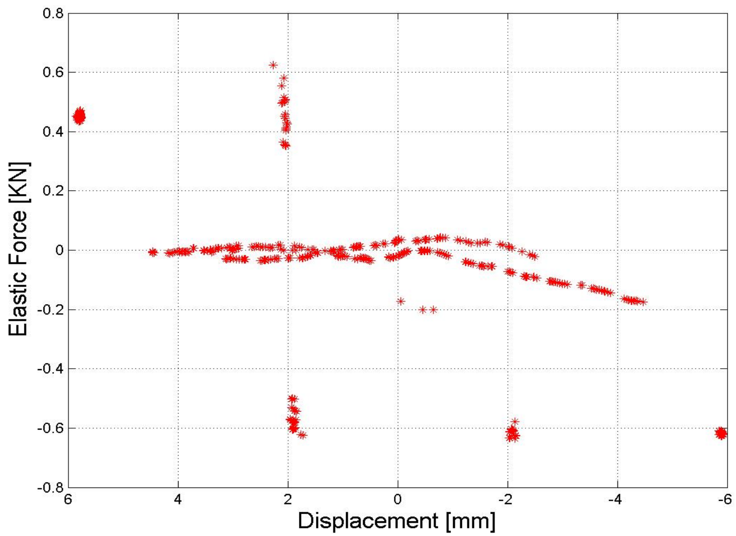
Elastic force estimation: estimated from triangular signals

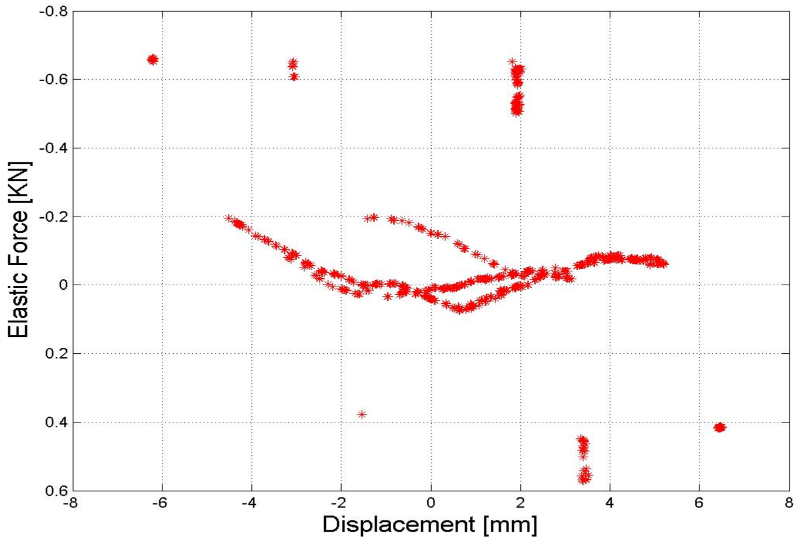
- A low-pass Fourier filter with a f_c 2-4 times the f_{cmd}
- Uses a synthesized velocity signal (from accelerometer and LVDT measurements)
- Signals used for estimation:

same applies to mass and dissipative force estimation

Test	Freq [Hz]	Amplitude [cm]	V_{max}[cm/s]
T5	0.4	1.0	1.6
T6	0.5	1.0	2

- Null or near zero elastic force
- Spurious estimates due to small SNR of the load cell force

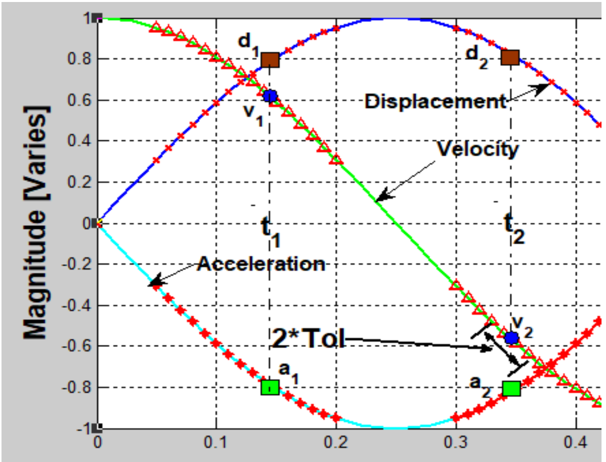




Mechanical system identification continued…

Effective horizontal mass estimation: estimated from sinusoidal signals

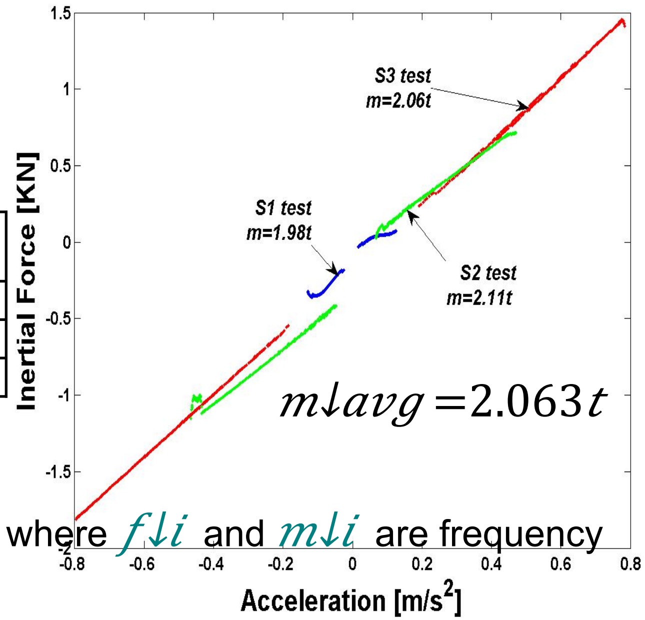
- Excludes velocities below $|\pm 5\,mm/s$
- Minimizing velocity sum at t1 and [T/2-t1-Tol, T/2-t1+Tol]



$$u_{min}(t_1)=\min\{|u_x(t_1)|+u_x(\underbrace{T/2-t_1}$$

- Test signals and estimates:

Test	Freq [Hz]	Amplitude [cm]	V_{max}[cm/s]	A_{max} [%g]	Mass estimates [ton]
S1	0.2	4.0	5.0265	0.6435	1.98
S2	0.4	4.0	10.1300	2.5760	2.11
S3	0.5	4.0	12.5660	4.0240	2.06



$$m_{avg}=2.063t$$

- Weighting estimated through a simplified approach:

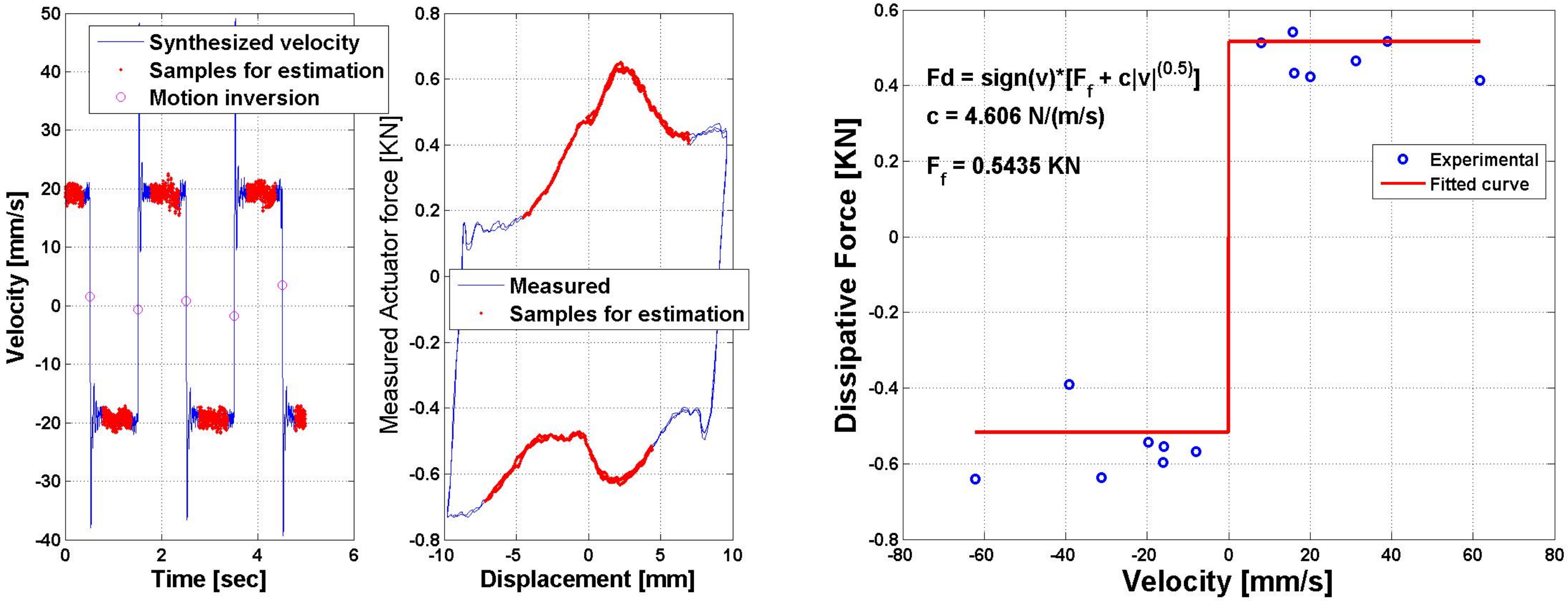
$w_i=f_i/\sum f_i$, $m_{avg}=\sum_{i=1}^{n}(w_i * m_i)$, where f_i and m_i are frequency and mass estimates of each test respectively.

(a deduction from *Lanese et al., 2012*)

Mechanical system identification continued…

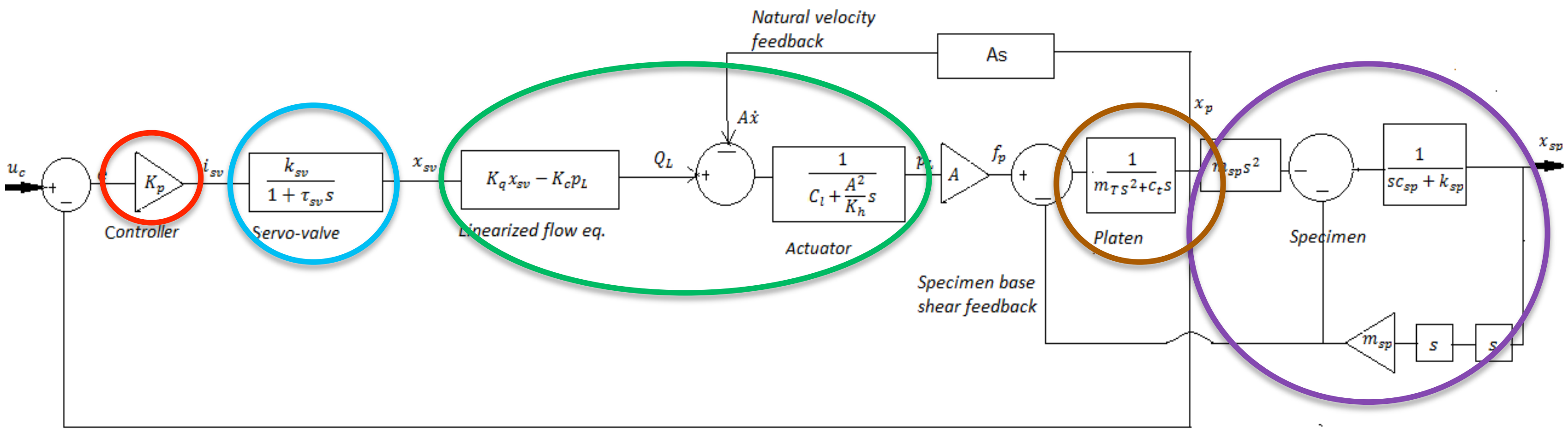
Dissipative force estimation: estimated from triangular signals

- Straightforward coupling of samples (no displacement sum minimization)
- Propagation of disturbance is pronounced after inversion of motion
- Each triangular test yield +ve and –ve estimates of dissipative force
- The coulomb friction force governs the dissipative force



Complete system identification

- ❖ Involves modelling:
  - ✓ Servo-controller - proportional gain only controller
  - ✓ Servo-valve - first order transfer function (good in 0-50Hz range)
  - ✓ Hydraulic actuator - linearized flow equation and oil-column frequency
  - ✓ Estimated effective mass of the platen (elastic force was set to zero)
  - ✓ Dissipative force (simplified to viscous damping only)
  - ✓ Payload (specimen) dynamic properties - no payload condition to produce a generalized system model; Model subject to changes under any experimental specimen



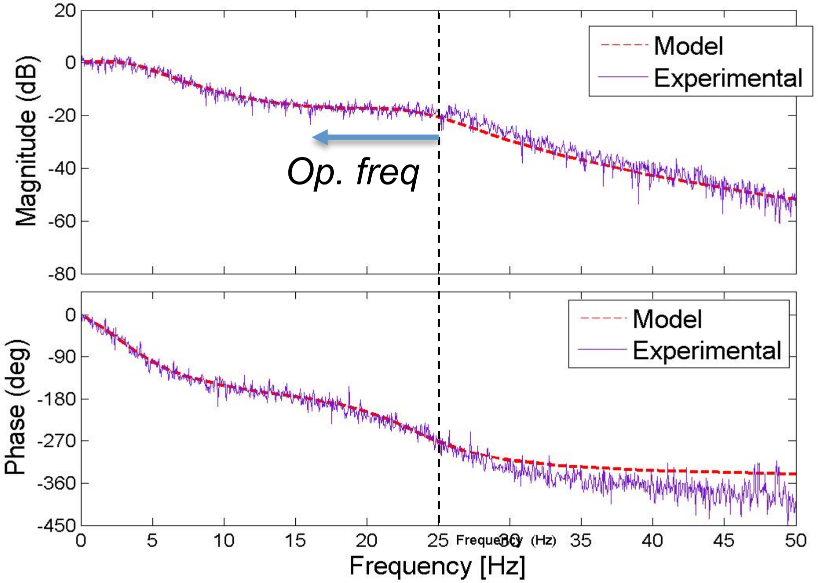
Where: ,

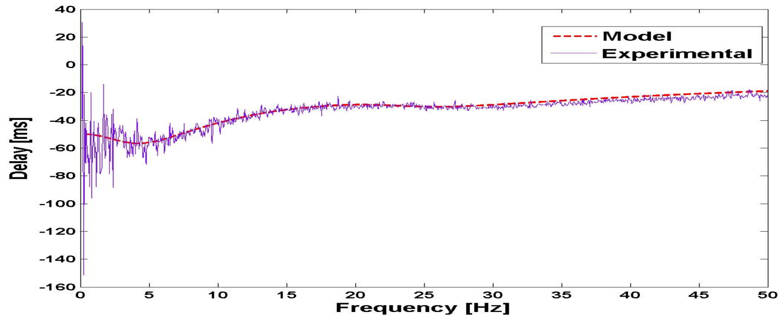
and

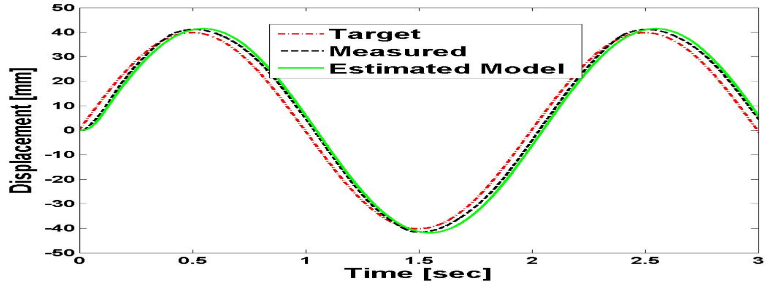
❖ $m_T^* = m_p$ under no payload condition, while, under payload m_{sp} and $H_{sp}(s)$ are non-zero te

Complete system identification continued…

- Test signal characteristics:
  - ✓ Band Limited White Noise (BLWN) ranging 0-50Hz
  - ✓ RMS value of 0.345 cm
- Parametric identification of the ST1D model in SIMULINK using:
  - ✓ Transfer system equation developed
  - ✓ Command signal, measured signal, and the gain error identified
  - ✓ Constrained non-linear least square solution
- Validation of estimated shake table parameters







$$H_{ST1D} = \frac{1.9485e+7}{s^4 + 92.29 s^3 + 2.5132e+4\, s^2 + 9.3478e+5 s + 1.8746e+7}$$

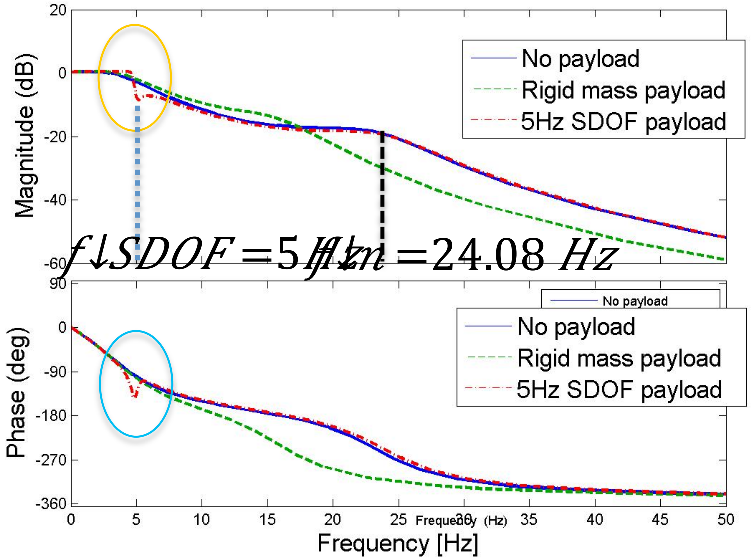
(4 poles without zeros)

Oil bulk modulus (β_e)	Servo-valve time constant (τ_{sv})	Valve pressure gain +Leakage factor (

ST1D model under payload & uncertainty of its parameters

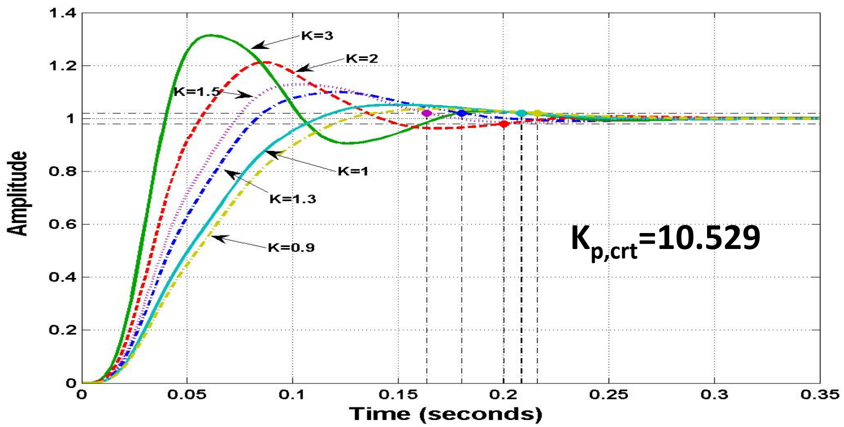
SDOF payload : Flexible and rigid

- ❖ Flexible SDOF comes with Control-structure interaction (CSI)
  - ✓ Behaves similarly to a no-payload condition, except it introduces:
    - ➢ A notch and a peak in the magnitude plot
    - ➢ A notch in the phase plot
- ❖ Rigid SDOF results in a stiffer response and a significant shift in the oil-column resonance

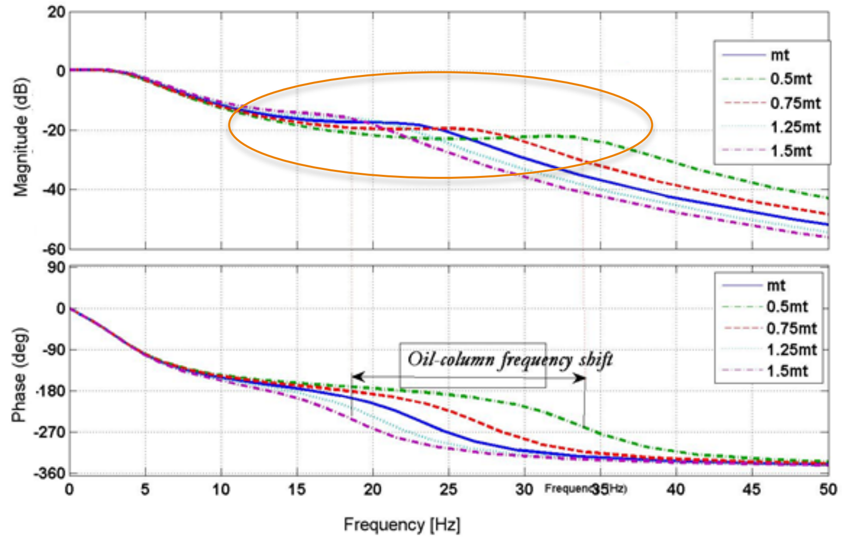


Uncertainty of ST1D parameters: No payload

- ❖ Undershooting mass results in a spongy response
- ❖ Overshooting apparently makes system stiffer
- ❖ Increase in K_p: stiffer but smaller margins of stability



$K_{p,crt}$=10.529



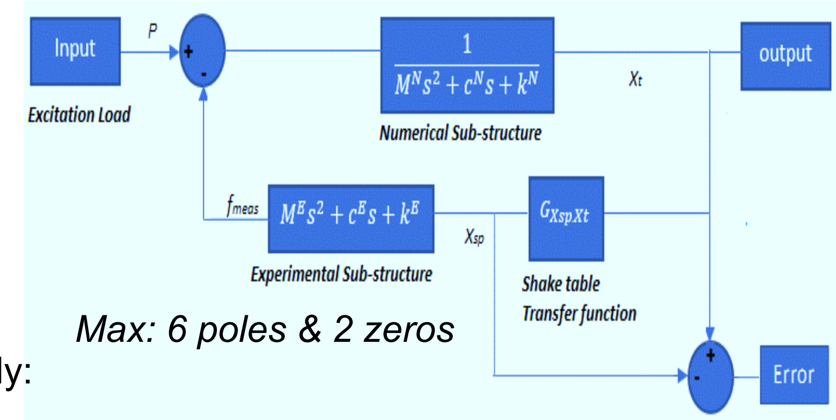
Stability analysis of a VHS using Routh-Hurwitz test

Why and how stability test:

- Delay in HS is interpreted as a '*negative damping*'
- Stability under a linear transfer system is dictated by:
  - ✓ Experimental stiffness
  - ✓ Experimental mass and
  - ✓ Experimental damping
- Shortcomings of Mercan & Ricles (2007) stability study:
  - ✓ A '*pure delay*' assumption in HS
- System delay is a function of frequency, hence it requires the system model
- Stability test using Routh-Hurwitz stability:
  - ✓ Sufficient for stability of LTI control systems
  - ✓ Allows parametric study under a linear response

Routh-Hurwitz method:

- Finds roots of the characteristic polynomial equation (D(s)) that fall in right-half S-plane
- Procedure:
  - ✓ Develop the Hurwitz matrix (upper triangular matrix)
  - ✓ Any sign change in 1st column indicates unstable test



$$G_{VHS}(s)=X_t(s)/P(s)=N(s)/D(s)$$

$$D(s)=b_m s^m+b_{m-1}s^{m-1}+b_{m-}$$

s^m	b_m	b_{m-2}	b_{m-4}	b_{m-6}	…
s^{m-1}	b_{m-1}	b_{m-3}	b_{m-5}	b_{m-7}	…
s^{m-2}	c_1	c_2	c_3	c_4	…
s^{m-3}	d_1	d_2	d_3	d_4	
⋮	⋮	⋮	⋮	⋮	⋮
s^0	e_1				

Where: $c_i=\left(b_{m-1}*b_{m-2i}-b_m*b_{m-2i-1}\right)/b_{m-1}$ and so on

Stability analysis of a linear SDOF

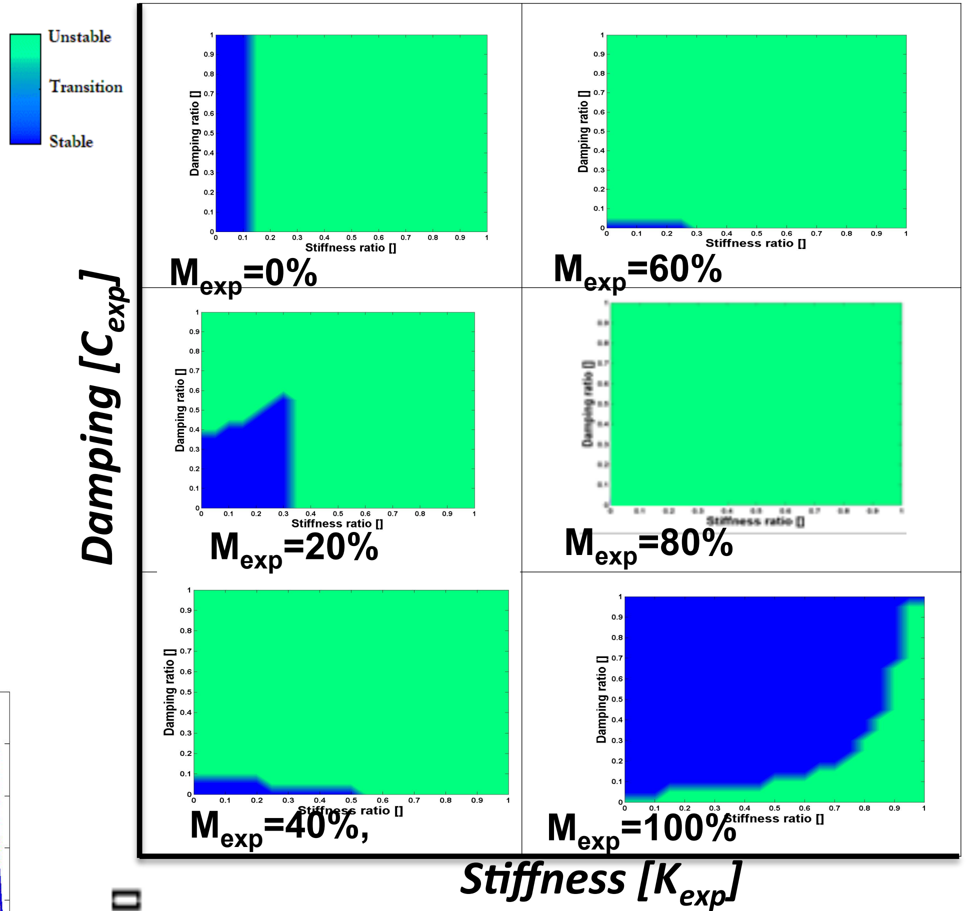
Analysis and results:

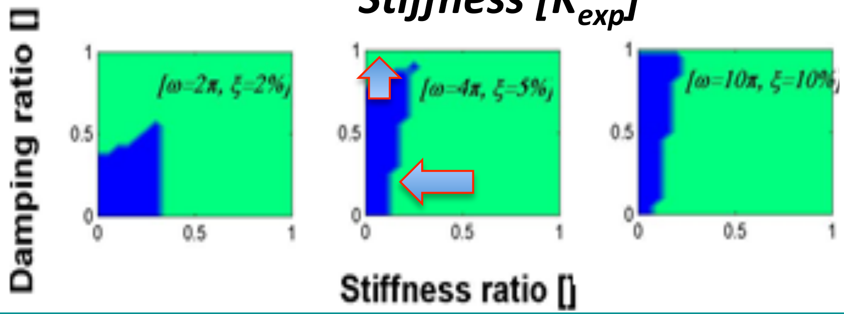
- A SDOF properties: *ω=1Hz, m=2t and ζ=2%*
- Generating stability surface by constantly changing the fractions of experimental and numerical sub-structures, e.g., M_{exp}= M_{exp}/m
- A percentage step of 5% was adopted for each parameter
- 2-parameter study, namely:
  - ***i. K_{exp} versus C_{exp} at:***
    - ***M_{exp}= 0, 20%, 40%, 60%, 80%, 100%***
  - ***ii. C_{exp} versus M_{exp} at:***
    - ***K_{exp}= 0, 20%, 40%, 60%, 80%, 100%***
  - ***iii. K_{exp} versus M_{exp} at:***
    - ***C_{exp}= 0, 20%, 40%, 60%, 80%, 100%***
- 2-parameter study at a fixed third parameter, but at varying values of stiffness and viscous damping of the SDOF (for cases *i* and *ii*)
  - ***m=2t (constant), ω=1,2,5 Hz, and ζ=2%, 5%, 10%***

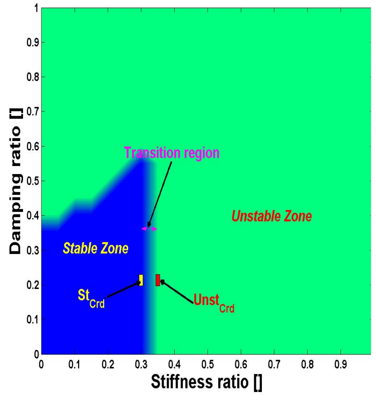
Stability analysis of a linear SDOF

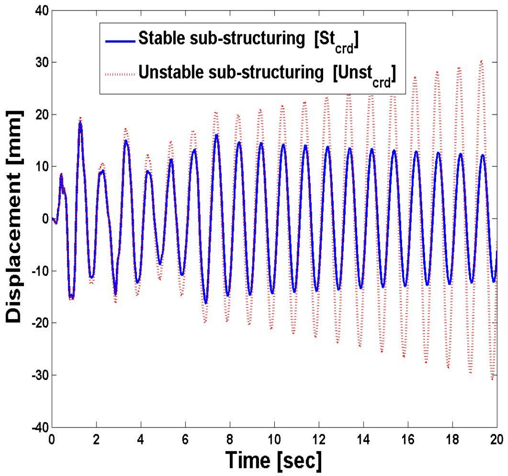
K_{exp} versus C_{exp} at constant M_{exp}:

- As M_{exp} increases (0-80%), C_{exp} falls
- As M_{exp} increases (0-40%), K_{exp} rises, but falls when M_{exp} increases further
- At full experimental inertial mass, system attains improved stability
- At 20% of $M_{exp,}$ as ω and ζ of the SDOF increase (constant mass),K_{exp} ↓, while C_{exp} ↑
- Validation of stability test at a transition zone, using El Centro at 0.3g PGA and a free-vibration tail, at M_{exp}=20%





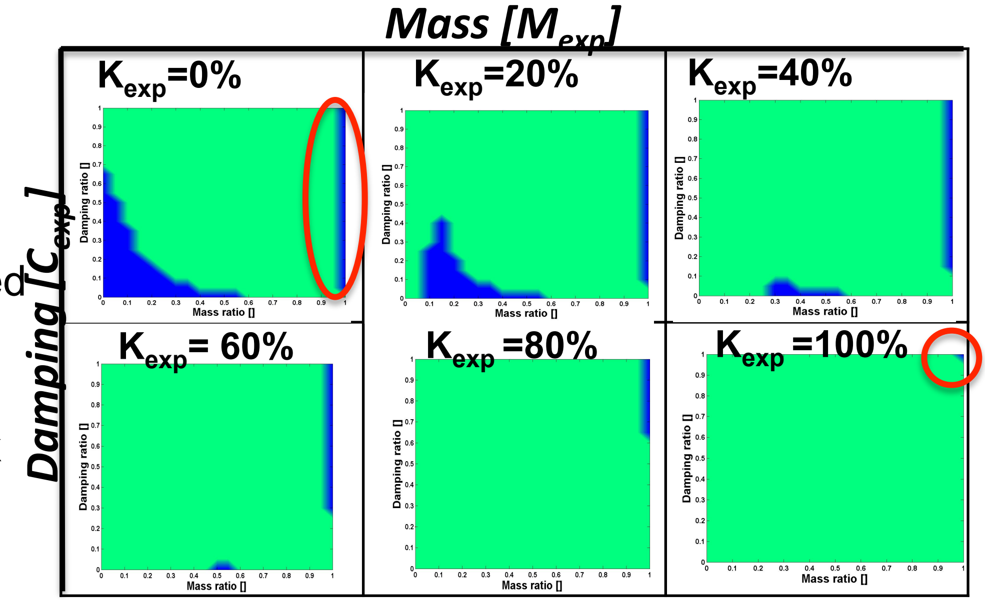


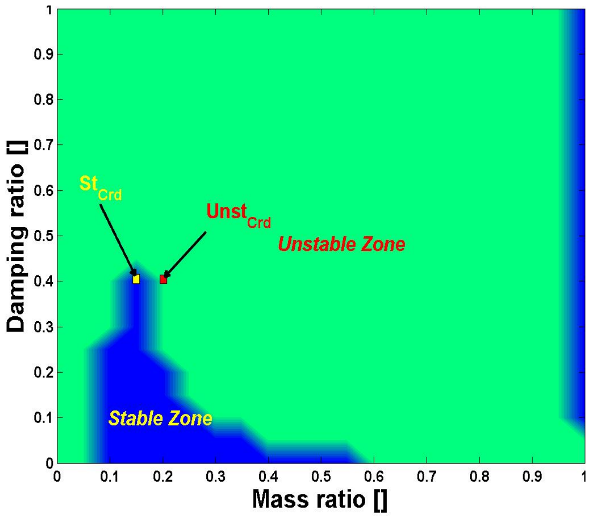


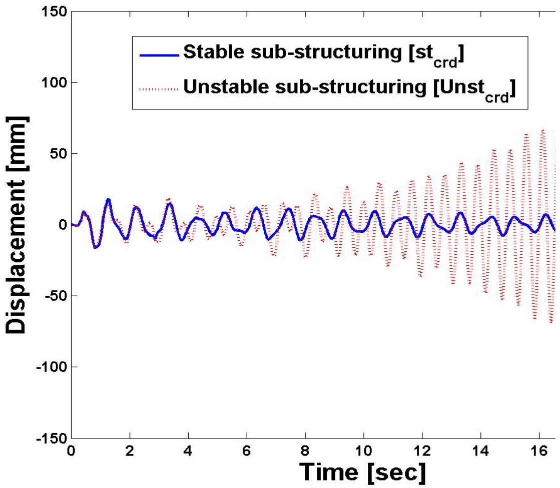
Stability analysis of a linear SDOF continued…

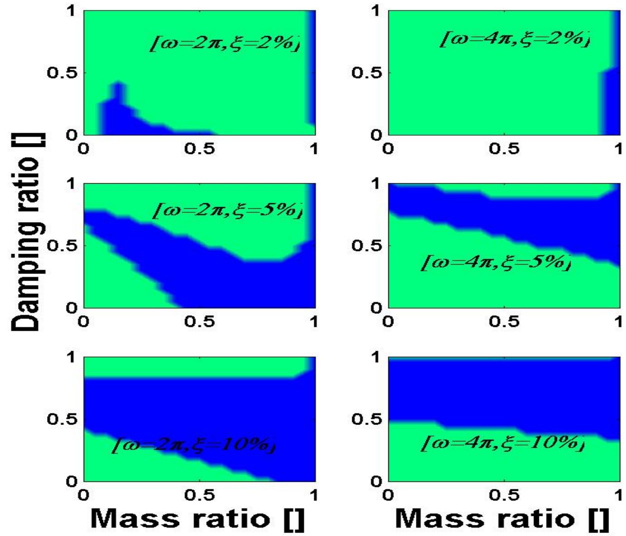
<u>C_{exp} versus M_{exp} at constant K_{exp}:</u>

- ❖ Increasing K_{exp}, C_{exp} falls and the minimum required M_{exp} rises
- ❖ M_{exp} needs to be bounded for stability
- ❖ At full experimental mass, stability is attained if the minimum C_{exp} is increased
- ❖ When ω and ζ of the SDOF increase (constant mass), system exhibits a complex pattern
- ❖ Validation of stability test at K_{exp}=20%





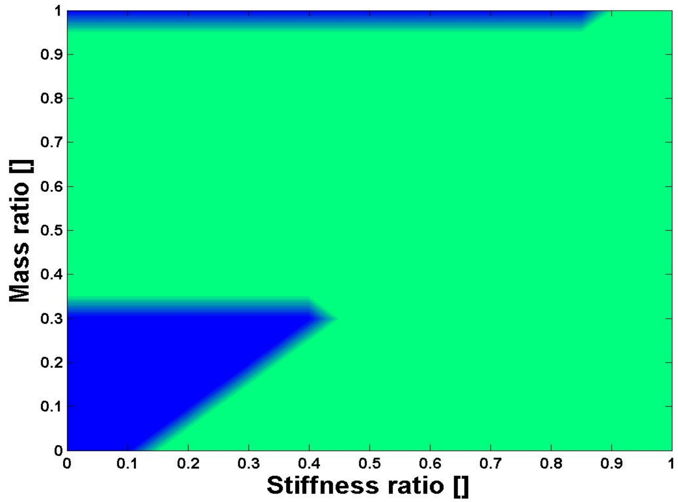




Stability analysis of a linear SDOF continued…

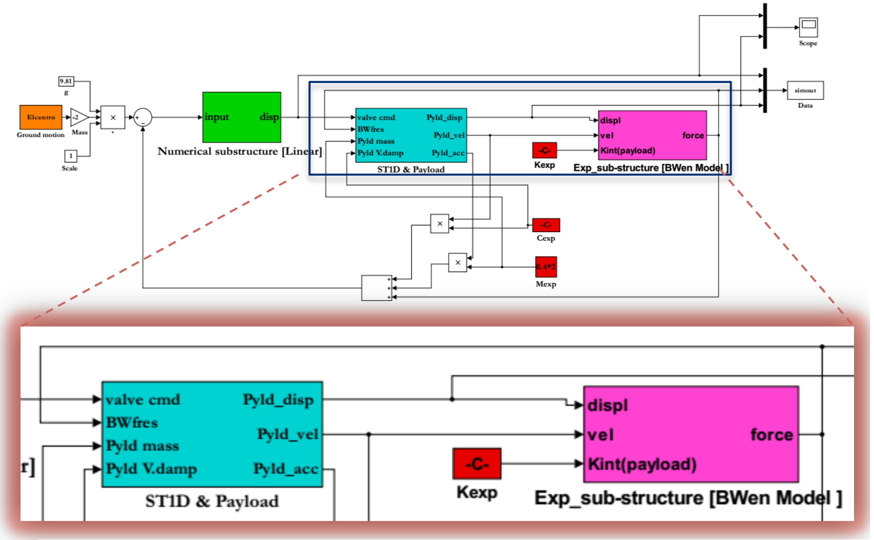
K_{exp} versus M_{exp} at varying C_{exp}:

- ✓ Regardless of the C_{exp}, stability contour is identical, except at C_{exp}=100%
- ✓ A positive linear relationship exists between M_{exp} and K_{exp} until 30% of M_{exp}
- ✓ Stable at right top corner of contour plot at C_{exp}=100%, i.e., shake table test
- ✓ Small deviations of the stable zone under varying frequencies and damping, but the linear relationship still prevails



Tekeste, G.G., Correia, A.A., Costa, A. G., [2017], "Stability analysis of a real-time shake table hybrid simulation for linear and non-linear SDOF systems", 7th international conference on Advances in Experimental Structural Engineering, EUCENTRE Foundation, Pavia, Italy

Stability analysis of a nonlinear SDOF



Why?

- To address the nonlinear range of response
- Study the relation between degree of nonlinearity and stability

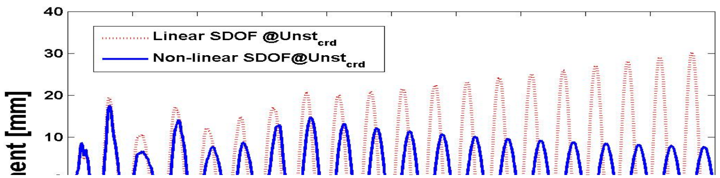
Methodology:

- Simulink model of VHS
  - ✓ Bouc-Wen model of the experimental part
  - ✓ Linear model of numerical part

$$f_{exp}(t)=\alpha k_{int}^{exp} x_{sp}(t)+(1-\alpha)k_{int}^{exp}\delta_y Z(t)+c^{exp} x_{sp}(t)$$

$$Z(t)=x_{sp}(t)\{A+\beta * sign(Z(t)x_{sp}(t))+\gamma|Z(t)|^{n}\}/\delta_y$$

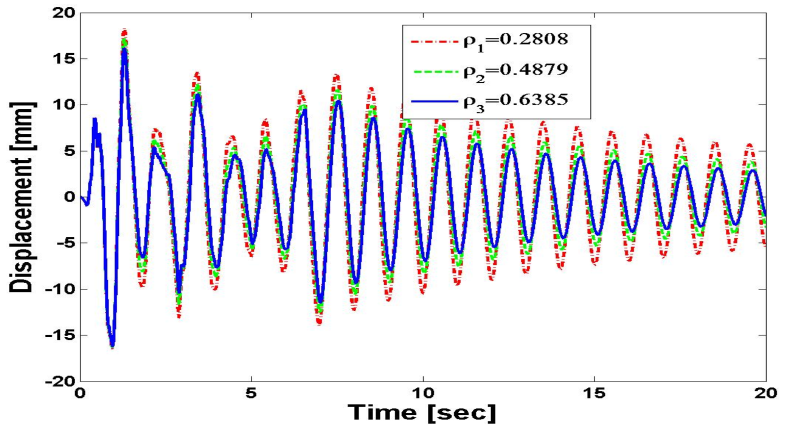
- The Simulink model calls a MATLAB .m function of Bouc-Wen at every step



Comparison:

- Validation at unstable coordinates of the linear case for comparison (M_{exp}=20%, C_{exp}=20% and K_{exp}=35%)
- Comments:
  - ✓ Improved stability under a non-linear response an or a material with high non-linear behaviour
  - ✓ A larger margin from instability with increasing degree of non-linearity, defined by

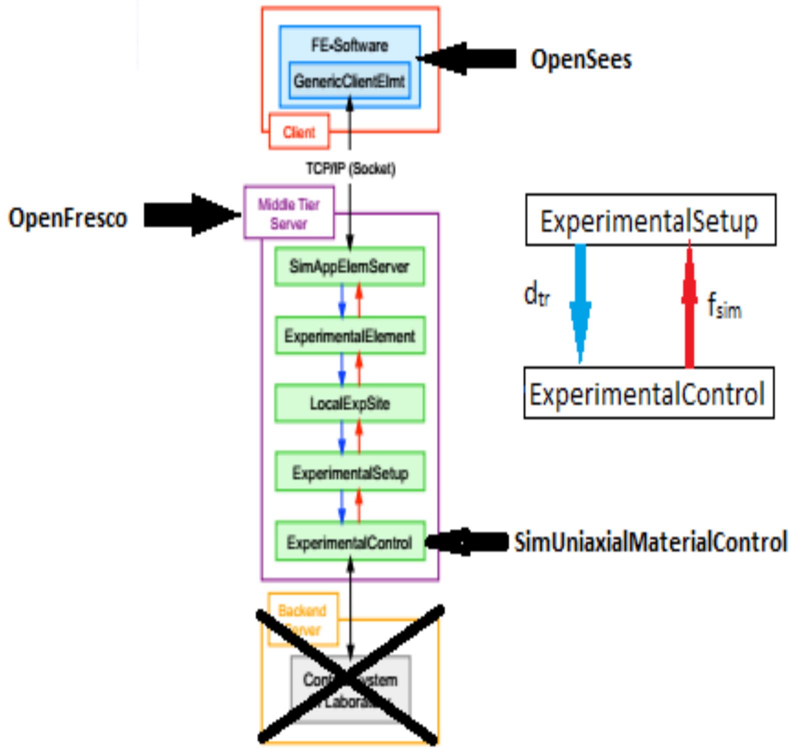
$$\rho=\max|f_r(t)|/f_{max}=|f_r(t)|/f_y[A/(\beta+\gamma)]^{1/n}$$



Virtual hybrid simulation using OpenSees and OpenFresco

- ❖ Virtual hybrid tests using OS as the computational driver and OF as the middleware

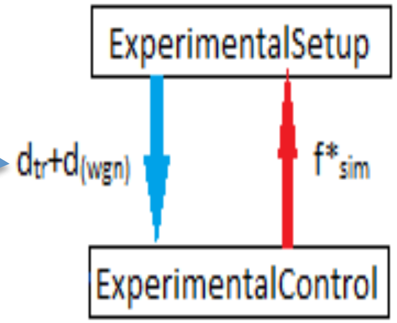
- ❖ Restoring forces returned to OS are simulated using OS uniaxial material (restoring forces in actual hybrid tests are measured in laboratory)



OpenFresco architecture (credit to Schellenberg et al., 2009)

Experimental errors modelling in OF:

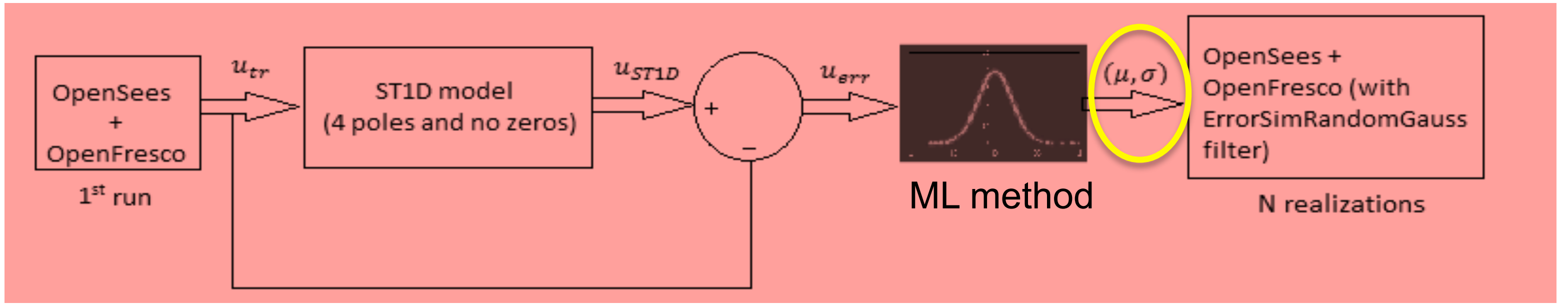
- ❖ Introducing experimental errors through ExpSignalFilter control object using:
  - ❖ ErrorSimUndershoot
  - ❖ ErrorSimOvershoot
  - ❖ ErrorSimRandomGauss (models random process in nature



Tekeste, G.G., Correia, A.A., Costa, A. G., [2017] "Virtual hybrid simulation tests accounting for experimental errors", OpenSees Days Europe: First European conference on OpenSees, Porto, Portugal

Modelling experimental errors in OSOF-VHS

❖ Offline estimation of error parameters using the ST1D model in Simulink

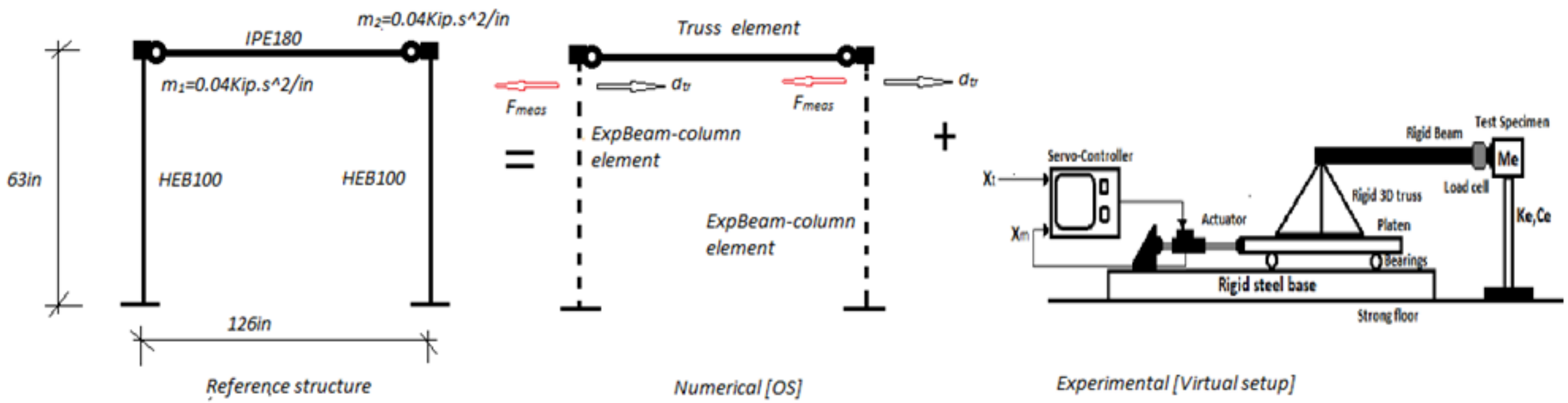



```
# expSignalFilter ErrorSimRandomGauss  $tag   $avg       $std
  expSignalFilter ErrorSimRandomGauss  1  -0.0003475  0.2051730
```

❖ One-bay frame with a truss element and two non-linear columns

❖ Columns are “experimental” substructures

❖ Truss element and masses are the “numerical” substructures

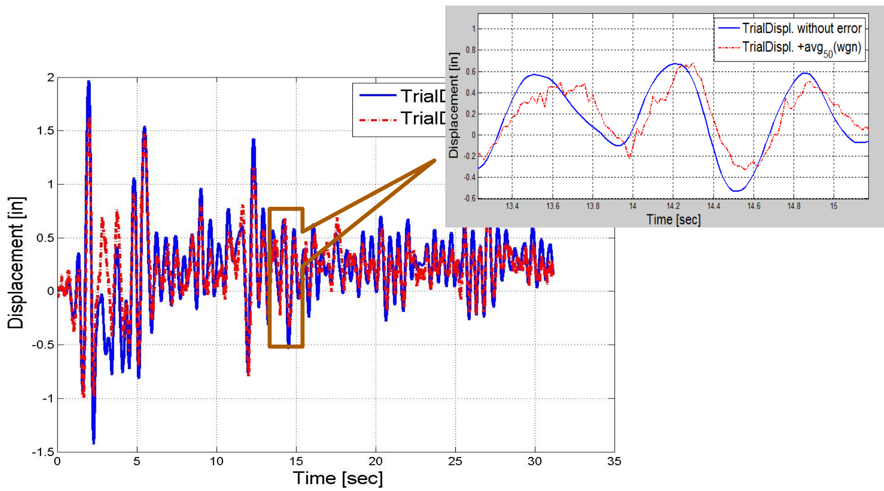


Modelling experimental errors in OSOF-VHS continued…

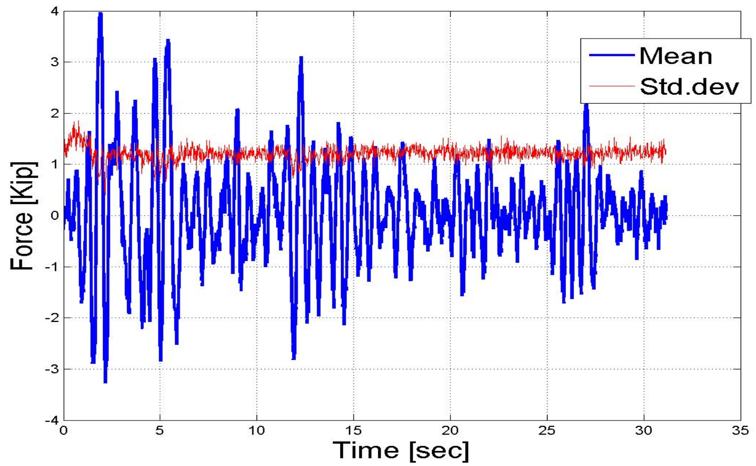
❖ OpenFresco definition:

```
expControl SimUniaxialMaterials 1 1 -ctrlFilters 1 0 0 0 0
expSetup OneActuator 1 -control 1 2 -sizeTrialOut 3 3 (only 1 ctrl and 1 out were used)
expSite LocalSite 1 1
expElement beamColumn 1 1 3 1 -site 1 –initStif . . .
```

- Since "real" experimental errors cannot be directly feedback to OS, a number of stochastic realizations were necessary to model the errors using WGN
- Expected response is found by averaging over 50 realizations
- The sub-space synchronization plot (SSP) shows both gain and time-lag errors



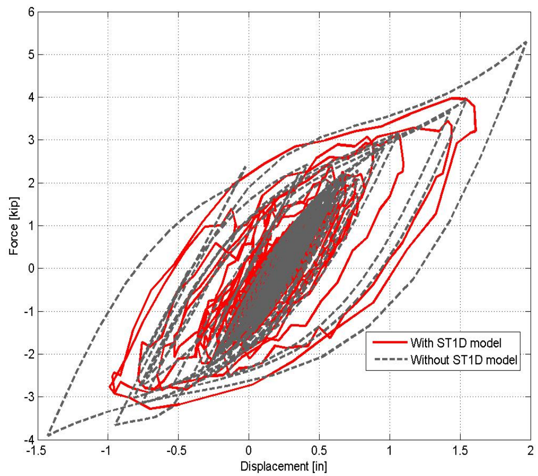
Trial displacements without error vs with WGN (averaged)



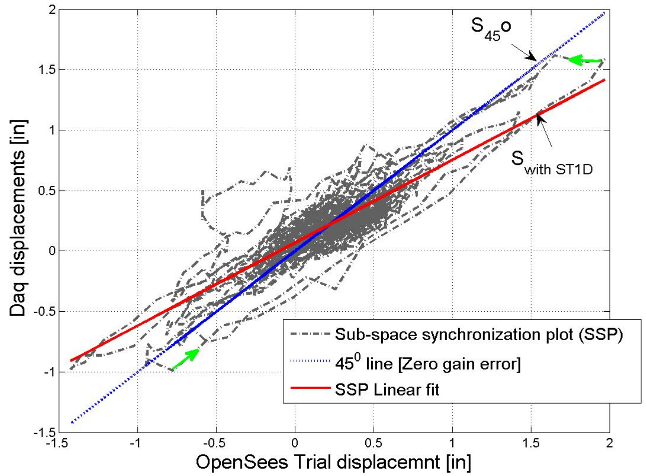
Coupling force returned to OS

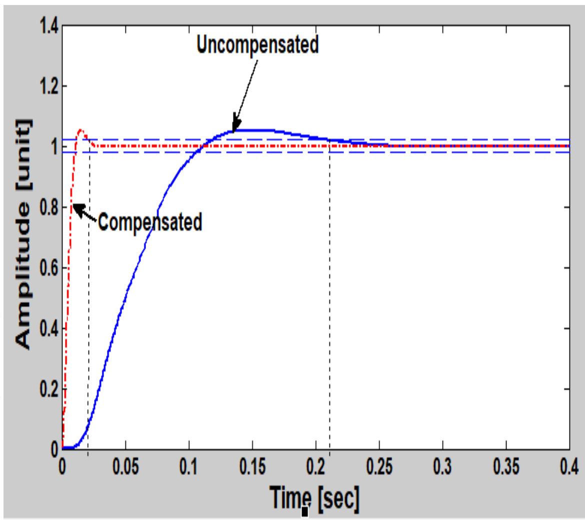
Modelling experimental errors in OSOF-VHS continued…

- Conclusions:
  - ✓ Experimental errors result in a coupled gain and delay (introduced by ST1D dynamics)
  - ✓ Reduction in energy dissipation capacity is prevalent
- Solutions sought:
  - Higher frequency of AD/DA conversion
  - Tuning the controller gains for minimal settling time and overshoot
  - Implementation of adaptive model based compensator in the outer loop



Column hysteresis of VHS with & without Exp. errors





Feedforward compensation of ST1D

Future directions

- Completion of a LabVIEW based testing software that works in conjunction with OF and OS
- Conducting hybrid simulations on a steel frame using the ST1D only
- Development of advanced control strategies for the RTHS framework
  - ✓ Shake table control
  - ✓ Force control of actuators
  - ✓ Model based and adaptive compensation

THANK YOU FOR YOUR ATTENTION!

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