Risk analysis of bridges using a new reliability-based robustness assessment methodology

Current Status: achievements and challenges

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Outline

- **Structural Reliability Analysis**
  - Achievements and challenges
- **Robustness as performance indicator**
  - Preliminary framework
- **Preliminary results – case study**

Framework for Structural Reliability Analysis

- **Goals**
  - ✓ Probability of failure / Reliability Index
  - ✓ Computational efficiency: accuracy, precision and computation time

- **Limitations**
  - ✓ For high dimension and complex systems, classical reliability methods do not yield a good efficiency in evaluating small probabilities of failure
    - ✓ Non linearity of Limit State Surface
    - ✓ Non linearity of system performance
    - ✓ …
  - ✓ FE based structural reliability analysis faces several difficulties
    - ✓ Closed-form state functions are not always easily obtained
    - ✓ Pointwise representation of simulations
    - ✓ …

Melchers (2001)
Bucher (2009)
Nowak and Collins (2012)

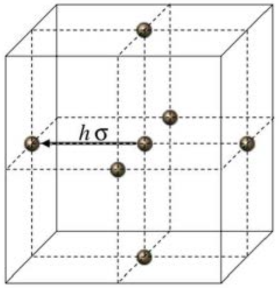
Adaptive Response Surface Methodology

Main steps of traditional RSM

1. Choice of a initial Experimental Design (ED);
2. Building a RS;
3. Searching the design point and the Reliability Index;
4. Adding new sampling points near the design point;
5. Repeating from step 2 until a convergence criterion is fulfilled;

$$X_i = \bar{X}_i + h \cdot \sigma_i$$

Rajashekhar and Ellingwood (1999)
Bucher (2009)



a) star

X_i Experimental point

$\bar{X}_i$ Central point

h Dispersion (1 to 3)

σ_i Standard Deviation

Adaptive Response Surface Methodology

- **Developed methodology description**
  - ✓ Combination of existing knowledge
    - ✓ Space-filling designs
    - ✓ Adaptive procedure to enrich ED in the Region of Interest (Roussouly et al. 2013)
    - ✓ Double weighted regression technique (Chen et al. 2010)
    - ✓ Computation of cross-validation error (Hanczar et al. 2013)
    - ✓ Confidence Intervals on P_f and β based on bootstrapping residuals approach (Lins et al. 2015)

- **Numerical applications**
  - ✓ Example 1 - Nonlinear analytic expression (explicit function)
  - ✓ Example 2 - Truss serviceability – mid-span displacement
  - ✓ Example 3 - Frame serviceability – top displacement

Adaptive Response Surface Methodology

1. **Stochastic simulation, sensitivity analysis and screening procedures**
   - ✓ Global sensitivity methods – sensitivity coefficients
   - ✓ Reduction of input random variables

2. **Choice of initial Experimental Design**
   - ✓ Space-filling design (Optimized Latin Hypercube Sample)

3. **Curve-fitting – RS according to a wise selection of terms**
   - ✓ Stepwise regression with double weighted technique for different criterion
   - ✓ Selection of RS according to best predictive coefficient Q^2 - cross validation error

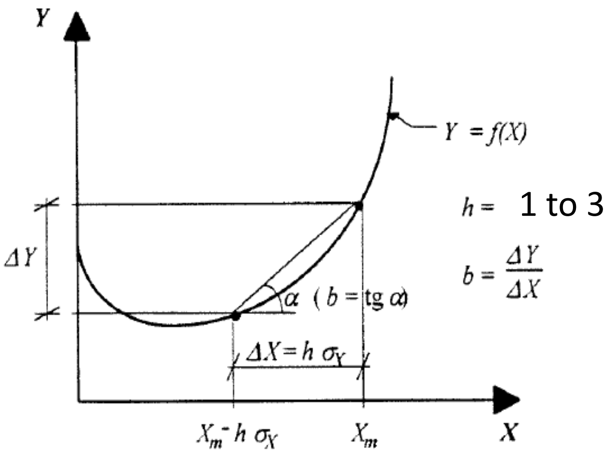
4. **Reliability analysis**
   - ✓ Importance Sampling and Definition of Region of Interest

5. **Enriching Design in the Region of Interest**
   - ✓ Augmenting ED in the region of interest keeping a LHS-design

6. **Repeat procedure until a convergence criterion is satisfied** $\left|\frac{\beta_i - \beta_{i-1}}{\beta_i}\right| \leq \varepsilon_\beta.$

Adaptive Response Surface Methodology

- **Sensitivity analysis**
  - ✓ Global sensitivity methods – measure the variability of the response of interest (Y) near the operating point (usually the mean).



Extending to a group k of n Random Basic Variables

Sensitivity Coefficients

$$b_k = \frac{1}{n} \cdot \sum_{i=1}^{n} \frac{\left(\Delta y_k / y_m\right)}{\left(\Delta x_{ik} / x_{im}\right)}$$

Importance measure (IM)

$$(b\sigma)_k = b_k \cdot \frac{1}{n} \sum_{i=1}^{n} \frac{\sigma_{Xi}}{x_{im}}$$

Relative Importance measure

$$(b\sigma)_i^{rel} = \frac{(b\sigma)_i}{(b\sigma)_{\max}} \times 100\%$$

… Screening procedure consists at neglecting groups of RV with a relative (IM) less than a predefined tolerance.

Adaptive Response Surface Methodology

- **Initial Experimental Design**

$$X_i = \bar{X}_i + h \cdot \sigma_i$$

h is no longer a fixed value but a LHS with a chosen dispersion (between 1 to 5). The value 3 is often recommended.

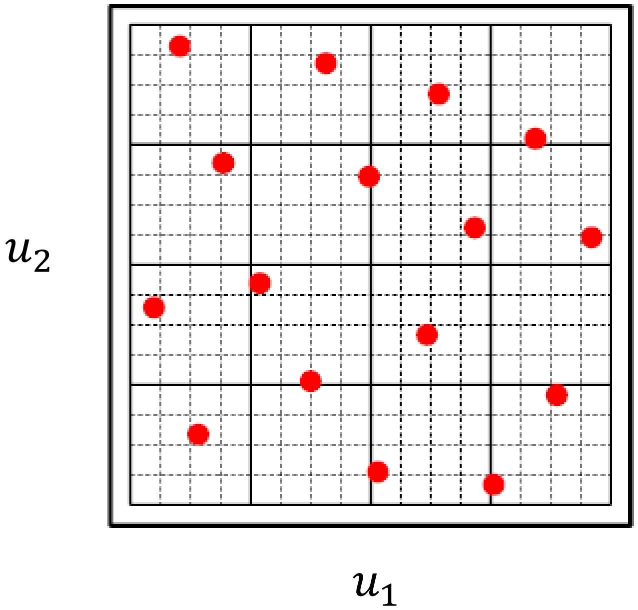
Optimized LHS:

- minimizing discrepancy
- maximizing the minimum distance between points

Experimental points are dispersed around the central points based on a LHS.

Dimension of initial ED is considered as equal to $N = 3M$, where M is the number of input variables.

Next, Additional experimental points are assumed $N_{add} = M + N_{redundant}$



Adaptive Response Surface Methodology

- **Curve Fitting**

… Response surface is built using a stepwise regression algorithm which consists in a systematic method for adding and removing terms from a linear or generalized linear model based on their statistical significance in explaining the response variable.

… `Stepwiselm` is a MATLAB function that uses **forward** and **backward** stepwise regression to determine a final model. At each step, the method searches for terms to add to or remove from the model based on the value of predefined criterion (SSE, AIC, BIC and $\boldsymbol{R^2_{adj}}$).

Adaptive Response Surface Methodology

- **Double weighted technique**

$$w_g = \alpha_g \frac{g_{best,g}}{\left| g(x_i) \right|}$$

$$w_d = \alpha_d \frac{g_{best,d} - \left\| x_i - x_d \right\|}{g_{best,d}}$$

$$g_{best,g} = \min_{i=1}^{N} \left| g(x_i) \right|$$

$$g_{best,d} = \max_{i=1}^{N} \left\| x_i - x_d \right\|$$

Benefit experimental points closer to limit state function.

Penalize points which are located far from the previous central point (design point).

$$\alpha_g + \alpha_d = 1$$

Trade-off between weights systems

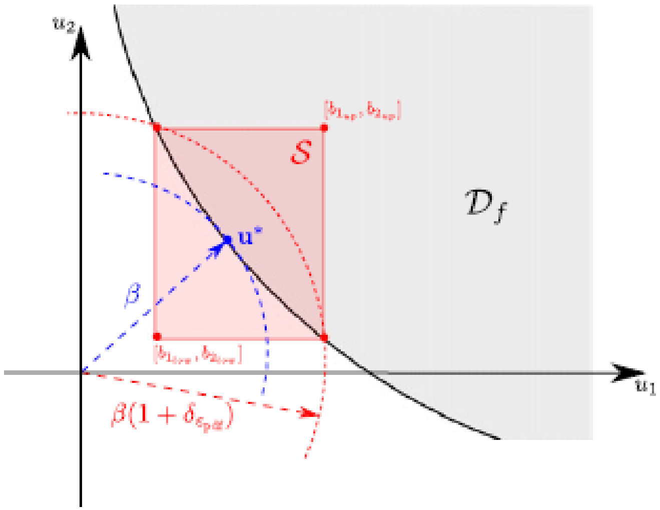
$$w = w_g + w_d$$

Chen et al. (2010)

Adaptive Response Surface Methodology

- **Definition of Region of Interest**

…. The region of interest is defined as the hypercube $\prod_{i=1}^{M}[b_{ilow,}b_{iup}]$ which framed the intersection between the ball $\mathcal{B}\left(0,\beta\left(1+\delta_{\varepsilon_{pdf}}\right)\right)$ and the limit state $H(u)=0$.



<u>*Lower bound*</u>

$$b_{i_{low}}=\arg\min u_i \quad \text{with} \begin{cases} \|\mathbf{u}\|-\beta(1+\delta_{\varepsilon_{pdf}})=0 \\ H(\mathbf{u})=0 \end{cases}$$

$$\delta_{\varepsilon_{pdf}}=\sqrt{1-\frac{2\ln(\varepsilon_{pdf})}{\beta^2}}-1$$

… The importance level, denoted ε_{pdf}, is defined in the sense that all points u with $\Phi(u)\leq\varepsilon_{pdf}\cdot\Phi(u^*)$ are considered to have a negligible probability density.

Roussouly et al. (2013)

Adaptive Response Surface Methodology

Example 1 - Analytical expression

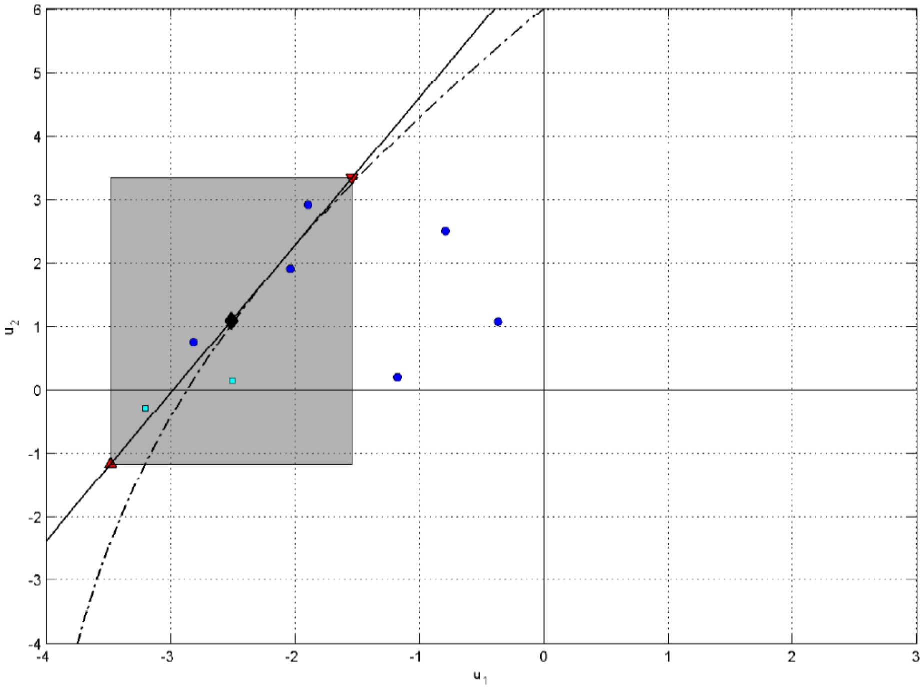
$$G(X) = \exp[(0{,}4 * (x_1 + 2) + 6{,}2] - \exp[0{,}3 * x_2 + 5] - 200$$

Random Variables

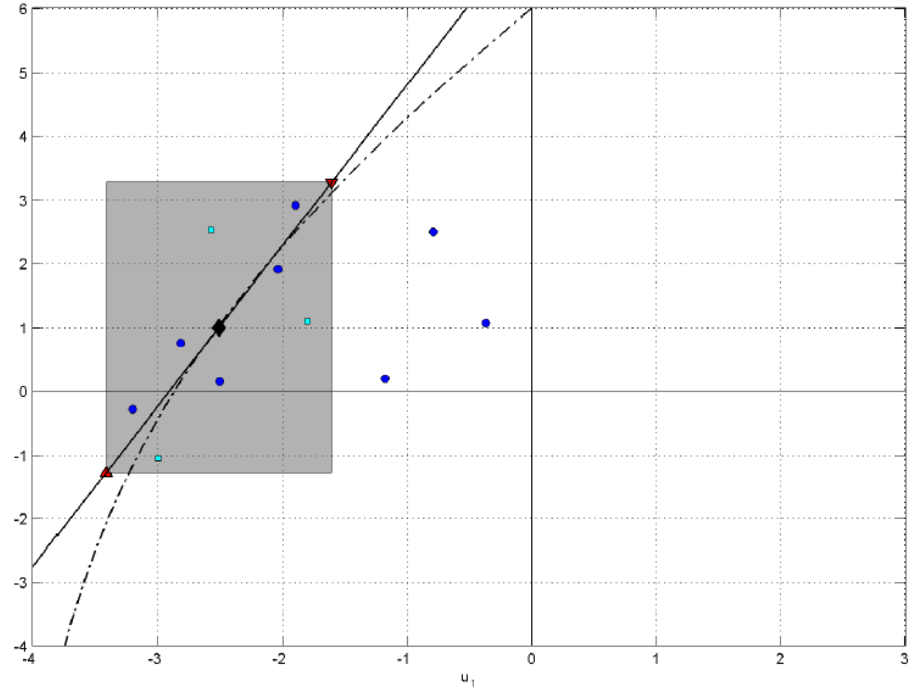
- x_1– standard normal random variable
- x_2– standard normal random variable

Reference /Method	Reliability Index, β	β, Error (%)	P_f	P_f, Error (%)	*N* (LSF)
COMREL / Importance Sampling	***2.686***	-	***3.616E-03***		*1 000 000*
Kim & Na (1997) / RSM projected sampling points	2.668	-0.67%	3.815E-03	5.52%	N.A
Kaymaz & McMahon (2005) / Weighted RSM	2.687	0.04%	3.605E-03	-0.30%	N.A
Nguyen et al. (2009) / Adaptive RSM with double weighted technique	2.707	0.78%	3.395E-03	-6.11%	12
Kanga et al. (2010) / RSM moving least squares	2.710	0.89%	3.364E-03	-6.96%	12
Roussouly et al. (2013) / Adaptive RSM	2.677	-0.34%	3.714E-03	2.73%	15
PROPOSED METHOD	***2.679***	***-0.26%***	***3.692E-03***	***2.11%***	***11***

Example 1 - Analytical expression



Iteration 1 – ***N*** = 6

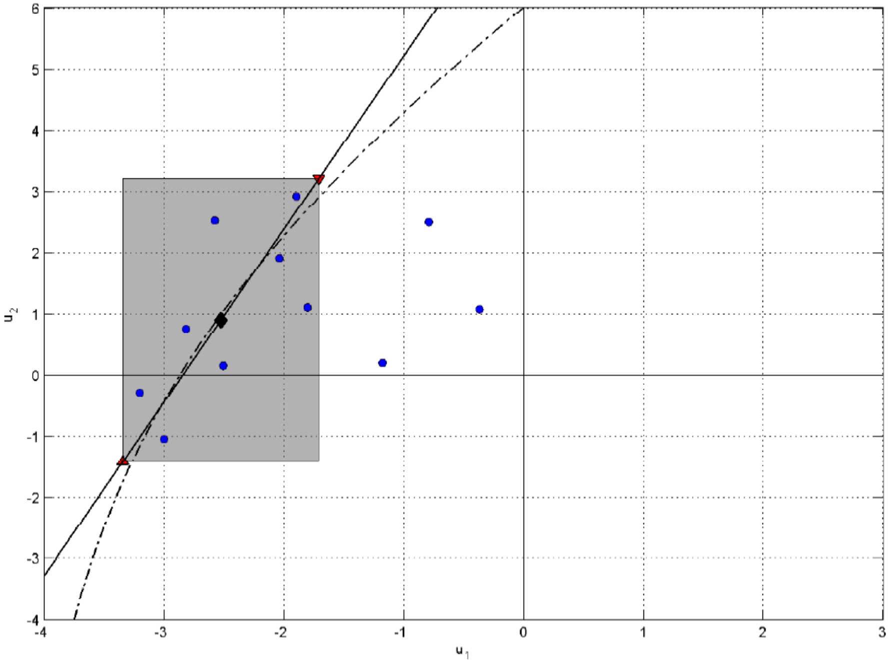


Iteration 2 – ***N* = 8**

Legend:

- sampling points, N
- Next sampling points
- Limit state function
- Response Surface
- Design Point
- Region of interest's boundaries

Example 1 - Analytical expression



Iteration 3 – ***N*** **=** 11

Iteration	Response surface	*N*	*β* (IS)
1	g(X)=b0+b1*x1+b2*x2	6	2.730
2	g(X)=b0+b1*x1+b2*x2	8	2.702
3	g(X)=b0+b1*x1+b2*x2	11	***2.679***

Cross-Validation

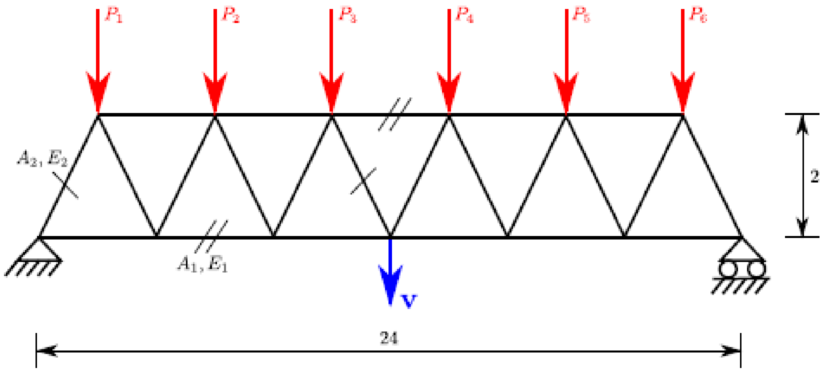
Q^2= 0.981

Bootstrapping residuals

β_{low}= 2.510

β_{up} = 2.782

Example 2 - Truss serviceability



$$G(X) = 0.14 - v(X)$$

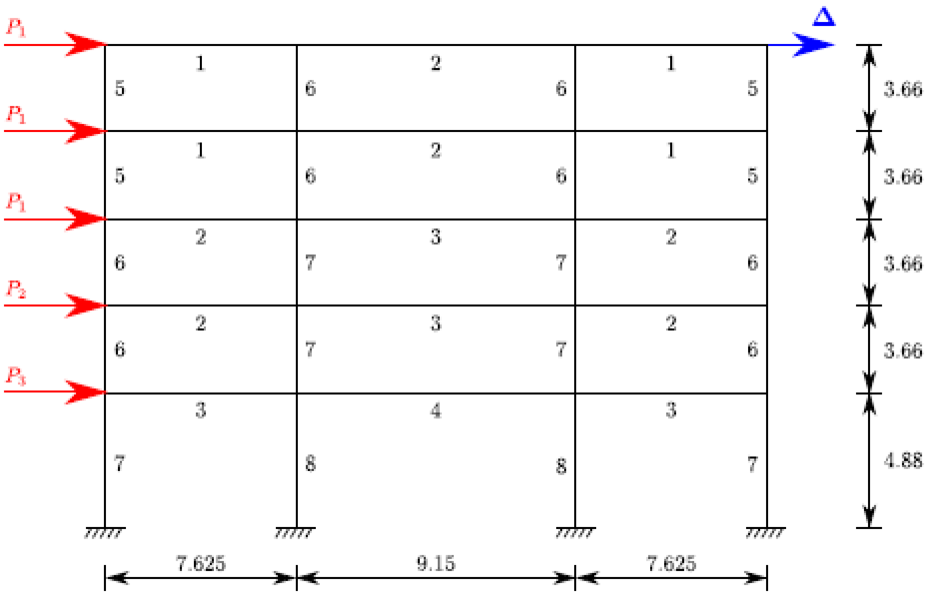
Variable	Distribution	Mean	Standard deviation
E_1, E_2 (Pa)	Lognormal	2.10×10^{11}	2.10×10^{10}
A_1 (m^2)	Lognormal	2.0×10^{-3}	2.0×10^{-4}
A_2 (m^2)	Lognormal	1.0×10^{-3}	1.0×10^{-4}
$P_1 - P_6$ (N)	Gumbel	5.0×10^4	7.5×10^3

Reference /Method	Reliability Index, β	β, Error (%)	P_f	P_f, Error (%)	*N* (LSF)
FERUM / Importance Sampling	3.990	-	3.304E-05	-	500 000
Blatman & Sudret (2010) / Full PCE	4.040	1.27%	2.673E-05	-19.31%	443
Blatman & Sudret (2010) / Sparse PCE	4.070	2.02%	2.351E-05	-29.03%	207
Roussouly et al. (2013) / Adaptive RSM	3.990	0.02%	3.304E-05	-0.25%	142
PROPOSED METHOD	***3.987***	***-0.05%***	***3.341E-05***	***0.89%***	***57***

Screening procedure: M = 6

Example 3 - Frame serviceability

$$G(X) = 0.061 - \Delta(X)$$



Cross section and moments of inertia of the same element with a coefficient $\rho_{A_i,I_i} = 0.95$;
All others geometrical properties with coefficient $\rho_{A_i,A_j} = \rho_{I_i,I_j} = \rho_{A_i,I_j} = 0.13$;

Correlation of Young's modulus is equal to $\rho_{E_1,E_2} = 0.9$;
All remaining variables have no correlation.

Elements	Young's modulus	Moment of Inertia	Cross section
1	E_1	I_5	A_5
2	E_1	I_6	A_6
3	E_1	I_7	A_7
4	E_1	I_8	A_8
5	E_2	I_1	A_1
6	E_2	I_2	A_2
7	E_2	I_3	A_3
8	E_2	I_4	A_4

Variable	Distribution	Mean	Standard deviation
P_1 (kN)	Lognormal	133.454	40.04
P_2 (kN)	Lognormal	88.97	35.59
P_3 (kN)	Lognormal	71.175	28.47
E_1 (kN/m^2)	Normal	2.1738×10^{7}	1.9152×10^{6}
E_2 (kN/m^2)	Normal	2.3796×10^{7}	1.9152×10^{6}
I_1 (m^4)	Normal	8.1344×10^{-3}	1.0834×10^{-3}
I_2 (m^4)	Normal	1.1509×10^{-2}	1.2980×10^{-3}
I_3 (m^4)	Normal	2.1375×10^{-2}	2.5961×10^{-3}
I_4 (m^4)	Normal	2.5961×10^{-2}	3.0288×10^{-3}
I_5 (m^4)	Normal	1.0812×10^{-2}	2.5961×10^{-3}
I_6 (m^4)	Normal	1.4105×10^{-2}	3.4615×10^{-3}
I_7 (m^4)	Normal	2.3279×10^{-2}	5.6249×10^{-3}
I_8 (m^4)	Normal	2.5961×10^{-2}	6.4902×10^{-3}
A_1 (m^4)	Normal	3.1256×10^{-1}	5.5815×10^{-2}
A_2 (m^4)	Normal	3.7210×10^{-1}	7.4420×10^{-2}
A_3 (m^4)	Normal	5.0606×10^{-1}	9.3025×10^{-2}
A_4 (m^4)	Normal	5.5815×10^{-1}	1.1163×10^{-1}
A_5 (m^4)	Normal	2.5302×10^{-1}	9.3025×10^{-2}
A_6 (m^4)	Normal	2.9117×10^{-1}	1.0232×10^{-1}
A_7 (m^4)	Normal	3.7303×10^{-1}	1.2093×10^{-1}
A_8 (m^4)	Normal	4.1860×10^{-1}	1.9537×10^{-1}

Example 3 - Frame serviceability

Reference /Method	Reliability Index, β	β, Error (%)	Pf	Pf,Error (%)	Neval (LSF)
FERUM / Importance Sampling	3.722	-	9.867E-05	-	500 000
Blatman & Sudret (2010) / Full PCE	3.600	-3.29%	1.591E-04	61.25%	443
Blatman & Sudret (2010) / Sparse PCE	3.610	-3.02%	1.531E-04	55.16%	207
Roussouly et al. (2013) / Adaptive RSM	3.630	-2.48%	1.417E-04	43.62%	142
PROPOSED METHOD	***3.726***	***0.10%***	***9.727E-05***	***-1.42%***	***103***

Screening procedure: M = 7

Cross-Validation (10-time 10-fold validation)

$Q^2 = 0.988$

Bootstrapping residuals

$\beta_{low} = 3.651$
$\beta_{up} = 3.797$

Robustness assessment

… Proposed measures

Most complete measure

	Frangopol and Curley (1987) Fu and Frangopol (1990)	Lind (1995)	Ghosn and Moses (1998)	ISO (2007)	Starossek (2008)	Baker et al. (2008)	Biondini and Restelli (2008)	Cavaco (2013)
Nature	Probabilistic	Probabilistic	Probabilistic	Deterministic	Deterministic	Risk-based	Deterministic	Det. or Prob.
Atribute	Redundancy Reliability	Vulnerability Damage Tolerance	Redundancy	Performance indicator	Stiffness-based Damage-based Energy released	Robustness index	Performance indicator	Performance indicator
Range	[0,∞] ← [0,∞] →	[1,α] ← [α^{-1},1] →	Target Reliabilities verification	[0,1] →	- [0,1] → -	[0,1] →	[0,1] →	[0,1] →
Scenario	Damaged vs Intact	Damaged vs Intact	Limit states	Damaged vs Intact	Damaged vs Intact	Multi hazard	Damaged vs Intact	Spectrum of Damage States

→ Increasing robustness

Robustness assessment

… Existing approaches' cons

- **Ghosn and Moses (1998, 2001, 2010)**
  - Deterministic reserve capacity factors
  - Redundancy factor to assess overall system safety

- **Baker et al. (2008)**
  - Quantification of consequences

- **Cavaco (2013)**
  - Deterministic approach when dealing with damage states' spectrum
  - Reliability must be treated with simplifications

Robustness assessment

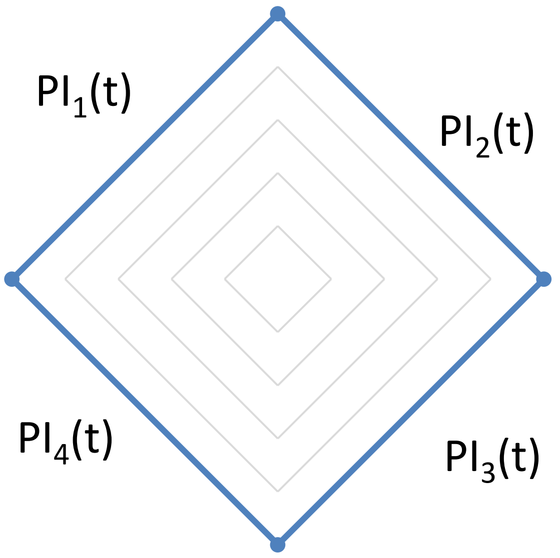
… Tentative framework

- **Main objective**
  - *Facilitate application* by practitioners to help decision making
  - *Normalization* from 0 (null) to 1 (full robustness)
  - *Combination* of existing knowledge
  - Application at two performance levels: *ultimate and service states*
  - Extension to *life-cycle performance*

Robustness assessment

… Tentative framework

Robustness is computed as equal to the area of a quadrilateral, whose sides' lengths represent a performance indicator (PI).

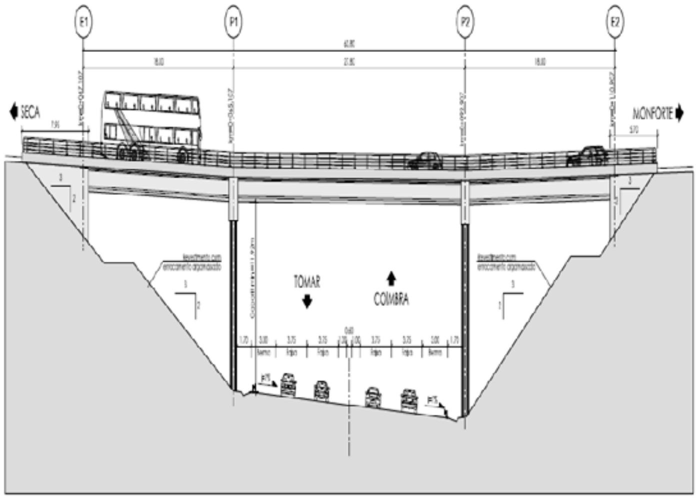


- PIs can be time dependent

- PIs must be also normalized from 0 to 1.

- PIs can be weighted according to their importance.

Case Study – Highway overpass (PS8)



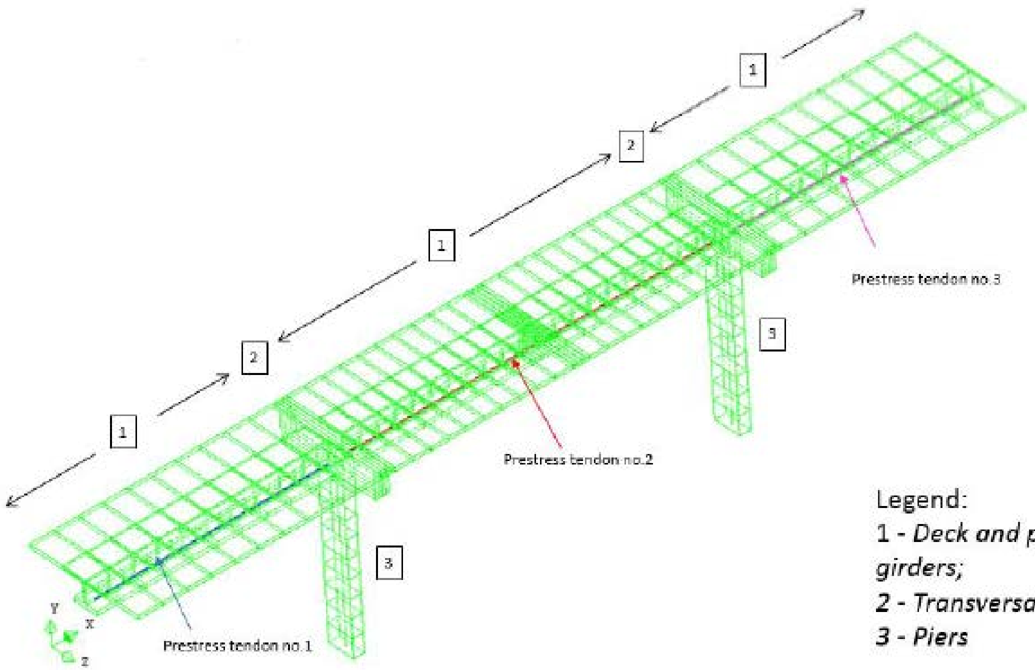
Bridge Deck:

- 8.9 m wide
- Two traffic lines (2.75m) and two 1.20m wide sideways

Geometry:

- Three-span prestressed concrete continuous rigid-frame with elastomer bearings at the abutments
- Cross-section: three precast and prestressed I-shaped concrete girders
- Precast concrete panels and cast-in place topping

Case Study – Highway overpass (PS8)



Legend:
1 - *Deck and precast I-shape girders;*
2 - Transversal Beams
3 - Piers

2D non-linear finite element analysis
- Class III beam elements based on Mindlin-Reissner theory
- Concrete behaviour - total strain fixed crack model
  - Parabolic diagram in compression
  - Linear tension with softening
- Steel tri-linear diagram

Conclusions

- **Structural Reliability Analysis**
  - ✓ An improved Adaptive Response Surface Methodology is described and applied to numerical examples found in the literature;
  - ✓ Promising results suggest that this methodology can be applied when no explicit functions are available.
  - ✓ Dealing with different failure modes must be treated carefully and properly separated at the construction of Response Surfaces.

- **Robustness Assessment**
  - ✓ A reliability-based robustness assessment framework to evaluate bridge's safety is briefly introduced;
  - ✓ The main goal is to facilitate the understanding of some attributes regarding robustness, aiming to propose a versatile framework to evaluate robustness according to a choice of key performance indicators;
  - ✓ The methodology seeks not only to obtain a normalized robustness index but also to visualize the influence of different attributes/ hazards;

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