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Direct Displacement Based Design of a RC Frame – Case of Study

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1. Introduction

Over the last years, as the importance of displacements, rather than forces, has become better appreciated, a growing interested appeared for methods based on displacements, in particular for what regards RC structures. Several contributions were made towards the development of Displacement Based Design (DBD) approaches, but it was only in the 1990's that formal proposals were made to implement the emerging ideas into formalized design procedures. One of these new design procedures is the Direct Displacement Based Design, which was developed on the base of Priestley works [1, 2]. The central idea of the Direct Displacement Based Design (DDBD) procedure is to design structures in order to achieve displacements corresponding to a given seismic hazard level.

The objective of this study is to apply the Direct Displacement Based Design to a simple case of study, a reinforced concrete frame building and to assess the applicability of the method and the needed of develop an automatic design tool [3].

2. Direct Displacement Based Design Method for Reinforced Concrete Frames

The step by step DDBD procedure is listed in the following:

Step 1: Definition of the target displacement shape and amplitude of the MDOF structure on the base of performance level considerations (material strain or drift limits) and then derive from there the design displacement Δ_d of the substitute SDOF structure of the MDOF.

Fig.1 presents a simplified model of a multi-storey frame building, where is shown the required variables in DDBD procedure.

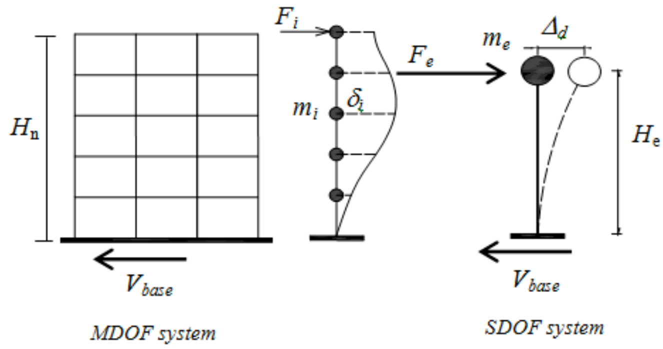


Figure 1: Simplified model of a multi-storey building

where, δ_i is the normalised inelastic mode shape, m_i are the masses at each significant storey i, m_e is the equivalent mass of the SDOF, H_i is the height of each storey, H_n are the total height of the building, H_e is the equivalent height and Δ_d is the equivalent SDOF design displacement.

- Displacement Shape

The normalised inelastic mode shape δ_i of the frame MDOF structure is defined in Ref. [2] and should be obtained according to the number of stories, *n*, as:

$$for\ n \le 4 : \delta_i = \frac{H_i}{H_n} \tag{1}$$

$$for\ n > 4 : \delta_i = \frac{4}{3} \cdot \left(\frac{H_i}{H_n} \right) \cdot \left(1 - \frac{H_i}{4H_n} \right) \tag{2}$$

The design storey displacements Δ_i are found using the shape vector δ_i, defined from Eq. (1) or Eq. (2), scaled with respect to the critical storey displacement Δ_c and to the corresponding mode shape at the critical storey level δ_c. According to Ref. [2], the design storey displacements for frame buildings will normally be governed by drift limits in the lower storey of the building (i.e. in general Δ_c=Δ_1 and δ_c=δ_1). Knowing the displacement of the critical storey (Δ_c) and the critical normalised inelastic mode shape (δ_c), the design storey displacements of the individual masses are obtained from:

$$\Delta_i = \omega_\theta \cdot \delta_i \cdot \left(\frac{\Delta_c}{\delta_c} \right) \tag{3}$$

where, ω_θ is a drift reduction factor to take into account the higher mode effects and is given by, $\omega_\theta = 1.15 - 0.0034 H_n \le 1.0$ (H_n in m, see Fig.1).

- Design Displacement of the equivalent SDOF structure

The equivalent design displacement can be evaluated as:

$$\Delta_d = \sum_{i=1}^{n} \left(m_i \Delta_i^2 \right) / \sum_{i=1}^{n} \left(m_i \Delta_i \right) \tag{4}$$

- Equivalent Mass of the SDOF structure

The mass of the substitute structure is given by the following equation:

$$m_e = \sum_{i=1}^{n} m_i \left(\frac{\Delta_i}{\Delta_d} \right) = \frac{\sum_{i=1}^{n} m_i \Delta_i}{\Delta_d} \tag{5}$$

- Equivalent Height of the SDOF structure

The equivalent height (see Fig.1) of the SDOF substitute structure is given by:

$$H_e = \sum_{i=1}^{n} \left(m_i \Delta_i H_i \right) / \sum_{i=1}^{n} \left(m_i \Delta_i \right) \tag{6}$$

Step 2: Estimation of the level of equivalent viscous damping ξ. To obtain the equivalent viscous damping the displacement ductility μ must be known. The displacement ductility is the ratio between the equivalent design displacement and the equivalent yield displacement Δ_y (see Fig.2). The equivalent yield displacement is estimated according to the considered properties of the structural elements, for example through the use of approximated equations proposed in Ref. [2], and based on the yield curvature.

- Displacement ductility of the SDOF structure

The SDOF design displacement ductility (see Fig.2) is given by Eq. (7) and is related to the equivalent yield displacement Δ_y:

$$\mu = \frac{\Delta_d}{\Delta_y} \tag{7}$$

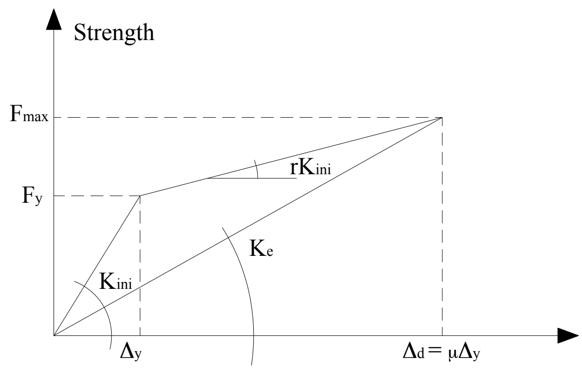


Figure 2: Constitutive law of the equivalent SDOF system

- Equivalent yield displacement

The equivalent yield displacement is given by the following equation:

$$\Delta_y = \theta_y H_e \tag{8}$$

where θ_y is the yield drift and for reinforced concrete frames is given by:

$$\theta_y = 0.5\varepsilon_y L_{j-1} / h_b \tag{9}$$

where, ε_y is the yield strain of steel, L_{j-1} is the beam length and h_b is the beam section depth.

- Equivalent viscous damping

To take into account the inelastic behaviour of the real structure, hysteretic damping (ξ_{hyst}) is combined with elastic damping (ξ_0). Usually, for reinforced concrete structures the elastic damping is taken equal to 0.05, related to critical damping. The equivalent viscous damping of the substitute structure for frames could be defined according to Ref. [2] by the following equation:

$$\xi = \xi_0 + 0.565\left(\frac{\mu - 1}{\mu\pi}\right) \qquad (10)$$

Step 3: Determination of the effective period T_e of the SDOF structure. The effective period of the SDOF structure at peak displacement response is found from the design displacement spectrum for the equivalent viscous damping ξ, i.e. entering the design displacement of the substitute SDOF structure Δ_d and determining the effective period T_e (see Fig.3).

The displacement spectra for other different levels of ξ than 5% can be found from the formulation defined in Eurocode 8 [4], as:

$$S_{D,\xi} = S_{D,5\%}\left(\frac{10}{5+\xi}\right)^{1/2} \qquad (11)$$

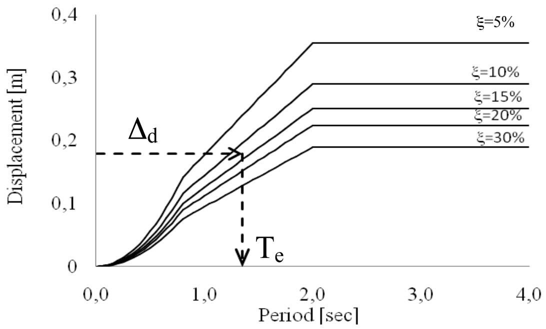


Figure: 3 Design Displacement Spectrum

Step 4: Derivation of the effective stiffness k_e of the substitute structure from its effective mass and effective period. Then it is possible to obtain the design base shear as the product of the effective stiffness by the design displacement of the substitute SDOF structure.

- Effective stiffness of the substitute SDOF structure

$$k_e = \frac{4\pi^2 m_e}{T_e^2} \qquad (12)$$

- Design base shear force

$$V_{base} = k_e \Delta_d \qquad (13)$$

- *P-Δ* effects in Direct Displacement-Based Design

As suggested in Ref.[2], for reinforced concrete structures *P-Δ* effects should be considered if the stability index θ_Δ is greater than 0.10, with a maximum value of 0.33. The stability index compares the magnitude of the *P-Δ* effect at expected maximum displacement (Δ_{max}) to the design base moment capacity of the structure (M_D). The structural stability index is given by:

$$\theta_\Delta = \frac{P\Delta_{\max}}{M_D} \tag{14}$$

Substituting in Eq. (14) $M_D = OTM$ and $\Delta_{max}=\Delta_d$, where *OTM* is the overturning moment at the base given by Eq. (19) and Δ_d is the design displacement of the substitute SDOF structure. *P* is the axial force due to gravity loads.

The design base shear force V_{base} to take into account the *P-Δ* effects is given by:

$$V_{base} = k_e\Delta_d + C\frac{P\Delta_d}{H_e} \tag{15}$$

where, k_e is the effective stiffness and H_e is the equivalent height of the SDOF substitute structure. The *C* parameter shall be taken as 0.5 for reinforced concrete buildings.

Therefore, the required base moment capacity is:

$$M_B = K_e\Delta_d H_e + CP\Delta_d \tag{16}$$

After the determination/actualization of the design base shear force, this is distributed between the mass elements of the MDOF structure as inertia forces.

Step 5: Distribution of the design base shear force V_{base} to the locations of storey mass of the building (MDOF structure).

The design base shear force is distributed to the storey levels as:

$$for\ n < 10 \qquad F_i = V_{base}\frac{(m_i\Delta_i)}{\sum_{i=1}^{n}(m_i\Delta_i)} \tag{17}$$

$$for\ n \geq 10 \qquad F_i = F_t + 0.9V_{base}\frac{(m_i\Delta_i)}{\sum_{i=1}^{n}(m_i\Delta_i)} \tag{18}$$

where, $F_t = 0.1V_{base}$ at roof level, and $F_t = 0$ at all other storey levels.

Step 6: Evaluation of design moments at potential hinge locations. To this purpose the method of analysis used is a simplified method based on equilibrium considerations (statically admissible distribution of internal forces).

Beam Moments

The lateral seismic forces F_i obtained with Eq. (17) or Eq. (18) produce in each of the columns axial forces (compression or tension) and column-base moments (M_{cj}). The seismic axial forces induced in each of the columns (*T* for tension or *C* for compression) by the seismic beams shears are the sum of seismic beam shears in each vertical alignment ($\sum V_{Bi}$). In Fig.4 is shown a typical distribution of seismic lateral forces F_i and the corresponding internal forces induced in a frame building. Considering the equilibrium at base level, the total overturning moment is given by:

$$OTM = \sum_{i=1}^{n} F_i H_i \tag{19}$$

Knowing that equilibrium should be assured between internal and external forces, the total overturning moment at the base of the structure, hence:

$$OTM = \sum_{j=1}^{m} M_{cj} + \sum_{j=2}^{m} \left[\left(\sum_{i=1}^{n} V_{Bj-1,i} \right) \times L_{j-1} \right] \tag{20}$$

where, M_{cj} are the column-base moments (*m* is the number of columns) and L_{j-1} is the length of each span.

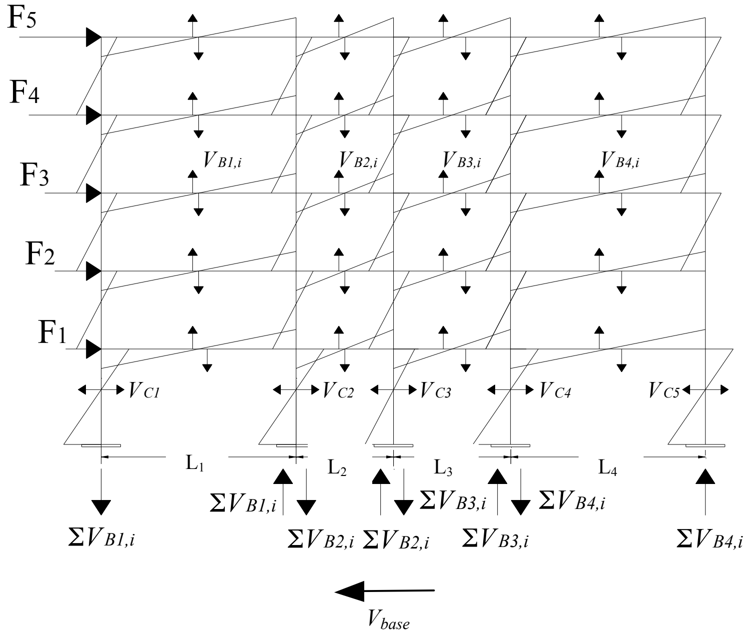


Figure 4: Seismic Moments from DDBD [adapted from Ref. [2]]

From Fig.4 and considering only a parcel of *OTM* regarding the seismic axial forces (*OTM**), the corresponding overturning moment is given by:

$$OTM^* = \sum_{j=2}^{m} \left(\sum_{i=1}^{n} V_{Bj-1,i} \right) \times L_{j-1} = \sum_{i=1}^{n} V_{B1,i} \times L_1 + \sum_{i=1}^{n} V_{B2,i} \times L_2 + \sum_{i=1}^{n} V_{B3,i} \times L_3 + \sum_{i=1}^{n} V_{B4,i} \times L_4 \tag{21}$$

where, $V_{B,1i}$, $V_{B2,i}$, $V_{B3,i}$ and $V_{B4,i}$ are the seismic beam shears at level i for bay 1 to 4, respectively. The seismic beam shears for each span is constant, thus, $V_{Bj-1,i} = 2M_{Bj-1,i}/L_{j-1}$, where $M_{Bj-1,i}$ is beam moment of each span at the storey i.

Replacing the seismic beam shears in Eq. (21), the overturning moment OTM^* will be:

$$OTM^* = 2\sum_{j=2}^{m}\left(\sum_{i=1}^{n} \mathrm{M}_{\mathrm{Bj\text{-}1,i}}\right) \tag{22}$$

According to the example of Fig.4, OTM* is thus:

$$OTM^* = 2\sum_{i=1}^{n}\left(M_{B1,i} + M_{B2,i} + M_{B3,i} + M_{B4,i}\right) \tag{23}$$

Considering a relationship between beam moments as $M_{B2,i} = \alpha M_{B1,i}$, $M_{B3,i} = \beta M_{B1,i}$ and $M_{B4,i} = \chi M_{B1,i}$ and replacing in turn in Eq. (23), the beam moments corresponding to the first span L_1 are given by:

$$\sum_{i=1}^{n} M_{B1,i} = \frac{OTM^*}{2(1+\alpha+\beta+\chi)} \tag{24}$$

If α, β and χ are replaced in Eq. (24) and then $\sum_{i=1}^{n} M_{B1,i}$, the seismic beam shears for the first span is:

$$\sum_{i=1}^{n} V_{B1,i} = \frac{2\sum_{i=1}^{n} M_{B1,i}}{L_1} = \frac{OTM^*}{L_1}\left(\frac{\sum_{i=1}^{n} M_{B1,i}}{\sum_{i=1}^{n} M_{B1,i} + \sum_{i=1}^{n} M_{B2,i} + \sum_{i=1}^{n} M_{B3,i} + \sum_{i=1}^{n} M_{B4,i}}\right) \tag{25}$$

Therefore, for each span the seismic beam shears due to OTM* are given by:

$$\sum_{i=1}^{n} V_{Bj-1,i} = \frac{\sum_{i=1}^{n} M_{Bj-1,i}}{\sum_{j=2}^{m}\sum_{i=1}^{n} M_{Bj-1,i}} \frac{OTM^*}{L_{j-1}} \tag{26}$$

Combining Eq. (19) and Eq. (20) and replacing the parcel of seismic axial forces due to OTM^* given by Eq. (26), the total sum of seismic axial forces is defined as:

$$\sum_{i=1}^{n} V_{Bj-1,i} = \frac{\sum_{i=1}^{n} M_{Bj-1,i}}{\sum_{j=2}^{m}\sum_{i=1}^{n} M_{Bj-1,i}}\left(\sum_{i=1}^{n} F_i H_i - \sum_{j=1}^{m} M_{cj}\right) / L_{j-1} \tag{27}$$

All distribution of the total required beam shear that assures Eq. (27) will result in a statically admissible equilibrium solution and can be chosen on the base of engineering judgment. However, in Ref. [2] it is suggested that the distribution of the total beam shear force could be done in proportion to the storey shears in the level below the beam under consideration as depicted in Fig.5.

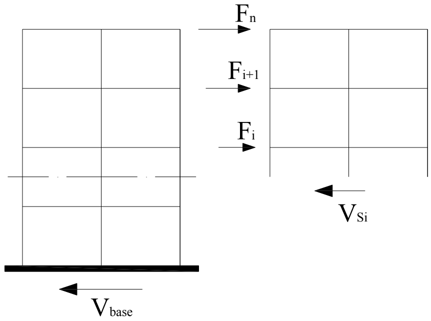


Figure 5: Storey shear forces

The distribution of the total beam shear force is thus:

$$V_{Bj-1,i} = \sum_{i=1}^{n} V_{Bj-1,i} \cdot \frac{V_{S,i}}{\sum_{i=1}^{n} V_{S,i}} \tag{28}$$

where the storey shear forces at level *i*, $V_{S,i}$ are given by:

$$V_{S,i} = \sum_{k=i}^{n} F_k \tag{29}$$

After the individual beam shear forces have been calculated, the beam design moments at the column centrelines are obtained by the following equation:

$$M_{Blj-1,i} + M_{Brj-1,i} = V_{Bj-1,i} \cdot L_{j-1} \tag{30}$$

where, $M_{Bl\,j-1,i}$ and $M_{Br\,j-1,i}$ are the beam moments at the column centrelines at the left and right end of the beam, respectively.

Column Moments

Knowing that structural analysis based on equilibrium considerations is actually an approximation of the real distribution, the designer gets some freedom in choosing distribution of the total storey shear force between the columns and the design moment at the column-base of first storey, provided the equilibrium is maintained between internal and external forces.

The total storey shear force given by Eq. (29) is shared between the columns. This could be done according to the following ratio: 1 for external columns and 2 for internal columns, as suggested in Ref. [2]; from the shear forces at the base of each column V_C, it is then possible to obtain the column-base moments at the base and top of the columns between the ground storey and 1st storey.

According to Ref. [2], for one-way frames the contra-flexure point for the 1st storey columns-base moment $M_{C01,b}$ could be considered around 60% of the height of the column H_1 (see Fig.6). Therefore, the column-base moments at the bottom and top of the 1st storey are given by:

$$M_{C01,b} = 0.6 V_{C01} \cdot H_1 \tag{31}$$

$$M_{C01,t} = 0.4V_{C01} \cdot H_1 \tag{32}$$

Once known the column-base moments of the first storey and the beam moments at each it is then possible to obtain the column moments distribution in height, considering the equilibrium from the 1st storey nodes and successively until the top level is reached, as illustrated in Fig.6.

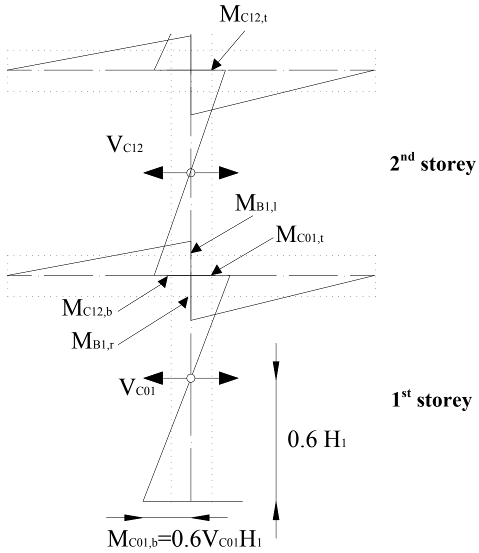


Figure 6: Determination of Column Moments from Considerations of Joint Equilibrium [adapted from, Ref. [2]].

Step 7: Capacity Design Requirements for Frames.

Capacity design rules must then be implemented to ensure that plastic hinges cannot develop at unintended locations and, that shear failure cannot occur for the desired mechanism (see Fig.7).

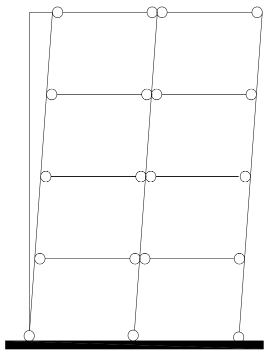


Figure 7: Beam-Sway Mechanism for moment resisting frame

For this purpose column flexural strengths at locations other than at the base or top and shear strengths at plastic hinge location must be amplified all through the structure. The relationship between design strength and basic strengths are given by the following equation:

$$\phi_S S_D \geq \phi^0 \omega_f S_E \tag{33}$$

where, S_D is the design strength defined according to the capacity design rules, ϕ_S is a strength reduction factor relating dependable and design strengths of the action (ϕ_S = 1 should be adopted for flexural design of plastic hinges and ϕ_S < 1 for other actions and locations), S_E is the basic strength, i.e. the value corresponding to the design lateral force distribution determined from the DDBD method, ϕ^0 is an overstrenght factor to account for the overcapacity at the plastic hinges and ω_f is the amplification due to higher mode effects. To apply capacity design rules an approximate method, as proposed in Ref. [2], is used.

Beam Flexure:

According to the desired inelastic mode, depicted in Fig.7, the plastic hinges should form at beam ends. For these regions the flexural design of plastic hinges is based on the larger of the moments due to factored gravity loads or corresponding to the design lateral forces from DDBD procedure (seismic moments).

For the regions between the beam plastic hinges, design moments are found from the combination of reduced gravity loads applicable for the seismic design combination, and overstrenght moment capacity at the beam hinges. Therefore, at a distance x from the left support, the total moment is given by:

$$M_x = M_{E,l}^0 + \left(M_{E,r}^0 - M_{E,l}^0\right) \times \frac{x}{L} + \frac{w_G^0 L}{2} x - \frac{w_G^0 x^2}{2} \quad (34)$$

where L is the beam span, $M^0_{E,l}$ ($=\phi^0(x)M_{Bi,l}$) and $M^0_{E,r}$ ($=\phi^0(x)M_{Bi,l}$) are the moments at left and right of column centrelines, respectively, and $w_G{}^0$ is the gravity load (dead and live) constant along the beam and amplified of 30% of seismic gravity moments are considered to account for elastic vertical response of the beam to vertical ground accelerations. Eq. (34) is defined taken into account that the beam moments cannot exceed M^0_E, the overstrenght values at the beam plastic hinges; thus the design moments are defined by adding the gravity moments for a simple supported beam to the seismic moments.

Beam Shears:

The seismic beam shears corresponding to the plastic hinges locations are constant along the beam. As recommended in Ref. [2] the design shear force along the beam, should considerer the effects of beam vertical response (combined seismic shears with reduced gravity shears applicable for seismic load combinations), therefore:

$$V_x = \frac{\left(M_{E,r}^0 - M_{E,l}^0\right)}{L_{j-1}} + \frac{w_G^0 L_{j-1}}{2} - w_G^0 x \quad (35)$$

Column Flexure:

Column end moments, other than at the base or top, and shears forces are amplified for both potential overstrenght capacity at beam plastic hinges (material strengths exceed the design values) and dynamic amplification resulting for higher mode effects, which are not considered in the structural analysis.

The required column flexural strength according to DDBD capacity design rules is given by:

$$\phi_f M_N \geq \phi^0 \omega_f M_E \tag{36}$$

where, ϕ_f is the strength reduction factor;

M_N is the design column moments;

ϕ^0 is the overstrenght factor;

ω_f is the dynamic amplification factor, defined in the following;

M_E is the column moments resulting from lateral seismic forces (see Fig.4).

The overstrenght factor ϕ^0 is the ratio of overstrenght moment capacity to required capacity of the plastic hinges, as referred previously and could be obtained by moment-curvature analysis or using a default value. The effort to obtain overstrenght factors by moment-curvature analysis maybe excessive for some structures and as suggested in Ref. [2] a default value should be considered. It is possible to adopt two values for different situations, if the design is based on a strain-hardening model for the flexural reinforcement ϕ^0 is taken as 1.25, if not, it is recommended a value of 1.60.

The dynamic moment amplification factor ω_f is height and ductility dependent, as shown in Fig.8. From the first storey until ¾ of the total height, for one-way frames $\omega_{f,c}$ is given by:

$$\omega_{f,c} = 1.15 + 0.13\left(\mu^0 - 1\right) \tag{37}$$

where, μ^0 is the reduced ductility corresponding to the average overstrenght capacity of the beam hinges.

The value at the base of the bottom storey and at the top should be taken as $\omega_{f,t} = 1.0$, where hinging at the column is acceptable, according with the desirable inelastic mode referred previously.

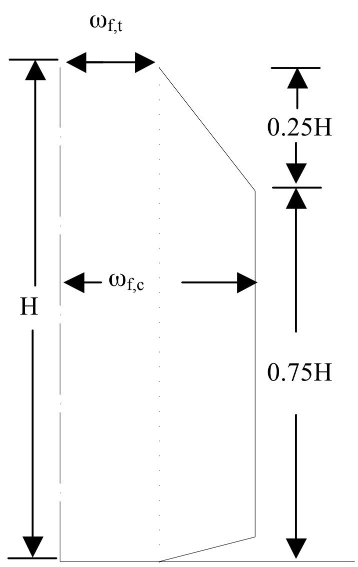


Figure 8: Dynamic amplification of frame column moments [adapted from, Ref. [2]]

Column Design Shear Forces:

According to Ref. [2] it has been stated that the dynamic amplification factor for column shear should be obtained by a constant offset of shear demand above design-force envelope with height, given by:

$$\phi_S V_N \geq \phi^0 V_E + 0.1\mu V_{E,base} \leq \frac{M^0_{Ci,t} + M^0_{Ci,b}}{H_{Ci}} \tag{38}$$

where,

V_E is the shear demands from lateral seismic forces;

$V_{E,\,base}$ is the V_E value at the base of the column;

μ is the displacement ductility;

$M_{Ci,t}^{o}$ and $M_{Ci,b}^{o}$ are the moments at the top and bottom of the column, respectively, corresponding to development of plastic hinging;

H_{ci} is the clear height of the column.

3. Case of study- Reinforced Concrete Frame

The DDBD procedure is applied [5, 6] to the interior frame of the four-storey reinforced concrete structure plotted as section A-A of Fig.9, with a global geometry (height and spans as well as beams and columns cross-section dimensions) defined in the context of the "Cooperative Research on the Seismic Response of the Reinforced Concrete Structures" [7]. The structure is irregular in terms of spans and the lateral resistance is provided by one-way frame action. The slab thickness is equal to 0.15 m. The reinforced concrete frames are made with concrete C25/30 (f_{cd} = 16.7 MPa). The reinforcement steel is a classical Tempcore steel B500 (f_y =500 MPa). In addition to the self-weight of the beams and the slab, due to floor finishing and partitions a distributed dead load of 2 kN/m^2 is considered, as well as an imposed live load with nominal value of 2 kN/m^2.

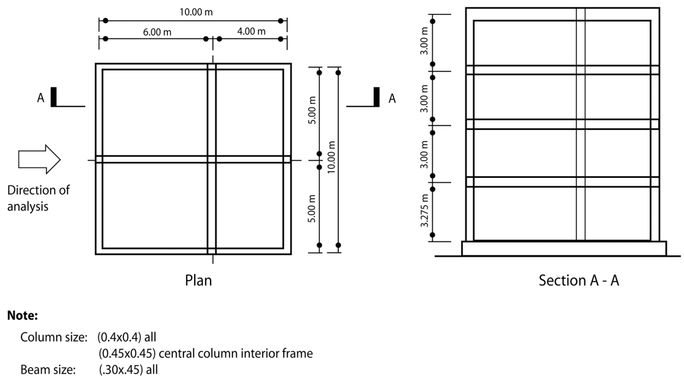


Figure 9: General Layout [adapted from, Ref. [7]]

In a first stage, the frame building is designed according to the Direct Displacement Based Design procedure considering being located in Continental Portugal (Algarve) seismic zone 1.1, as an ordinary building, class of importance II. The seismic action was defined according to Eurocode 8 [4] and Portuguese National Annex [8] with the elastic acceleration response spectrum S_a for subsoil class D. The value of peak ground acceleration a_g used in the definition of the response spectrum is 0.35g. The elastic displacement spectrum S_{De} used for DDBD, shown in Fig.10, is the one defined in Eurocode 8 by:

$$S_{De}(T) = S_a(T)\left[\frac{T}{2\pi}\right]^2 \tag{39}$$

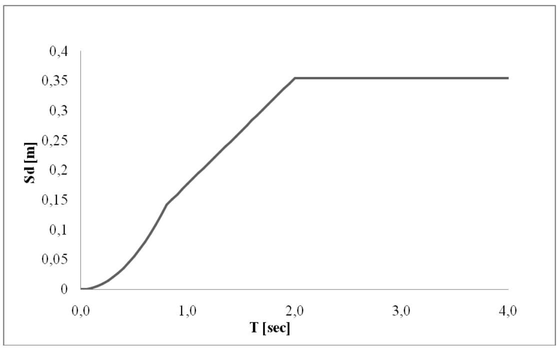


Figure 10: Design Displacement Spectrum

The second stage regards the application of the DDBD procedure when the displacement capacity exceeds the spectral demand. For this purpose the DDBD procedure was applied again to the same case of study but considering the building located in Continental Portugal (Lisbon) seismic zone 1.3, for a design ground acceleration a_g of 0.27g.

The seismic performance of the designed structure for peak ground acceleration (a_g) of 0.35g was assess by means of non-linear dynamic analyses and it is presented in Section 3.3.

3.1. Direct Displacement Based Design for a peak ground Acceleration of 0.35g

In this section is presented the frame design according to DDBD procedure and for a_g equal to 0.35g. The step-by-step procedure defined in section 2 is following in this case study.

Step 1: Definition of the design storey displacement, design displacement of the SDOF structure, equivalent mass and equivalent height

The normalised inelastic mode shape of the MDOF frame structure for this case of study is given by Eq. (1), with n=4. According to Ref. [2], for frame buildings the design displacement of the substitute SDOF structure will usually be governed by a specified drift limit in the lower storeys of the building. This shape implies that the maximum drift occurs between the ground and first storey. For design purpose and according Ref. [2] the drift limit was considered as 2.5 %. The critical design storey displacement for the first storey (H_1= 3.275 m) is thus Δ_c = $\Delta_1 = 0.025 \times 3.275 = 0.0818m$ and the critical normalised inelastic mode shape δ_c = δ_1= 0.267.

The design storey displacement profile is found from Eq. (3), reproduced herein by convenience:

$$\Delta_i = \omega_\theta \left(\frac{\Delta_c}{\delta_c} \right) = 1.0 \times \frac{0.082}{0.267} \delta_i = 0.307 \delta_i \tag{40}$$

where, ω_θ is taken as 1.0.

Table 1 presents the calculations to obtain the design displacement of the equivalent SDOF structure and equivalent height.

Table 1: Calculations to obtain design displacement of the SDOF structure

Storey, i	Height, H_i (m)	Mass, m_i (ton)	δ_i	Δ_i (m)	$m_i \Delta_i$	$m_i \Delta_i^2$	$m_i \Delta_i H_i$
4	12.275	46.59	1.00	0.307	14.30	4.39	175.51
3	9.275	46.59	0.76	0.232	10.80	2.51	100.21
2	6.275	46.59	0.51	0.157	7.31	1.15	45.87
1	3.275	46.95	0.27	0.082	3.84	0.31	12.59
Σ					36.26	8.35	334.18

In Fig.11 is depicted the design storey displacements profile according to the selected target drift limit, where the top target displacement Δ_{target} (roof displacement) is equal to 0.307m.

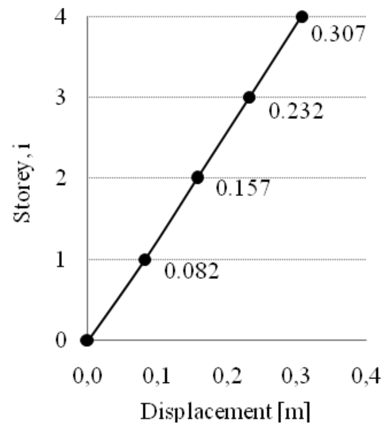


Figure 11: Design storey displacements

From Eq. (4) to Eq. (6) and from the values presented in Table 1 it is possible to derive the design displacement Δ_d, the equivalent mass m_e and equivalent height H_e of the SDOF structure. Therefore, the design displacement of the SDOF structure is 0.23m, the equivalent mass is 157.34 tonne and the equivalent height is 9.217m (75.1% of building height).

Step 2: Estimation of the level of equivalent viscous damping

The design displacement ductility is given by Eq. (7), reproduced herein by convenience:

$$\mu = \frac{\Delta_d}{\Delta_y} \tag{41}$$

The equivalent yield displacement is the product between the yield rotation (see Eq. (9)) and the equivalent height of the SDOF structure. In this case of study, with beam depths for spans 1 and 2 h_{b1} = h_{b2} = 450 mm, the yield rotation θ_y is given by:

$$\theta_{y1,i} = 0.5 \times \varepsilon_y \frac{L_1}{h_{b1,i}} \tag{42}$$

$$\theta_{y2,i} = 0.5 \times \varepsilon_y \frac{L_{b2,i}}{h_{b2,i}} \tag{43}$$

The yield strain is thus:

$$\varepsilon_y = f_{ye} / E_s = 1.1 \times 500/200000 = 0.00275 \tag{44}$$

where the design yield strength of steel is $f_{ye} = 1.1\, f_y$, according to the recommendations in Ref. [2] for design material strengths for plastic hinge regions.

The equivalent yield displacement is given by:

$$\Delta_y = \frac{M_{1,i}\theta_{y1,i} + M_{2,i}\theta_{y2,i}}{M_{1,i} + M_{2,i}} \cdot H_e = \frac{0.018 + 0.012}{2} \times 9.217 = 0.141\, m \tag{45}$$

The $M_{1,i}$ and $M_{2,i}$ are the contribution from both bays, and the considered relationship between them is $M_{1,i}/ M_{2,i} = 1$.

Replacing in Eq. (41) the design displacement of the SDOF structure and the equivalent yield displacement, the SDOF system design displacement ductility is μ=1.64.

The equivalent viscous damping of the SDOF structure was obtained by Eq. (10), reproduced herein by convenience:

$$\xi = 0.05 + 0.565\left(\frac{\mu - 1}{\mu\pi}\right) = 11.99\% \tag{46}$$

Step 3: Determination of the effective period

The effective period at peak displacement response is found from the design displacement spectrum defined for the equivalent viscous damping of ξ=11.99% through Eq. (11) and Eq. (39), entering the design displacement of the equivalent SDOF structure Δ_d and determining the effective period T_e (see Fig.12).

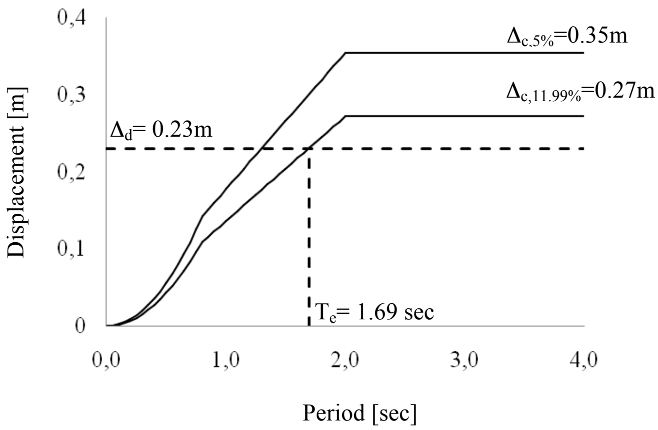


Figure 12: Design Displacement Spectrum

The effective period of the SDOF structure is T_e = 1.69sec.

Step 4: Derivation of the effective stiffness and design base shear force

Knowing the effective period it is possible to derive the effective stiffness and the design base shear force of the SDOF structure from Eq. (12) and Eq. (13), respectively. The effective stiffness of the SDOF structure is $k_e = 2164.30 kN/m$ and the design base shear force is $V_{base} = 498.70 kN$.

Table 2 presents a summary of the results obtained previously.

Table 2: Results of DDBD in terms of displacement, equivalent yield displacement, ductility, effective mass, effective period and design base shear force

Δ_{target} (m)	Δ_d (m)	Δ_y (m)	μ	m_e (tonne)	ξ (%)	T_e (s)	V_{base} (kN)
0.307	0.230	0.141	1.64	157.34	11.99	1.69	498.70

Step 5: Distribution of the design base shear force

The next step of the DDBD procedure involves the distribution of the design base shear force obtained for the SDOF structure in the real structure (MDOF structure), in a variation of the equivalent lateral force-based. The distribution of the design base shear through the real structure was obtained by Eq. (17) and the values presented in Table 3.

Step 6: Design actions for MDOF Structure

The real structure is then analyzed under these forces (defined in step 5) and then the design moments are obtained.

Beam Moments

Table 3 shows the calculations to obtain the distribution of the design base shear through the real structures, the value of column shear forces in each alignment (shared between the exterior and interior columns in proportion 1:2 as suggested in Ref. [2]). Storey shear forces V_{Si} obtained from Eq. (29) are defined by summing the storey shear forces above the storey (see Fig.5). The last column of Table 3 presents the overturning moment *OTM* given by Eq. (20).

Table 3: Calculations

Storey, i	Height, H_i (m)	$m_i\Delta_i$	F_i (kN)	$V_{Ci,1}$ Col 1 (kN)	$V_{Ci,2}$ Col 2 (kN)	$V_{Ci,3}$ Col 3 (kN)	$V_{S,i}$ (kN)	*OTM* (kNm)
4	12.275	14.30	196.69	49.17	98.34	49.17	196.69	0
3	9.275	10.80	148.62	37.15	74.31	37.15	345.30	590.06
2	6.275	7.31	100.55	25.14	50.27	25.14	445.85	1625.96
1	3.275	3.84	52.88	13.22	26.44	13.22	498.73	2963.49
0	0			0.00	0.00	0.00		4596.82
Sum		36.26	498.73	124.68	249.36	124.68	1486.56	

- *P-Δ* effects

According to Eq. (14) the stability index θ_Δ for this example is 0.094, therefore there is no need to consider *P-Δ* effects, because θ_Δ< 0.10.

Thus, the value of the design base shear force V_{base} to use in DDBD procedure is 498.73kN and the values presented in Table 3 are used in further calculations.

Based on Eq. (31) the total resisting moment provided at the column base is thus:

$$\sum M_{cj} = 0.6 H_1 V_{base} = 498.7 \times 0.6 \times 3.275 = 980.0 kNm \quad (\approx 21.3\% \text{ OTM}) \tag{47}$$

According to Eq. (27), beam seismic shears corresponding to design lateral forces, admitting a relationship between beam moments $M_{B1,i}/M_{B2,i}$=1, for span 1 and 2, respectively, are given by:

$$\sum_{i=1}^{n} V_{B1,i} = \frac{1}{2}(4596.8 - 980.0)/6 = 301.4kN \tag{48}$$

$$\sum_{i=1}^{n} V_{B2,i} = \frac{1}{2}(4596.8 - 980.0)/4 = 452.1kN \tag{49}$$

These forces are distributed to the beams in proportion to the storey shears directly below the beams considered according to Eq. (28) and Eq. (29).

$$V_{B1,i} = 301.4 \times V_{S,i} / 1486.56 = 0.203 V_{S,i} \tag{50}$$

$$V_{B2,i} = 452.1 \times V_{S,i} / 1486.56 = 0.304 V_{S,i} \tag{51}$$

The resulting seismic beam shears for each span is presented in Table 4.

Table 4: Calculations for seismic beam shears

Storey, i	$V_{B1,i}$ (kN)	$V_{B2,i}$ (kN)
4	39,90	59,80
3	70,00	105,00
2	90,40	135,60
1	101,10	151,70

The beam design moments at the column centrelines are given by Eq. (30) and at column faces by:

$$M_{Bj-1,i} = V_{Bj-1,i}(L_{j-1} - h_c)/2 \tag{52}$$

In Tables 5 and 6 are presented the values of the seismic design beam moments at the centreline and at the column face, respectively.

Table 5: Beam seismic moments at the centreline (ignoring gravity loads)

	Span 1 (L_1=6m)		Span 2 (L_2=4m)	
Storey, i	$M_{B1,i.l}$ (kNm)	$M_{B1,i,r}$ (kNm)	$M_{B2,i,l}$ (kNm)	$M_{B2,i,r}$ (kNm)
4	119.63	119.63	119.63	119.63
3	210.03	210.03	210.03	210.03
2	271.19	271.19	271.19	271.19
1	303.35	303.35	303.35	303.35

Table 6: Beam seismic moments at the face of the column (ignoring gravity loads)

	Span 1 (L_1=6m)		Span 2 (L_2=4m)	
Storey, i	$M_{B1,i.l}$ (kNm)	$M_{B1,i,r}$ (kNm)	$M_{B2,i,l}$ (kNm)	$M_{B2,i,r}$ (kNm)
4	111.66	-110.66	106.18	-107.67
3	196.03	-194.28	186.40	-189.03
2	253.11	-250.85	240.68	-244.07
1	283.13	-280.60	269.23	-273.02

According to Ref. [2] the flexural design of the beam plastic hinges is based on moments due to factored gravity loads or seismic moments corresponding to the design lateral forces (seismic case). Both values should be compared and the larger should be adopted for the design. Therefore, it is presented the calculations for these two cases. The factored gravity moments were obtained considered three load cases: 1) the dead and live loads applied to both spans at the same time, 2) and 3) considering alternate live loads acting in the spans. Fig.13 shows the larger beam moment from these three combinations.

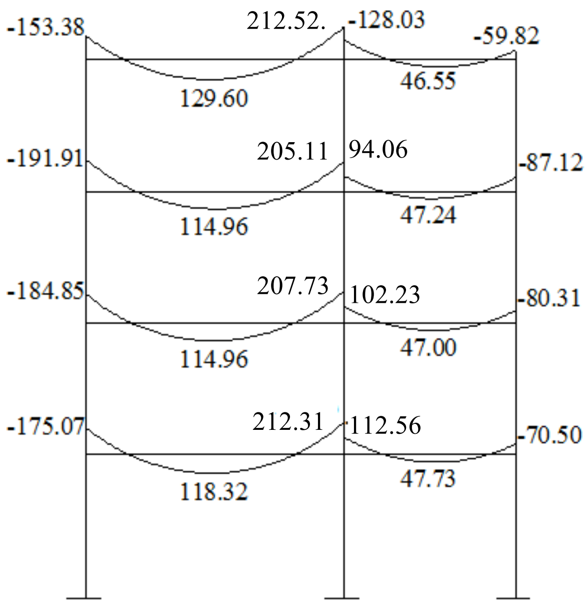


Figure 13: Beam moment distribution due to factored gravity loads [units in kNm]

In Table 7 is presented the larger beam factored gravity moments for the three combinations.

Table 7: Beam moments due to factored-gravity loads [units in kNm]

Storey, *i*	Span 1			Span 2		
	left end	mid span	right end	left end	mid span	right end
4	153.38	129.57	212.52	128.03	46.55	59.82
3	191.91	114.96	205.11	94.06	47.24	87.12
2	184.85	114.96	207.73	102.23	47.00	80.31
1	175.07	118.32	212.31	112.56	46.73	70.50

The beam ends (plastic hinge locations) should be designed for the larger of the moments presented in Table 6 and 7, respectively. In Table 8 is shown the design beam moments for plastic hinges locations; the moments bold marked is due to factored gravity loads.

Table 8: Design beam moments [units in kNm]

Storey, *i*	Span 1		Span 2	
	left end	right end	left end	right end
4	**153.38**	**212,52**	**128.03**	-107.67
3	196.03	**205.11**	186.40	-189.03
2	253.11	250.85	240.68	-244.07
1	283.13	280.60	269.23	-273.02

Column moments

The column moments presented in Table 9 corresponds to the design lateral forces and they were obtained by equilibrium considerations as described previously in section 2.

Table 9: Column moments [units in kNm]

Storey, *i*		Col 1	Col 2	Col 3
4	Top	119.63	239.30	119.63
	bottom	-27.90	-55.80	-27.90
3	Top	182.15	364.30	182.15
	bottom	-76.82	-153.64	-76.82
2	Top	194.40	388.72	194.40
	bottom	-140.02	-280.03	-140.02
1	Top	163.33	326.70	163.33
	bottom	-245.00	-490.00	-245.00

Step 7: Capacity design requirements for frames

In the following it is presented the application of the capacity design rules.

Beam Flexure

In DDBD procedure recommendations [2] the material design strengths for design locations of intended plastic hinges, for concrete and reinforcement should be f'_{ce}=1.3f'_c and f_{ye} =1.1f_y, respectively. Where, f'_c is the specified (28 days) concrete compression strength, f'_{ce} is the expected compression strength of DDBD, f'_y is the specified minimum characteristic yield strength of steel and f_{ye} is the expected yield strength of steel for DDBD.

Thus, for a concrete C25/30 and reinforcement steel B500 the following values will apply:

- Concrete compression

$$f'_{ce} = 1.3 \times 25 = 32.5MPa \tag{53}$$

$$f_{cd} = \frac{f'_{ce}}{\gamma_c} = 21.7MPa \qquad (\gamma_c = 1.5\,[10]) \tag{54}$$

$$f_{ctm} = 0.30 f'_{ce}{}^{(2/3)} = 3MPa \tag{55}$$

- Steel reinforcement

$$f_{ye} = 1.1 \times 500 = 550MPa \tag{56}$$

$$f_{yd} = \frac{f_{ye}}{\gamma_s} = 478.5MPa \qquad (\gamma_s = 1.15\,[10]) \tag{57}$$

The required longitudinal reinforcement for beams ends is shown in Table 10. The longitudinal reinforcement was obtained for simple flexure; the values of Table 8 are reproduced for convenience.

Table 10: Longitudinal reinforcement for beam plastic hinges (tension zone)

Design details:

b= 0.30m; d=0.42m b=width; d=effective depth

	Storey, i	Location	M_{sd} (kNm)	μ	ω	$A_s(cm^2)$	ϕ	$A_{sprovided}(cm^2)$	ρ (%)
Span 1	1	left end	283.13	0.247	0.290	16.60	9ϕ16	18.10	1.44
		right end	280.60	0.245	0.287	16.38	9ϕ16	18.10	1.44
	2	left end	253.11	0.221	0.254	14.50	8ϕ16	16.08	1.28
		right end	250.85	0.219	0.251	14.34	8ϕ16	16.08	1.28
	3	left end	196.03	0.171	0.189	10.80	6ϕ16	12.06	0.96
		right end	205.11	0.178	0.200	9.95	6ϕ16	12.06	0.96
	4	left end	153.38	0.134	0.145	8.30	6ϕ16	12.06	0.96
		right end	212.52	0.185	0.207	10.30	6ϕ16	12.06	0.96
Span 2	1	left end	269.23	0.235	0.273	15.57	8ϕ16	16.08	1.28
		right end	273.02	0.238	0.277	15.83	8ϕ16	16.08	1.28
	2	left end	240.68	0.210	0.239	13.64	8ϕ16	16.08	1.28
		right end	244.07	0.213	0.243	13.90	8ϕ16	16.08	1.28
	3	left end	186.40	0.163	0.179	10.22	6ϕ16	12.06	0.96
		right end	189.03	0.165	0.181	10.40	6ϕ16	12.06	0.96
	4	left end	128.03	0.112	0.119	6.80	6ϕ16	12.06	0.96
		right end	107.67	0.094	0.100	5.70	6ϕ16	12.06	0.96

The reinforcement values for beams were obtained considering the requests specified in Eurocode 8 [4] for Ductility Class Medium (*DCM*) structures at plastic hinges locations. To verify the local ductility conditions according to EC8, the following requirements must be fulfilled:

- The value of the curvature ductility factor μ_φ shall be at least equal to:

$$\mu_\phi = 2q_0 - 1 \quad if \quad T_1 \geq T_C \tag{58}$$

$$\mu_\phi = 1 + 2(q_0 - 1) \quad if \quad T_1 < T_C \tag{59}$$

  in this case of study, $\mu_\phi = 2q_0 - 1 = 2\times 3 - 1 = 5 \qquad (q_0 = 3)$

- In the compression zone, reinforcement should be at least half of the reinforcement provided in the tension, in addition to any compression reinforcement needed for the *ULS* verification of the beam in the seismic design situation.
- The reinforcement ratio of the tension zone ρ does not exceed a value ρ_{max} equal to:

$$\rho_{\max} = \rho' + \frac{0.0018}{\mu_\phi \varepsilon_{s,yd}} . \frac{f_{cd}}{f_{yd}} = \rho' + \frac{0.0018\times 21.7\times 10^3}{5\times 0.0024\times 478.5\times 10^3} = \rho' + 0.68\% \tag{60}$$

  where, ρ' is the reinforcement ratio of the compression zone and $\varepsilon_{s,\,yd} = 0.0024$

For this case of study the reinforcement of the compression zone will be equal to the reinforcement of the tension zone.

Through the entire length of a primary seismic beam the reinforcement ratio of the tension zone ρ shall be not less than the following minimum value:

$$\rho_{\min} = 0.5 \times \frac{f_{ctm}}{f_{yk}} = 0.5 \times \frac{3}{550} = 0.30\% \tag{61}$$

Table 11: Reinforcement ratios for beam plastic hinges

	Storey, i	Location	ρ (%)	ρ' (%)	ρ_{max} (%)	ρ_{min} (%)
Span 1	1	left end	1.44	1.44	2.12	0.30
		right end	1.44	1.44	2.12	0.30
	2	left end	1.28	1.28	1.96	0.30
		right end	1.28	1.28	1.96	0.30
	3	left end	0.96	0.96	1.64	0.30
		right end	0.96	0.96	1.64	0.30
	4	left end	0.96	0.96	1.64	0.30
		right end	0.96	0.96	1.64	0.30
Span 2	1	left end	1.28	1.28	1.96	0.30
		right end	1.28	1.28	1.96	0.30
	2	left end	1.28	1.28	1.96	0.30
		right end	1.28	1.28	1.96	0.30
	3	left end	0.96	0.96	1.64	0.30
		right end	0.96	0.96	1.64	0.30
	4	left end	0.96	0.96	1.64	0.30
		right end	0.96	0.96	1.64	0.30

According to Table 11 the local ductility requirements in the critical regions are fulfilled.

The maximum diameter of longitudinal beam bars crossing a beam-column connection should be set by an upper limit of the diameter of the longitudinal bars of the beam, d_{bL}, that pass through interior beam-column joints or are anchored at exterior ones (to prevent bond failure), as:

- In interior beam-column joints:

$$\frac{d_{bL}}{h_c} \leq \frac{7.5 f_{ctm}}{\gamma_{Rd} f_{yd}} \cdot \frac{1 + 0.8\nu_d}{1 + 0.75 k_D \, \rho' / \rho_{\max}} \tag{62}$$

- In exterior beam-column joints:

$$\frac{d_{bL}}{h_c} \leq \frac{7.5 f_{ctm}}{\gamma_{Rd} f_{yd}} \left(1 + 0.8\nu_d\right) \tag{63}$$

for DCM $\begin{cases} \gamma_{Rd} = 1 \\ k_d = 0.75 \end{cases}$

$\nu_d = \frac{N_{ED}}{A_c f_{cd}}$, N_{ED} is the axial column force (from seismic design situation)

Therefore, the diameter of the longitudinal bars of the beams for column alignment 1 is 19.83mm, for column alignment 2, 17.51mm and for column alignment 3, 22.43mm. According to this requirement the maximum diameter of longitudinal beam bars shall be limited to d_{bL} = 16mm. From Table 11 it could be stated that these requirements are met.

The design beam moments at mid span due to seismic loads are given in Table 12 and the correspondent beam longitudinal reinforcement at Table 13. These are obtained from the combination of reduced gravity loads applicable for the design seismic combination, and overstrenght moment capacity at beam hinge location, according to Eq. (34). The overstrenght factor ϕ^0 is considered equal to 1.25. The design material strengths used are the characteristic material strengths, without amplification.

Table 12: Design beam moments at mid span [units in kNm]

Storey, i	Span 1	Span 2
4	216.16	214.60
3	216.63	213.89
2	216.95	213.42
1	217.11	213.16

Table 13: Beam longitudinal reinforcement for mid span

Design details:

Concrete C25/30 b= 0.30m; d=0.42m

Steel: A500 NR b=width; d= effective depth

	Storey, i	M_{sd} (kNm)	μ	ω	$A_s(cm^2)$	ϕ	$A_{s\ provided}(\text{cm}^2)$	ρ (%)
Span 1	1	217.11	0.246	0.288	13.96	7ϕ16	14.07	1.12
	2	216.95	0.245	0.288	13.96	7ϕ16	14.07	1.12
	3	216.63	0.245	0.288	13.96	7ϕ16	14.07	1.12
	4	216.16	0.244	0.287	13.88	7ϕ16	14.07	1.12
Span 2	1	213.16	0.241	0.282	13.63	7ϕ16	14.07	1.12
	2	213.42	0.241	0.282	13.63	7ϕ16	14.07	1.12
	3	213.89	0.242	0.283	13.70	7ϕ16	14.07	1.12
	4	214.60	0.243	0.284	13.75	7ϕ16	14.07	1.12

Column Flexure

The required column flexural strength according to DDBD capacity design rules is given by Eq. (36), reproduced herein by convenience.

$$\phi_f M_N \geq \phi^0 \omega_f M_E \tag{64}$$

ϕ^0 is the overstrenght factor considered as 1.25;

ω_f is the dynamic amplification factor - Eq. (37);

M_E is the column moments resulting from design forces (given in Table 9);

ϕ_f is the strength reduction factor considered as 0.9.

The design column moments and axial forces are shown in Table 14 and 15, respectively. Fig.14 presents a scheme of the design column moments.

Table 14: Design column moments [units in kNm]

Storey , *i*	Location	ω_f	Col 1	Col 2	Col 3
4	Top	1	166.15	332.31	166.15
	bottom	1	-45.92	-91.84	-45.92
3	Top	1.190	300.02	600.03	300.02
	bottom	1.190	-126.99	-253.98	-126.99
2	Top	1.186	321.28	642.56	321.28
	bottom	1.186	-231.45	-462.90	-231.45
1	Top	1	269.99	539.97	269.99
	bottom	1	-306.24	-612.50	-306.24

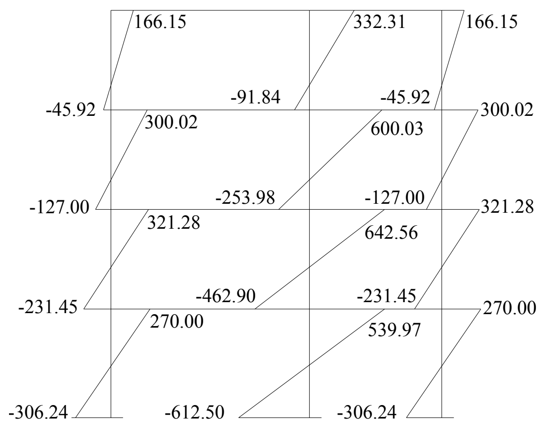


Figure 14: Design column moments distribution [units in kNm]

Table 15: Axial forces in columns [units in kN]

Storey, *i*	Location	Col 1	Col 2	Col 3
4	Top	-99.90	-213.03	-153.00
	bottom	-99.90	-213.03	-153.00
3	Top	-169.68	-410.99	-351.20
	bottom	-169.68	-410.99	-351.20
2	Top	-219.06	-598.77	-579.98
	bottom	-219.06	-598.77	-579.98
1	Top	-258.81	-782.97	-825.56
	bottom	-258.81	-782.97	-825.56

The required longitudinal reinforcement for the rectangular column sections was obtained considering composed bending and it is presented in Table 16. The required longitudinal reinforcement was obtained using simplified equations for rectangular cross sections with symmetric reinforcement [9]. The recover rebar was considered as 3 cm.

The design material design strengths used are the characteristic ones, without amplification, except for the column base, where it is expected the formation of plastic hinges (beam-sway mechanism).

Table 16: Longitudinal reinforcement bars /face

Design details:

Concrete C25/30 exterior col. b= 0.40m;h=0.40m

Steel: A500 NR interior col. b= 0.45m;h=0.45m

Storey, *i*	Col 1	Col 2	Col 3
4	3ϕ25 (14.73cm^2)	3ϕ32 (24.13cm^2)	3ϕ25 (14.73cm^2)
3	4ϕ25 (19.63cm^2)	4ϕ32 (32.17cm^2)	4ϕ25 (19.63cm^2)
2	4ϕ25 (19.63cm^2)	4ϕ32 (32.17cm^2)	4ϕ25 (19.63cm^2)
1	4ϕ25 (19.63cm^2)	4ϕ32 (32.17cm^2)	4ϕ25 (19.63cm^2)

Table 17: Reinforcement ratios (ρ (%))

Storey, *i*	Col 1 (A_c=0.16m^2)	Col 2 (A_c=0.2025m^2)	Col 3 (A_c=0.16m^2)
4	1.84	2.38	1.84
3	2.45	3.18	2.45
2	2.45	3.18	2.45
1	2.45	3.18	2.45

According to EC8 for a structure with a class ductility medium (DCM) the longitudinal reinforcement ratio for columns should be greater than 0.01 (1%) and not less of 0.04 (4%). From Table 17 it can be stated that these requirement are fulfilled.

3.2. Direct Displacement Based Design for a peak ground acceleration of 0.27g

The DDBD procedure is applied again to the same frame with the objective of illustrating the design situation when the displacement capacity exceeds the maximum possible spectral displacement demand. In this situation, the building was considered being located in Continental Portugal (Lisbon) seismic zone 1.3, as an ordinary building, class of importance II. The value of the peak ground acceleration a_g used in the definition of the response spectrum is 0.27g. Some of the results were obtained previously in section 3.1 and are herein reproduced for convenience. In Table 18 is presented a summary of the results obtained previously from the DDBD procedure, steps 1 and 2.

Table 18: Results of DDBD in terms of design displacement, equivalent yield displacement, ductility, effective mass and equivalent viscous damping

Δ_{target} (m)	Δ_d (m)	Δ_y (m)	μ	m_e (tonne)	ξ (%)
0.307	0.230	0.141	1.64	157.34	11.99

The step 3 of the DDBD procedure involves the determination of the effective period at peak displacement response and it is found from the design displacement spectrum as referred in section 3.1 (see Fig.15).

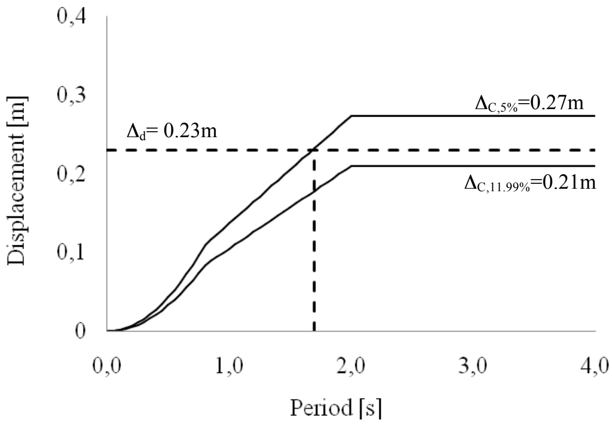
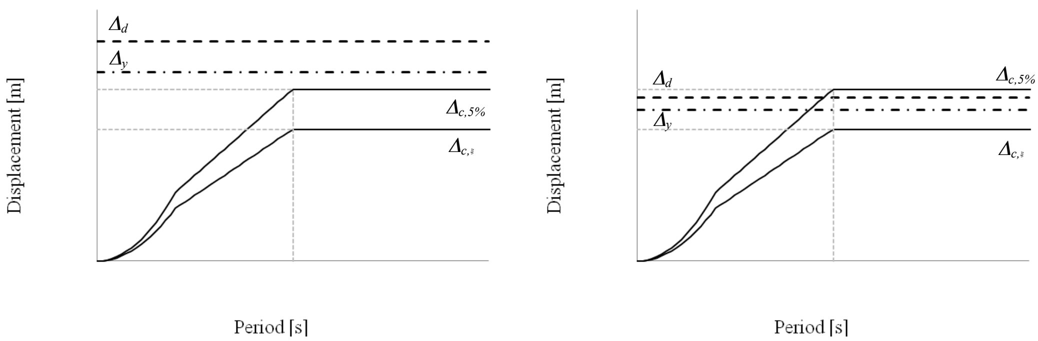


Figure 15: Design Displacement Spectrum

From Fig.15 it can be observed that the design displacement capacity for the SDOF structure exceeds the maximum possible spectral displacement demand for the considered damping level. This design situation can occur for very tall or flexible structures or when peak ground acceleration is too low. For these cases two possibilities must be considered: a) equivalent yield displacement may exceed 5% damping corner displacement, or b) equivalent yield displacement is less than 5% damping corner displacement, see Fig.16 a) and b), respectively.



a) $\Delta_y > \Delta_{c,5\%}$ b) $\Delta_y < \Delta_{c,5\%}$

Figure 16: Design situations when the displacement capacity exceeds the spectral demand

a) Yield displacement exceeds 5% damping value at the period corner $(\Delta_y > \Delta_{c,5\%})$

The structure will respond elastically and the response period T_{el} will be larger than the corner period $T_D \left(T_{el} > T_D\right)$. As suggested in Ref. [2] the response displacement will be taken equal to $\Delta_{c,5\%}$, thus the required design base shear force is given by:

$$V_{base} = k_{el} \times \Delta_{c,5\%} \tag{65}$$

where, k_{el} is the elastic stiffness and $\Delta_{c,5\%}$ is the 5% damping elastic response displacement.

In this design situation the solution is not unique. In fact the stiffness k_{el} depends on the elastic period, which depends in turn, on the strength. This leads to an uncertainty in choosing an acceptable value for the strength. Minimum strength requirements for *P-Δ* effects or gravity loads will governs the required strength.

b) Yield displacement is less than 5% damping value at the period corner $(\Delta_y < \Delta_{c,5\%})$

An inelastic response will occur but not at the level of ductility corresponding to the displacement or drift capacity of the structure. As in the previous case there was not a unique solution and elastic stiffness could correspond to any period larger than the period corner. Herein, the elastic period T_{el} is taken with the value of the period corner T_D, because if T_{el} is less than T_D the displacement capacity value will be incompatible with equivalent damping. An iterative method is required and the following procedure is recommended in Ref. [2]:

- **a.** Calculate displacement capacity, Δ_d, and the corresponding damping ξ.
- **b.** Calculate approximately the final displacement response Δ_{df} (one possibility to a first guess is to consider $\Delta_{df} = (\Delta_{c,\xi} + \Delta_d)/2$).
- **c.** Calculate the displacement ductility demand corresponding to Δ_{df} ($\mu = \Delta_{df}/\Delta_y$).
- **d.** Calculate the damping ξ corresponding to ductility demand μ.
- **e.** Calculate the displacement response $\Delta_{c,\xi}$ at T_D corresponding to ξ.
- **f.** Use this new value $\Delta_{c,\xi}$ as a new estimation of the final displacement Δ_{df}, iteratively.
- **g.** Cycle trough steps c. to f. until a stable solution it found.

From Fig.15 it can be stated that for this case of study the design displacement capacity exceeds the spectral demand (case b). Then, the inelastic response develops at a lower ductility than the structural capacity. Thus the effective period is T_D (2.0sec) with compatible displacement and damping. To find the final design displacement Δ_{df} of the SDOF structure it is necessary to follow the iterative procedure described previously.

In the following, the step-by-step procedure is applied to the frame building of the case of study and the results presented in Table 19.

Step a.

$\Delta_d = 0.23$m and ξ=11.99%

$\Delta_{c\xi} = 0.21$m

Step b.

$$\Delta_{df} = \frac{0.21 + 0.23}{2} = 0.22m$$

Step c.

$$\mu = \frac{\Delta_{df}}{\Delta_y} = \frac{0.22}{0.14} = 1.56$$

Step d.

$$\xi_{hyst} = \xi_0 + \left(\frac{\mu - 1}{\mu\pi}\right) = 11.47\%$$

Step e.

$$\Delta_{(2,11.47)} = 0.27\left(\frac{10}{5+11.47}\right)^{0.5} = 0.213m$$

This will be a new estimate for the final displacement.

Step f.

go to step c. to f. until a stable solution is found (see Table 19).

Table 19: Iterative procedure results

	Δ_{df} (m)	μ	ξ (%)	$\Delta_{(c.\xi)}$ (m)
1º	0.220	1.57	11.47	0.213
2º	0.213	1.51	11.10	0.216
3º	0.216	1.53	11.23	0.215
4º	0.215	1.52	11.18	0.215

The final design displacement of the SDOF structure is 0.215m.

Fig.17 shows the design displacement spectrum for 5% damping and for the initial damping (ξ=11.99%), as the initial (Δ_d= 0.230m) and final displacement response (Δ_{df}= 0.215m).

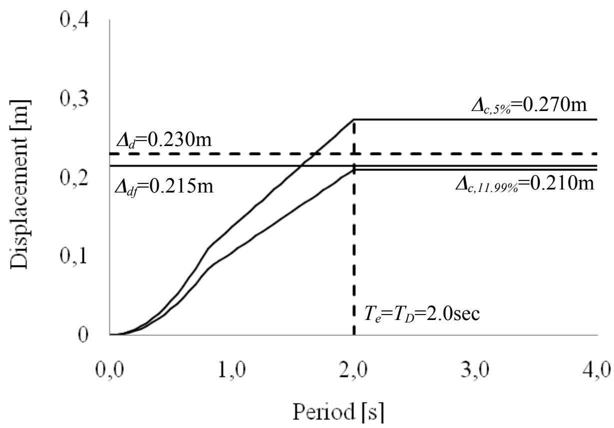


Figure 17: Design Displacement Spectrum

The design base shear force is thus:

$$V_{base} = \Delta_d \times k_{el} = \Delta_d \times \frac{4\pi^2 m_e}{T_D^2} = 333.87kN \tag{66}$$

Any value less than V_{base} will satisfy the design assumption.

4. Final remarks

In this report has been presented a brief summary of the Direct Displacement Based Design (DDBD) procedure. This design procedure was applied to a simple case of study, a reinforced concrete frame building. Different seismic intensities were considered: peak ground accelerations of 0.35g and 0.27g were adopted. For the peak ground acceleration of 0.35g, the design displacement capacity of the frame structure obtained through the DDBD procedure is less than the maximum possible spectral displacement demand for the considered damping level. For the low seismicity case (0.27g) the displacement capacity exceeds the maximum possible spectral displacement demand.

It can be stated that the DDBD procedure leads to an easy design than the traditional force-based procedures [3]. However, the DDBD procedure is based on hand calculations and throughout the design process some design choices must be done based on engineering judgment. Moreover, the DDBD procedure can be more difficult to apply, becoming an iterative procedure in some cases (for very flexible structures or/and low seismic intensity - see section 3.2). Thus, it is herein suggested to develop an automatic program.

5. References

[1] Priestley, M.N.J. and Kowalsky, M.J. (2000) "Direct Displacement-Based Seismic Design of Concrete Buildings", Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 44(2). 145–165

[2] Priestley, M.N.J., Calvi, G.M. and Kowalsky, M.J. (2007) "Displacement-Based Seismic Design of Structures", IUSS Press, Pavia, Italy

[3] Powell, G.H. (2008) "Displacement-Based Seismic Design of Structures – Book Review", Earthquake Spectra, Vol.24. No.2

[4] EN 1998-1:2004 "Eurocode 8: Design of Structures for Earthquake Resistance - Part 1: General Rules, Seismic Actions and Rules for Buildings", CEN, Brussels, Belgium.

[5] Massena, B., Degée, H. and Bento, R. (2010) "Consequences of design choices in Direct Displacement Based Design of RC frames", 14th ECEE, Ohrid, Republic of Macedonia

[6] Massena, B., Bento, R. and Degée, H. (2010) "Implications of different design assumptions in Direct Displacement Based Design of RC frames", Sísmica 2010, Aveiro (Portugal).

[7] CRSRRCS 1992, Negro, P. Varzeletti, G., Magonette, G. E. and Pinto, A., V. (1995) "Tests on the Four-Storey Full Scale Reinforced Concrete Frame with Masonry Infills: Preliminary Report", Joint Research Centre- Ispra, Italy

[8] NP EN 1998-1:2007, "Portuguese National Annex"

[9] D'Arga e Lima J. Monteiro and V. Mun M. (1996) "Betão Armado - Esforços Normais e de Flexão", LNEC, Lisboa, Portugal