Lecture 1: Transport networks design and evaluation
Case study presentation and data acquisition

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CASE STUDY PRESENTATION

- **Network design problems** include a large class of various problems related to the design of networks (graphs) and their usage.

- In **transport network design** problems can occur at strategical, tactical and operational level.

- This course will focus in **transport network design** at a **strategical** level and a **regional** scale, used as a planning tool for regional **accessibility** and **mobility**.

- The problems formulation will be a simplified model of the reality that will help decision makers at two levels:
  - Which are the main connections in the region that must be assured and how they interact with the other smaller centralities
  - Which must be the most adequate infrastructure type for each link in the network (capacity, free flow speed)
NETWORK DESIGN PROBLEMS

- There are several network design problem with different formulations:
  - Minimum Spanning Tree Problem
  - Minimum Cost Flow Network Problem
  - Minimum Cost Flow Multi-Terminal Network Problem

- A large set of real world problems result from the combination or adaptation of these basic NP-hard or NP-complete problems.
  - In computational complexity theory, the complexity class NP-complete (abbreviated NP-C or NPC) is a class of problems which no fast solution to them is known; that is, the time required to solve the problem using any currently known algorithm increases very quickly as the size of the problem grows.
KEY CONCEPTS OF GRAPHS THEORY

A graph is formed by a set of nodes and arcs that establish some connections between them. Being in our problem:

- **Nodes** - the cities or places in the regional system to be analyzed
- **Arcs** – the transport infrastructure that connects them (road or rail)

**Glossary**

- **Oriented arc** – Arc that establishes connection between nodes just in one direction
- **Oriented graph** – Graph formed by oriented arcs
- **Adjoining arcs** – Arcs that finish where other arc starts
- **Chain** – Sequence of adjoining arcs
- **Path** – Chain formed by oriented arcs
KEY CONCEPTS OF GRAPHS THEORY

- **Glossary**
  - **Cycle** – Chain with coincident initial and final node
  - **Circuit** – Path with coincident initial and final node
  - **Conexant graph** – Graph where it's possible to establish a chain between all the pair of nodes
  - **Tree** – Conexant graph without a cycle
  - **Spanning tree** – Tree formed by arcs of a graph that contains all the nodes of the graph

- The techniques used in this domain result from the application of linear programming problems
The techniques used in this domain result from the application of linear programming problems:

- With an objective function to be optimized (generally minimized)
- And a set of linear constraints

**Linear constraints**

\[ a_{11}x_1 + a_{12}x_2 \leq b_1 \]

**Objective function**

\[ f(x_1, \ldots, x_n) = c_1x_1 + c_nx_n \]
MODELING THE PROBLEM

In order to develop a model of the optimization problem we need to:

- Establish the framework of the optimization problem
  - Define decision variables to be optimized and their nature
  - Define auxiliary variables of the model
  - Define objective function of the problem
  - Define constraints of the problem

- Establish input data of the problem
  - Define data needed for the problem
  - Data acquisition
  - Build input data file (import to auxiliary variables)
VARIABLES DEFINITION

- The variables definition should establish:
  - the number of dimension of the variable (vector or matrix)
  - the domain of each dimension
  - the nature of the variable (integer, real, binary, string, etc.)

\[
\begin{bmatrix}
  x_1 \\
  \vdots \\
  x_n
\end{bmatrix}
\begin{bmatrix}
  x_{11} & \cdots & x_{1n} \\
  \vdots & \ddots & \vdots \\
  x_{n1} & \cdots & x_{nn}
\end{bmatrix}
\]

- In a minimum spanning tree problem
  - the main decision variable should present 2 dimension (NxN), being N the number of centralities being analyzed
  - N should be defined also as a range variable to be used in the problem
OBJECTIVE FUNCTION DEFINITION

- The objective function of the problem should contain all the concerning variables that the decision maker wants to minimize.
- For the network design problem normally the are considered:
  - Construction costs
  - Operation costs
  - User costs
- These components are obtained by a unit cost multiplied by the length of the arc or link (normally using the Euclidean distance or Manhattan distance).
- If the variables considered in the objective function present different dimensions they should be standardized using a common unit (e.g. monetary) and use a multi-criteria approach or converted them into a utility function.
- The final objective equation will result in
  \[ \min \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \alpha \times D_{ij} \times x_{ij} + \beta \times D_{ij} \times x_{ij} \right) \]
- Where \( x_{ij} \) is the decision variable for each arc of the graph of the problem.
 CONSTRAINTS DEFINITION

- A constraint is a condition that a solution to an optimization problem must satisfy.
- There are two types of constraints: equality constraints and inequality constraints.
- The set of solutions that satisfy all constraints is called the feasible set.
- The constraints in an optimization problem have two main functions:
  - Avoid non-trivial solutions of decision variables
  - Lead the objective function to acceptable solutions in different dimensions not introduced in the objective function

\[ \sum_{ij} x_{ij} \leq y_i + b \]
Example of constraint
MINIMUM SPANNING TREE PROBLEM

Goal: Design a network that connects all the nodes of the graph at minimum cost

Decision Variables:

- \(X\): array (NODES, NODES) – binary
- Level: array (NODES) - integer

Objective function:

\[ \sum_{i,j=1}^{\text{Nodes}} \alpha \cdot \text{Length}_{ij} \]

Constraints:

- Number of connections: \( \sum_{i,j=1}^{\text{Nodes}} x_{ij} = N - 1 \)
- Avoid Subcycle:
  \[ \forall_{i,j} \text{level}_j \geq \text{level}_i + 1 - N + N \times x_{ij} \]
- Direct all connections towards the root:
  \[ \forall_j \sum_{i=1}^{N/j>1} x_{ij} = 1 \]
MINIMUM COST FLOW NETWORK PROBLEM

- **Goal:** Assign demand flow from an existing network from a single source to a single sink node at minimum cost considering capacity constraints on the links of the graph.

- **Decision Variables:**
  - **Flow:** array (ARCS) – integer

- **Objective function:**
  \[ \sum_{i=1}^{\text{Arcs}} \alpha \cdot \text{Flow}_i \cdot \text{Length}_i \]

- **Constraints:**
  - **Total source flow:**
    \[ \sum_{i=1}^{\text{Arcs}} \text{Flow}_i \geq \text{Demand} \]
  - **Node equilibrium:**
    \[ \forall \text{nodes} \sum_{i=1}^{\text{Arcs}} \text{Flow}_i = \sum_{k=1}^{\text{O Arcs}} \text{Flow}_k \]
  - **Capacity constraint:**
    \[ \forall_{\text{arcs}} \text{Flow}_a \leq \text{Capacity}_a \cdot \text{MaximumRatio} \]
  - **Positive flow:**
    \[ \forall_{\text{arcs}} \text{Flow}_a \geq 0 \]
MINIMUM COST FLOW
MULTI-TERMINAL NETWORK PROBLEM

- **Goal:** Assign demand flow the an existing network from a several sources to a several sink nodes at minimum cost considering capacity constraints on the links of the graph.

- **Decision Variables:**
  - **Flow:** array \((ARCS, ODPairs)\) – integer

- **Objective function:**
  \[
  \sum_{i=1, j=1}^{Arcs,ODPairs} a \cdot Flow_{ij} \cdot \text{Length}_i
  \]

- **Constraints:**
  - **Total source flow:**
    \[
    \forall_{j, ODPairs} \sum_{i=1}^{Arcs/O_j} Flow_{ij} \geq \text{Demand}_j
    \]
  - **Node equilibrium:**
    \[
    \forall_{nodes, jODPairs} \sum_{i=1}^{Arcs/D_i=n} Flow_{ij} = \sum_{k=1}^{Arcs/O_k=n} Flow_{k,j}
    \]
  - **Capacity constraint:**
    \[
    \forall_{odpairs} \sum_{i=1}^{ODPairs} Flow_{ai} \leq \text{Capacity}_a \cdot \text{MaximumRatio}
    \]
  - **Positive flow:**
    \[
    \forall_{arcs, jODPairs} Flow_{aj} \geq 0
    \]

Spain Network Example
HOW TO EVALUATE THE SOLUTIONS

- The evaluation of a transport network can be done considering different aspects:
  - design (comparison between the existing network and the optimal planning network)
  - operational performance and reliability of transport networks considering individual and collective objectives (level of service, safety)
  - economic justification of the recourse to network hierarchisation (i.e. different capacity, speed and level of service for each arc)
  - interaction between Land Use and Transport Networks (evaluation of accessibility to activities)

- All these aspects can be considered in the optimization process (in the objective function or as constraints) increasing its complexity
TRANSPORT NETWORKS EVALUATION DESIGN

- This evaluation is performed normally after the optimization process to compare the results with the existing situation
  - Evaluate reasons for the different physical features of the network
  - Economic evaluation of difference in costs between the optimal solution and the existing one (or time saving measures)

- This analysis is normally prior a network update or expansion (similar problem but with some $x_{ij}$ variables already fixed)

Existing network

Optimal network

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hierarchy 1</td>
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<td>Hierarchy 2</td>
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<td>Hierarchy 3</td>
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</tbody>
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Diagram:

- Red: Hierarchy 1
- Blue: Hierarchy 2
- Green: Hierarchy 3
The operational performance of a transport network can be measured from several points of view (some conflicting).

Usually its assessment is made using indicators:

- **Capacity used ratio**
- **Level of service**
- **Safety** – Usually measured indirectly at a strategic level through the congestion levels (capacity used ratio, level of service) and the arc network hierarchy (speed, number of lanes, etc.)
- **Reliability** – Can be measured from two points of view:
  - **Vulnerability** of a network as the network’s or the links’ susceptibility to failure, where the term failure expresses a considerable deviation from the normal functioning state of the link or network and the damages produced by its malfunctioning.
  - **Reliability** of a network as the probability that the arcs within a network will function - degree of stability of the quality of service that a system offers.
The accessibility to activities measures the interaction between the exiting land uses and the transport network. The accessibility can be assessed through different indicators depending the analysis level (strategical, tactical, operational) and the case study scale (neighborhood, city, regional, national).

This course focuses in the strategical level and regional scale, being the main indicators:

- Hansen integral accessibility index (Ai)
  \[ A_i = \frac{\sum_j B_j f(c_{ij})}{\sum_j B_j} \quad f(c_{ij}) = \frac{1}{x_{ij}} \]

- ARIA Index [0,15]
  \[ ARIA_{iL} = \sum_L \min\left\{3, \frac{x_{iL}}{\bar{x}_L}\right\} \]

- Gravity model Index
  \[ w_{ij} = \frac{g_{ij}}{\sum_i g_{ij}} \quad g_{ij} = \frac{B_i B_j}{x_{ij}^2} \]

where Bi is the population of city i and xij the travel time between i and j.

### ARIA service center categories

<table>
<thead>
<tr>
<th>Service centre category</th>
<th>Population</th>
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<tbody>
<tr>
<td>A</td>
<td>( \geq 250,000 )</td>
</tr>
<tr>
<td>B</td>
<td>48,000–249,999</td>
</tr>
<tr>
<td>C</td>
<td>18,000–47,999</td>
</tr>
<tr>
<td>D</td>
<td>5,000–17,999</td>
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<tr>
<td>E</td>
<td>1,000–4,999</td>
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</tbody>
</table>
ACCESSIBILITY - A MAJOR CONCEPT AND INSTRUMENT (I)

- Accessibility impedance index ($A(i)$)

$$A(i) = \sum_j M(j) \cdot f[c(i, j)]$$

Where:
- $M(j)$ is the “mass” of partners at location $j$
- $c(i, j)$ is the cost of travel between
- $f$ is a decay function describing the reduction of willingness to interact as the travel cost increases
- Traditional choices are for $f[c(i, j)]$
  - Power function (not usable for zero cost)
  - Exponential function
  - Inverted Logistic function
The first step to perform an efficient data acquisition process is to identify previously what data is needed to model the specific problem we are dealing.

In a network design and evaluation problem there is a great range of data that can be assessed, although we should focus on the input variables needed for the optimization process and some specific evaluation we need to perform:

- Cities/centralities coordinates
- Existing or estimated Transport/traffic demand
- Real network
- Social-demographic data and Land Use data
In the Portuguese context the main data source is the Instituto Nacional de Estatística (INE):

- Social-demographic data and Land Use data for different geographical levels (NUT1 to BGRI ~ city block) – Census or other studies
  - Resident population (Population pyramid)
  - Gross domestic Product (GDP)
  - Purchasing power parity (PPP) index
  - Economic activity
    - Number of companies per economic activity (CAE)
    - Number of employees per economic activity (CAE)
    - Capital of companies seeded at a location
DATA ACQUISITION

- INE (cont.)
  - Existing or estimated Transport/traffic demand – regional transport studies
    - Existing/estimated traffic flows

- GIS data (cities coordinates, existing networks)
  - Several Portuguese entities have their own GIS databases that can be asked upon request (what can take some time depending on the entity)
  - All the GIS files needed for the problems resolution will be given upon request from the working group with the respective metadata file (data source, coordinate system, year, etc.)