Compendium of Papers

An optimisation algorithm to establish the location of stations of a mixed fleet biking system: an application to the city of Lisbon

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Abstract

This paper presents the design and deployment of a bike-sharing system developed for Lisbon. The design of this new service is performed through an heuristic, encompassing a Mixed Integer Linear Program (MILP), that simultaneously optimises the location of shared biking stations, the fleet dimension and measuring the bicycle relocation activities required in a regular operation day. The results obtained for several tested scenarios provided better insights into knowing how to improve the design and operation of these systems.

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Keywords: bike-sharing systems, electric bikes, mixed integer linear optimisation problem.

1. Introduction

Bike-sharing, or public bicycle programs, have gained increasing attention in the last decades as a viable mobility option in dense urban areas to perform short trips. They are not only considered an alternative to conventional public transport or private car, but a viable option for the first/last mile of other public transport solutions.

The bike-sharing concept is a bike short term rental network formed by three main components: the docking stations, the bicycles and the information technology (IT) interfaces, which have been recently added to increase the quality of the service, providing information as the location of stations and the number of bicycles available.

It is possible to identify three main generations of bike-sharing programs. Originally the concept emerged with the White Bike program in Amsterdam (1965), where some white bikes were spread around the city randomly. Few days later, all the bikes had been stolen or damaged. Other systems like the vélos jaunes in La Rochelle, France (1974) and the Green Bike Scheme in Cambridge, UK (1993) are some forerunners of this generation (Home, 1991).

In 1995 the municipal government of Copenhagen, in Denmark, in an attempt to reduce exposure to theft, created the first Coin-deposit Bike program called Bycyken. This system was also the first large-scale program with 1,100 bicycles. Each station had a locking system, which unlocked with a coin, refunded upon delivery (Commons, 2009). These programs were a new step on the bike-sharing history for two reasons. Firstly, the
program operation was carried out by a non-profit organisation and, secondly, bicycles were now parked in specific docking stations. Nevertheless, bicycles still experienced theft due to the anonymity of the user.

These modifications led also to additional costs of implementation and operation (docking station infrastructures and maintenance), increasing the risk for investors. Adding to this, some studies showed that the impact on mobility patterns was insignificant (Krykewycz, Puchalsky, Rocks, Bonnette, & Jaskiewicz, 2010).

This gave rise to a new generation of bike-sharing with improved customer tracking. The third generation appeared primarily in Portsmouth University (England) in 1996 and allowed students to rent a bicycle with a magnetic card, providing an easy access to this transport mode with low operating costs (Black & Potter, 1999). Despite the success of this system, bike-sharing programs did not flourish until 2005, when the Vélo’v program, in Lyon, France, was born. This was operated by JCDecaux that used a new business model, serving as a launching ramp for many other bike-sharing programs (Henley, 2005). The majority of today’s systems are either operated by advertising companies or financed by public authorities with non-profit oriented goals.

DeMaio (2009) and Shaheen et al. (2010) have done an exhaustive survey about the existing bike-sharing programs. Both the authors have identified the need of a new bike-sharing generation. This new generation should offer an optimised network, in order to solve problems like the high costs of bicycle redistribution, and have a more demand responsive system well fit to a city with traffic congestion problems, scarce urban space and a high concentration of air pollution.

The goal of this paper is to propose a new formulation to design a bike-sharing network, incorporating the main dimensioning factors: uncertainty in demand estimation, fixed and variable costs of the network infrastructure (i.e. bicycle fleet, docking stations and relocation fleet costs). The proposed approach improves significantly prior formulations by modelling individual trips discretised in space and time, while considering a mixed fleet of regular and electric bikes. Moreover, several fare collection methods will be considered: an individual annual pass to provide access to regular bicycle users, with an additional charge for the use of electric bicycles; and, an individual hourly trip fare. This model will use the city of Lisbon as a test bed.

The remainder of the paper is organised as follows: the next section will present a brief literature review over the main approaches that have been followed in the design optimisation models for transportation sharing systems; afterwards, the description of the methodology used to design the new optimisation model for a bike-sharing network will be presented; this will be followed by the presentation of the results obtained from the developed model for the study area; the last section discusses the model formulation and future challenges of this research prior to service deployment.

2. Literature review

The recent momentum of vehicle sharing systems, as a new mobility option in dense urban areas, has not been followed by an intense research on the optimal design of these systems.

The research about shared vehicle systems design has been focusing in one-way carsharing systems, whose financial viability remains uncertain. Fan et al. (2008) suggested a multistage stochastic linear integer model that integrates demand stochasticity in the model formulation to support the decision making related to vehicle fleet management. Kek et al. (2009) focused the relocation operations and applied a combined mixed integer programming (MIP) formulation, with heuristics, to minimise staff costs resulting from the fleet relocation movements. More recently, Correia and Antunes (2012) have developed a MIP model to determine the system configuration that maximises the profits of the carsharing organisation for different trip selection schemes.

Most of the developments in the field of operations research models for bike-sharing systems aim to optimise the system configuration of this new alternative transport mode. Lin and Yang (2011) presented an integer non-linear program aiming to determine the optimal location of docking stations and the necessary bicycle lanes. Based on a minimum-cost flow problem for a space-time network, Contardo et al. (2012) presented a bicycle fleet dimensioning demand equilibrium model. Shu et al (2010), proposed a linear programming model to
measure the importance of features such as the bicycle utilisation rate, the operational costs of fleet relocation and the capacity of each docking station.

However, none of these papers applies their formulation to a real-scale case study. Furthermore, demand is normally considered static and deterministic and aggregated into OD flows between city areas.

There is clearly a gap in the literature that should be explored in order to enable the development of a decision support tool for the deployment of real bike-sharing systems.

3. Model formulation

3.1. Problem description

In this paper, we address the problem of a bike-sharing system design. More specifically, the location of shared biking docking stations, the required bicycle fleet to operate and the bicycle relocation activities required in a regular operation day.

This formulation aims to encompass several decisions under the same model, integrating the key operational issues that may emerge during regular operation of the system. This problem is NP-Complete, hardly analytically solvable for medium to large instances. For this reason, an innovative heuristic that divides the problem for the whole operation day in several steps was created within a MILP formulation. Figure 1 presents the general model framework that computes several days of operation, maintaining the dimensioning data from previous iterations, re-computing the hour operation MILP model and updating the system design, until the configuration reaches a net revenue equilibrium, producing a stable and “optimal” system configuration. The relocation operations are only considered as an additional term of the costs of the system, not being included explicitly as a decision variable in the MILP hourly problem.

In order to include demand uncertainty in the model, the individual trips are represented as a willingness to travel using this transport mode, and later aggregated into a person probability to perform, at least, a bicycle trip during the day. The resulting outputs will be the expected bicycle mobility, revenue and required fleet, during a regular operation day.

The evaluation of the system net revenue results from the fares collected from users that may cover the system costs resulting from three main components: the establishment of the bicycle docking stations, the acquisition of the bicycle fleet and the relocation operations of the bicycles during the day.

The following section presents the mathematical modelling of the hourly optimisation model, which is further refined into sub-operation periods compatible with regular bicycle trips travel times.

3.2. Mathematical formulation

As stated above, this problem was formulated as a MILP and was solved through a branch-and-bound procedure using the FICO Xpress optimiser.

The model presents the following formulation:

Sets: $N = \{1,...,i,...,N\}$ set of all candidate sites for the location of docking stations, where $N$ is the total number of docking stations; $T1 = \{1,...,t1,...,T1\}$ set of operational time steps in one day; $T2 = \{1,...,t2,...,T2\}$ set of relocation time steps in one day; $D=\{1,...,p,...,P\}$ set of demand from site $i$ to $j$ at time step $t$; $V=\{1,...,v,...,V\}$ set of candidate trips during one day of operation; $K=\{1,...,k,...,K\}$ set of persons that perform trips $v \in V$

Decision variables: $X_{u_ij}$: binary variable that defines if the trip $v \in V$ uses the station $i \in N$ and $j \in N$ with a type of bicycle $e$; $S_{let}$: integer variable that identifies the balance of bicycles in station $i \in N$ of type $e \in \{0,1\}$ at time step $t1 \in T1$ during the time step $t2 \in T2$; $Y_i$: binary variable that identifies the active docking stations; $Z_{ie}$: integer variable that identifies the capacity of station $i \in N$ of type $e \in \{0,1\}$; $F_e$: integer variable that identifies the total bicycle fleet of type $e \in \{0,1\}$; $I_{etiz}$: integer variable that identifies the number of bicycles of type $e \in \{0,1\}$.
{0,1} in station \( i \in N \) at time step \( t2 \in T2 \); \( M_{e,t2} \): integer variable that identifies the number of bicycles that of type \( e \in \{0,1\} \) in station \( i \in N \) that need to be relocated; \( H_k \): binary variable that defines if the person \( k \in K \) performs a trip

**Data:** \( A_{ije} \): matrix that represent the travel time estimates to travel from docking station \( i \in N \) to depot \( j \in N \) with a bicycle of type \( e \in \{0,1\} \); \( U_{t2} \): binary variable that identifies if the user \( k \in K \) already used the system in an interval \( t2' < t2 \); \( \tilde{Z}_{t2e} \): integer variable that identifies the capacity of depot \( i \in N \) of type \( e \in \{0,1\} \) obtained in an interval \( t2' < t2 \); \( F_{t2e} \): integer variable that identifies the total bicycle fleet of type \( e \in \{0,1\} \) obtained in an interval \( t2' < t2 \); \( B_{t2je} \): integer variable that identifies the number of bicycles that did not arrive to the depot in the previous interval \( t2 \)

**Constants:** \( Z_{\text{max}} \): maximum capacity of docking stations; \( Z_{\text{min}} \): minimum capacity of docking stations; \( P_0 \): fare rate of a normal bicycle per time step \( t1 \in T1 \); \( P_1 \): fare of an electric bicycle per time step \( t1 \in T1 \); \( P_s \): type of fare system annual pass vs. trip fare; \( P_d \): daily fare per user; \( C_s \): space cost in station \( i \in N \) for one bicycle per time step \( t1 \in T1 \); \( C_d \): fixed docking stations cost per time step \( t1 \in T1 \); \( C_q \): cost of a normal bicycle per time step \( t1 \in T1 \); \( C_q \): cost of an electric bicycle per time step \( t1 \in T1 \); \( E \): elasticity of the probability of riding the bike share per percentage of travel time saved for using an electric bicycle; \( F_{\text{max}} \): maximum number of bicycles in the system.

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**Figure 1** Bike-sharing planning system model framework

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Test several days
Day Operation
---

Hour Operation
Bicycle Fleet and depot location (MILP)
Transition variables
User (\( U_t \))
Depot capacity (\( Z_{\text{im}} \))
Total bicycle fleet (\( F_{\text{in}} \))
Travelling bicycle fleet (\( B_{tje} \))
---

Relocation Operations

Difference in Net Revenue
Stop (\(< 1\%\))
---

Optimal Configuration
---

Yes
No
With this notation, the objective function is described by the following expression:

Revenue:

\[ P_p P_0 \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} X_{vij0} D_v A_{ij0} + P_1 \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} X_{vij1} D_v A_{ij1} E \left( 1 + E \frac{A_{ij0} - A_{ij1}}{A_{ij0}} \right) + P_2 (1 - P_3) (1 - U_{t2}) \sum_{k \in K} H_k \]  

(1)

Depot cost:

\[ C_d \sum_{i \in N} Y_i - C_s \sum_{i \in N} \sum_{e \in \{0, 1\}} Z_{ie} \]  

(2)

Fleet costs:

\[ C_0 F_0 + C_1 F_1 \]  

(3)

**Net Revenue = Revenue − Depot Costs − Fleet Costs**

(4)

Where the goal is to maximise the net revenue of the operation period \( t2 \) of the bicycle sharing system. This solution space is subject to the following constraints:

\[ \sum_{i \in N} \sum_{j \in N} \sum_{e \in \{0, 1\}} X_{vije} \leq 1 \ \forall \ v \in V \]  

(5)

Ensures that each trip is assigned only to one pair of origin-destination depots \((i, j) \in N\)

\[ \sum_{v \in V} \sum_{j \in N} X_{vije} D_v \left( 1 + E \frac{A_{ij0} - A_{ij1}}{A_{ij0}} \right) \]

\[ \leq l_{te2} - s_{te1} + M \left( 1 - \sum_{v \in V} \sum_{j \in N} X_{vije} \right) \ \forall \ i \in N, t1 < T1, t2 > T2, e \in \{0, 1\} \]  

(6)

Warrants that a bicycle sharing trip can only be started at station \( i \in N \) if a bicycle is available at moment \( t1 \in T1 \). This constraint is only active when trip is performed between \( i \) and \( j \in N \), otherwise a relaxation to this constraint is introduced by a parameter \( M \) set to a high value.

\[ \sum_{v \in V} \sum_{j \in N} X_{vije} D_v \left( 1 + E \frac{A_{ji0} - A_{ji1}}{A_{ji0}} \right) \]

\[ \leq Z_{ie} - l_{te2} - s_{te1} + M \left( 1 - \sum_{v \in V} \sum_{j \in N} X_{vije} \right) \ \forall \ i \in N, t1 < T1, t2 > T2, e \in \{0, 1\} \]  

(7)

Guarantees that a bicycle sharing trip can only arrive at station \( i \in N \) if a free space is available at that depot at moment \( t1 + a_{ije} \in T1 \). This constraint is only active when trip is performed between \( i \) and \( j \in N \), otherwise a relaxation to this constraint is introduced by a parameter \( M \) set to a high value.
\[ \sum_{e \in \{0,1\}} Z_{ie} \leq Z_{\text{max}} \quad \forall \, i \in N \]  

\text{(8)}

Ensures that the maximum depot capacity is not exceeded.

\[ I_{let2} + S_{let1} \leq Z_{le} \quad \forall \, i \in N, \, t1 \in T1, \, t2 \in T2, \, e \in \{0,1\} \]  

\text{(9)}

Warrants that the instantaneous fleet available at depot \( i \in N \) does not exceed the depot capacity.

\[
S_{let1} \geq \sum_{j \in N} \sum_{v \in V} D_v X_{vijle} \left( 1 + E \frac{A_{ij0} - A_{ij1}}{A_{ij0}} \right) + \sum_{j \in N} \sum_{v' \in V} \sum_{v \in V} D_{v'} X_{v'jile} \left( 1 + E \frac{A_{ij0} - A_{ij1}}{A_{ij0}} \right) \quad \forall \, i \in N, \, t1 \in T1, \, e \in \{0,1\} 
\]  

\text{(10)}

Guarantees that the balance of bicycles related to the initial time step \( t2 \in T2 \) is greater or equal than the sum of bicycles departed and arrived at the instant \( t1 \in T1 \). The arriving trips \( v' \in V \) started a trip at \( t_1 < t_1 \).

\[ I_{let2} + S_{let1} \geq 0 \quad \forall \, i \in N, \, t1 \in T1, \, t2 \in T2, \, e \in \{0,1\} \]  

\text{(11)}

This constraint is related to the fleet conservation. It ensures that the total number of bicycles docked at the depot or moving has to remain constant for the entire interval \( t2 \in T2 \).

\[
F_e \geq \sum_{t \in N} \left[ I_{let2} - S_{let1} \right] + \sum_{v \in V} \sum_{t \in N} \sum_{j \in N} \sum_{v \in V} D_v X_{vijle} \left( 1 + E \frac{A_{ij0} - A_{ij1}}{A_{ij0}} \right) \forall \, t1 \in T1, \, t2 \in T2, \, e \in \{0,1\} 
\]  

\text{(12)}

Warrants that the number of bicycles at any depot is always greater or equal than zero.

\[ \sum_{e \in \{0,1\}} F_e \leq F_{\text{max}} \]  

\text{(13)}

Guarantees that the estimated fleet is smaller than a maximum threshold.

\[ \sum_{e \in \{0,1\}} Z_{le} \geq Z_{\text{min}} Y_{t} \quad \forall \, i \in N \]  

\text{(14)}

Ensures that the capacity of a depot is greater than a minimum threshold.

\[ Y_{t} \leq \sum_{e \in \{0,1\}} \frac{Z_{ls}}{Z_{\text{max}}} \quad \forall \, i \in N \]  

\text{(15)}

Warrants that a depot is only considered when it presents capacity.

\[ \sum_{t \in N} I_{let2} = F_e \quad \forall \, t2 \in T2, \, e \in \{0,1\} \]  

\text{(16)}
Guarantees that the fleet of time interval $t_2 \in T2$ is distributed in its initial time step $t_1 \in T1$.

$$H_k \leq \sum_{\mathcal{P} \in \mathcal{E}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \sum_{e \in (i,j)} X_{eij} \tag{17}$$

Ensures that a potential user is only considered as a client after performing a trip during a time interval $t_2 \in T2$.

4. Lisbon Case-study

Lisbon is the Capital city of Portugal and is the largest city of the country with approximately 565 thousand inhabitants in an area of 84.6 km$^2$. The city is situated on the Atlantic Ocean coast on the Tagus estuary, being the most western capital in mainland Europe. Lisbon is the centre of the Lisbon Metropolitan Area (LMA), which has approximately 2.8 million inhabitants, representing roughly 25 percent of Portugal population, with an area of 2,962.6 km$^2$, formed by 18 municipalities.

![Figure 2 Spatial and temporal distribution of demand.](image)

The LMA generates daily, approximately, 3.23 million trips, from which, approximately, 882 thousand are performed inside the municipality of Lisbon. This large number of trips presents an average length of 3.2 km, which are fairly compatible with the natural space-time range of bicycle trips. The formulated model was tested to a subarea of the city of Lisbon, formed from its main activity centre that contains 38.08 percent of the city
traffic, with an area of 23.81 km$^2$. This subarea presents an average trip length of 1.80 km, which represents an attractive mobility range for the bicycle. Figure 2 illustrates the temporal and spatial distribution of demand of bicycle trips in the study area. This demand presents a considerable variability in intensity and geographical location during the day, following the traditional commuting patterns from the morning peak towards work and school locations and the inverse in the afternoon. Yet, there is a significant share of trips performed between the peak periods in the study area resulting from other trip purposes (e.g. leisure, shopping). During the night, there is a considerable sprawl of demand, leading to a more complex configuration of the supply to satisfy properly the estimated demand.

The selected study area presents a limited number of dedicated bicycle paths and lanes, which would not influence significantly the paths chosen by users to perform trips. For this reason the two existent dedicated bicycle lanes were not regarded in this study.

4.1. Data settings

In order to estimate the potential demand of a bike-sharing system for the city of Lisbon, a database was generated through a synthetic travel simulation model developed and calibrated for the Lisbon Metropolitan Area, presenting all the trip extremes discretised both in space (at the census block level) and in time (different trip departure and arrival times) (Viegas & Martínez, 2010). After filtering the trips contained in the study area, an evaluation of the propensity or willingness of each trip to be performed on bicycle was estimated, using a calibrated discrete choice mode share model for the city of Lisbon (Eiró & Martínez, 2012).

The model generated a total demand in the study area of 197,324 trips performed by 98,740 travellers, from which 29,639 trips of 15,274 persons are willing to use bike as their mobility option.

The location of the potential depots was determined using a traditional capacitated p-median problem, whose objective is to minimise the walking distance of the candidate users to depots, given a minimum distance between depots, and a maximum walking time of a candidate user to its assigned depot. With this model, it was obtained 565 potential depots located in the study area.

The first step in the application of the model formulation described above for the city of Lisbon is the data preparation, which includes a considerable pre-processing work:

- Development of a combined rule based model with a calibrated choice model that defines the willingness of users to select a determined source/sink depot and type of bicycle for a specific trip;
- Geocode the potential depots for electric bicycles, using the current location of electric car charging stations;
- Computation of travel times of the OD matrix that contains all the possible paths between depots using a GIS network shortest path algorithm with a digital elevation model. The differentiation of the shortest path for the electric bicycles was included by disregarding the roads altimetry variation;
- Computation of walking times for each candidate user from the determined location to the different potential depots;

4.2. Design of the possible system configurations

To better evaluate the viability of a large-scale bike-sharing system for the Lisbon city, several scenarios were designed and tested. These scenarios present several possible system attributes, like the lifespan of its infrastructures and bicycles, the fare system applied and the variation on the willingness of using an electric bike for each travel time minute saved.

From the demand side, there are three possible variations: one is considering an annual card for regular bicycles and an additional fee for electric bikes using, a second considering a fee for each individual trip and a third one with an increase to the base willingness for using electric bicycles (measured with an elasticity relative to the time saved). The established cost structure of the system considers a lifespan of three years for the bicycles
docking stations, one year for the regular bikes and 1.5 years for the electric bikes. These low values results from a benchmark of system already under operation were the theft and vandalism towards these systems has proven to be high. Table 1 presents in detail the values used for each attribute tested.

### Table 1: Specification of the cost and fare parameters of the tested scenarios for the Lisbon bike-sharing network

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Annual card with additional fee for electric bikes riding</td>
<td>3</td>
<td>1</td>
<td>1.0</td>
<td>0.15</td>
<td>0.00</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Individual trip fee</td>
<td>3</td>
<td>1</td>
<td>1.0</td>
<td>0.00</td>
<td>0.60</td>
<td>1.80</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Electric bikes oriented system</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
<td>0.15</td>
<td>0.00</td>
<td>1.00</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Full demand coverage</td>
<td>3</td>
<td>1</td>
<td>1.0</td>
<td>0.15</td>
<td>0.00</td>
<td>1.00</td>
<td>1</td>
</tr>
</tbody>
</table>

### 4.3. Results

In this section we will analyse all the resulting outputs from all the scenarios tested with the developed model. Table 2 presents the general dimensioning outputs of the model, where a clear difference of system optimal layout resulting from the fare system used can be observed. The annual card fare system leads the system to increase the efficiency of the client per bike rate, producing the lowest fleet of the estimated scenarios and not warranting the commuting nature of some trip chain with a trip-client ratio close to one. The individual trip fee leads to a system configuration with a high trip-client ratio (2.54), a large fleet and a good docking station spatial coverage. The increase in the willingness to use electric bicycles (Scenario 3) to travel showed a small impact in the system configuration, mainly derived from the higher costs of this type of bicycles, but also due to the limited coverage of the existing electric car charging stations. The full demand coverage scenario (Scenario 4) with an annual fee fare system resulted in a similar configuration to the trip based fee system (Scenario 2), although producing a smaller spatial coverage of the docking stations.

The spatial distribution of the estimated docking stations is presented in Figure 3, showing a good spatial distribution of the system capacity although with less intensity in the Easter area due to a smaller estimated demand. The active electric bike docking stations were placed in areas of great demand density and where the altimetry variation in the surrounding areas may pay-off using an electric bike.

### Table 2: Outputs of the model of each scenario

<table>
<thead>
<tr>
<th>Sc. Code</th>
<th>Total trips served</th>
<th>Total clients</th>
<th>Regular bikes fleet</th>
<th>Electric bikes fleet</th>
<th>Stations</th>
<th>Regular bikes capacity</th>
<th>Electric bikes capacity</th>
<th>Trips /bike</th>
<th>Client/bike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc 1</td>
<td>15,520</td>
<td>15,181</td>
<td>2,033</td>
<td>5</td>
<td>272</td>
<td>3,627</td>
<td>20</td>
<td>7.62</td>
<td>7.45</td>
</tr>
<tr>
<td>Sc 2</td>
<td>28,071</td>
<td>11,022</td>
<td>2,096</td>
<td>0</td>
<td>276</td>
<td>4,444</td>
<td>0</td>
<td>13.39</td>
<td>5.26</td>
</tr>
<tr>
<td>Sc 3</td>
<td>15,453</td>
<td>15,041</td>
<td>2,112</td>
<td>15</td>
<td>228</td>
<td>3,875</td>
<td>62</td>
<td>7.27</td>
<td>7.07</td>
</tr>
<tr>
<td>Sc 4</td>
<td>29,639</td>
<td>15,274</td>
<td>2,335</td>
<td>0</td>
<td>264</td>
<td>4,464</td>
<td>0</td>
<td>12.69</td>
<td>6.54</td>
</tr>
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</table>
Table 3 presents the results of the daily revenue, total costs, net income and the annual balance for each tested scenario. In terms of financial performance the different systems tested, the trip based fee scenario (Scenario 2) proved to be more profitable, not needing any additional financial support from public entities or related business support of this type of systems as publicity. The scenarios 1 and 3 did also lead to balanced financial configurations, although slightly negative in the first situation, requiring an annual public contribution of 21,896 euros. The full demand coverage scenario would require a considerable contribution from alternative funding sources above mentioned.

Table 3 Scenarios balance

<table>
<thead>
<tr>
<th>Sc. code</th>
<th>Revenue [€]</th>
<th>Total costs [€]</th>
<th>Net income [€]</th>
<th>Annual balance [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>-60</td>
<td>2,331</td>
<td>2,391</td>
<td>- 21,896</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>2,037</td>
<td>5,383</td>
<td>3,346</td>
<td>743,471</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>180</td>
<td>2,836</td>
<td>2,656</td>
<td>65,598</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>-632</td>
<td>2,683</td>
<td>3,314</td>
<td>- 230,553</td>
</tr>
</tbody>
</table>
Analysing the cost structure of the different tested scenarios, it can be easily perceived the significant burden introduced from the fleet relocation operations during the day (between 30 and 40 percent of the costs), especially for the Scenario 4 where all trips intend to be served (see Figure 4). This fact shows the importance of this component on the overall system performance, and the relevance of its inclusion on the design process of the system. A finer optimisation of these costs may be performed during the system operation introducing a variable fare system that aids to the system to move to more auto-balance configurations. This operational optimisation will be pursued in further developments of this research using simulation-optimisation techniques used in the literature for similar problems (Chen & Lee, 2011).

The estimated infrastructure costs are also significant, presenting values close to 30 percent in all the tested scenarios. This component may be also improved by introducing vandalism mechanisms close to the docking stations location as video surveillance systems. Moreover, the fleet costs, which represent the base cost of the system, present a high value derived from the reduced expected lifespan of the fleet, increasing significantly in the electric bike oriented scenario. A greater lifespan of the fleet is also a key for the financial sustainability of the system, as regarded in the literature (e.g. (Shaheen, et al., 2010)), which may be improved also by introducing video surveillance systems and a greater control and liability towards the system’s users.

5. Summary and concluding remarks

Shared bike systems have emerged around the world as a viable urban mobility alternative, being already widely spread. These systems have been quickly evolving in the last decades and currently they are integrated with other existing transportation modes of many cities. This study addresses the design problem of a mixed sharing system with regular and electric bicycles. The conceived approach integrates the users demand, the required investment, as well as the operational costs and different types of fare schemes. The developed mathematical formulation resulted in a heuristic encompassing an operational MIP problem, being applied to a real case study: the city of Lisbon. The model evaluation comprises a sensitivity analysis to test the influence of the considered parameters in the system design.

Regarding the formulation, this proved to be efficient and sensitive to different operational configuration for a medium to large case study, showing great potential for further development.

The tested model configurations presented a good performance of the system in the city of Lisbon, obtaining almost financially balance scenarios with a required short contribution from additional revenue sources. Nevertheless, some of the key issues identified in the literature as the main drawback of this type of systems were...
confirmed, showing that solving the vandalism and the theft problem of the system infrastructure are the main challenge of bike-sharing programs.

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References