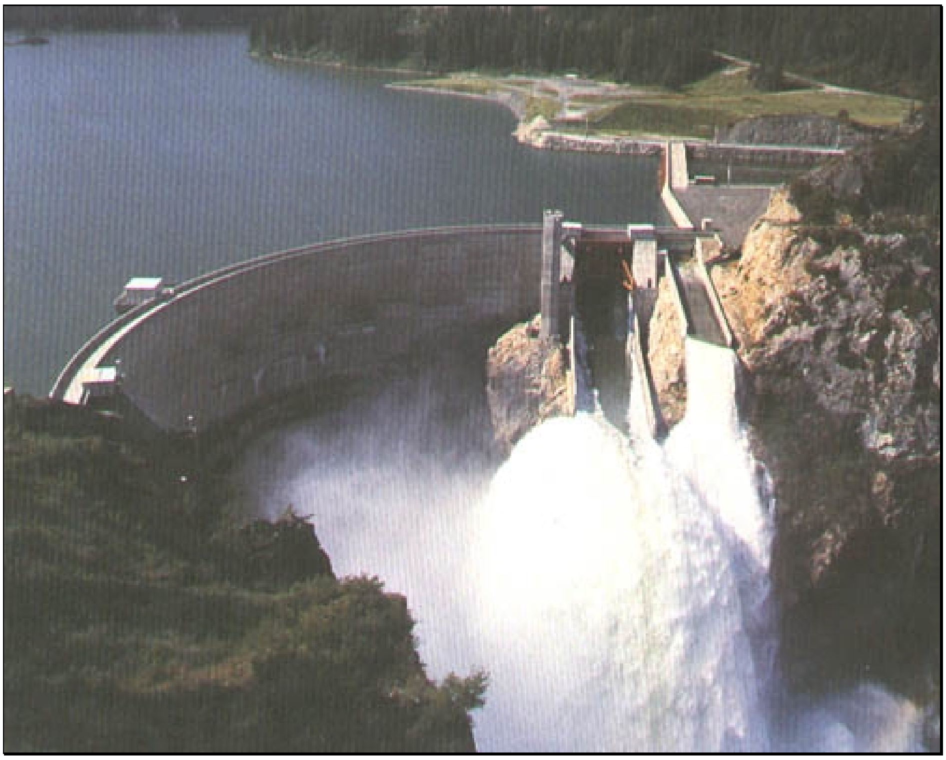
WAVELETS ON THE INTERVAL

APPLICATION TO ELASTICITY PROBLEMS

9 –12 April 2001
Marseille, France

- 3D
    - Dams (concrete)



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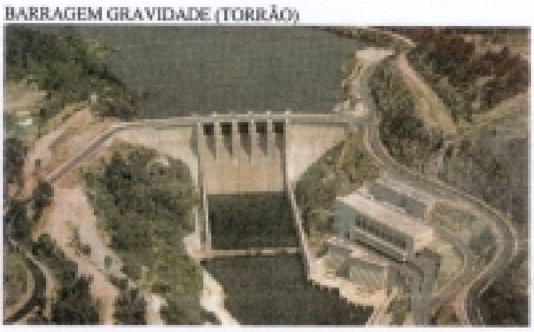
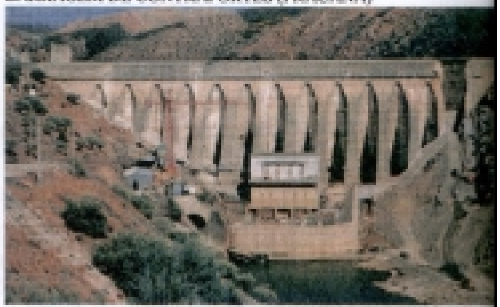
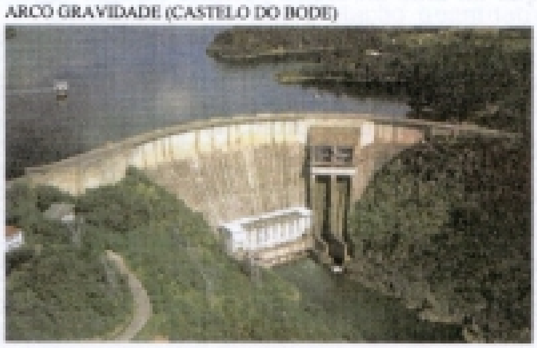
BARRAGEM GRAVIDADE (TORRÃO)



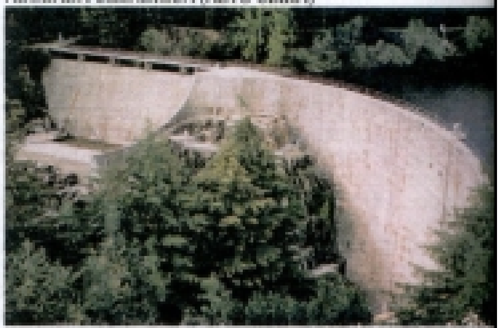
BARRAGEM DE CONTRAFORTES (PRACANA)



ARCO GRAVIDADE (CASTELO DO BODE)

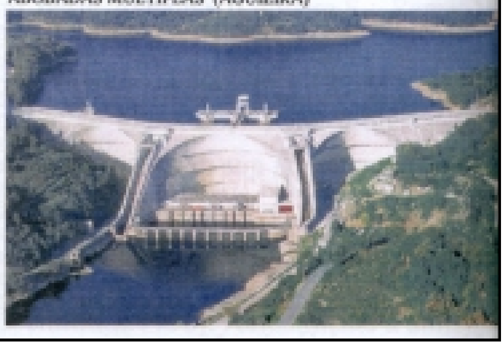


ABÓBADA CILÍNDRICA (ALTO CEIRA)



ABÓBADA DE DUPLA CURVATURA (CABRIL)

ABÓBADAS MÚLTIPLAS (AGUIEIRA)



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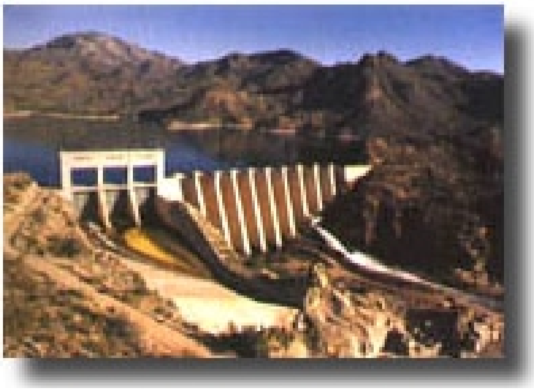
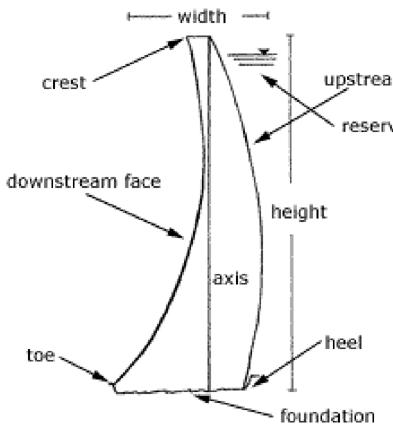
ARCH DAMS

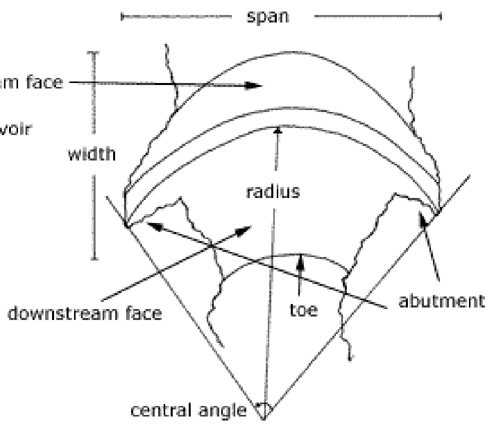
BUTTRESS DAMS

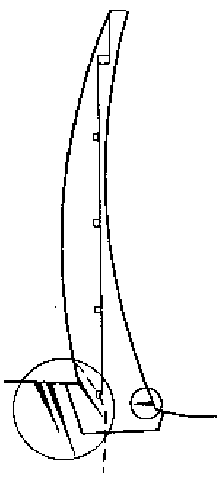


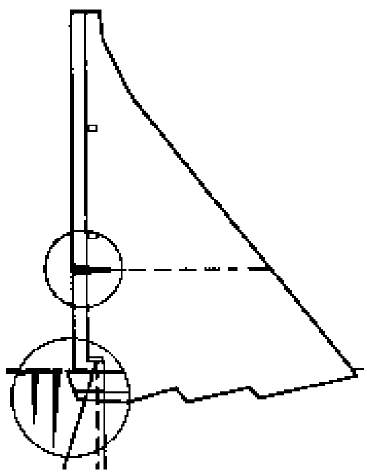
cross section

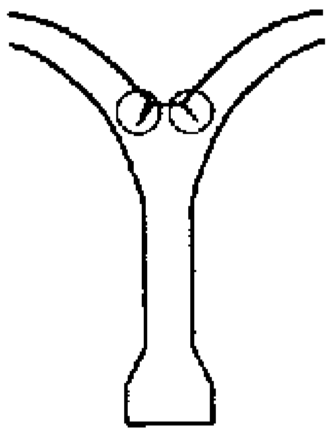


plan view



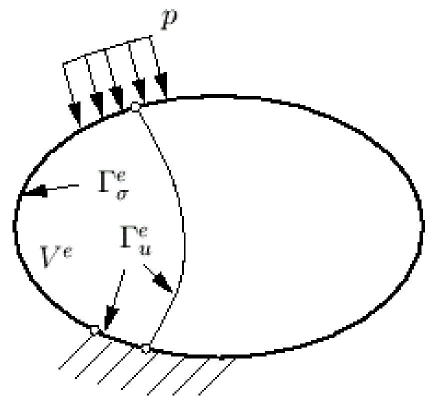






1. Introduction to the problems we study

- the domain V^e
- the surface Γ divided in two complementary parts:
  - Dirichlet boundary Γ_u^e
  - Neumann boundary Γ_σ^e

	EQUILIBRIUM CONDITIONS	**ELASTICITY CONDITIONS**	**COMPATIBILITY CONDITIONS**
on V_e	$\sigma_{ij,i} + b_j = 0$ $\mathbf{D\sigma} + \mathbf{b} = 0$	$\sigma_{ij} = -\dfrac{E}{1+\upsilon}\varepsilon_{ij} + \dfrac{E\nu}{(1+\upsilon)(1-2\upsilon)}\varepsilon_{kk}\delta_{ij}$ $\boldsymbol{\varepsilon} = \mathbf{f\sigma}$	$\varepsilon_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i})$ $\boldsymbol{\varepsilon} = \mathbf{D}^*\mathbf{u}$
on Γ	$\sigma_{ij}n_i = t_j$ $\mathbf{N\sigma} = \mathbf{t}$		$u_i = \overline{u_i}$ $\mathbf{u} = \overline{\mathbf{u}}$

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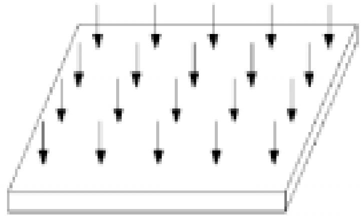
STRETCHING PLATES	BENDING PLATES
	

APPROXIMATION CRITERIA		
$\sigma = S \cdot X$	$u = U_V \cdot q_V$	$u = U_\Gamma \cdot q_\Gamma$

EQUILIBRIUM IN THE DOMAIN	EQUILIBRIUM ON THE BOUNDARY
$\int_V U_V^T (D\sigma + b) dV = 0$	$\int_{\Gamma_\sigma} U_\Gamma^T (N\sigma - t) d\Gamma_\sigma = 0$
$A_V = \int (DS)^T U_V \, dV$	$A_\Gamma = \int_{\Gamma_\sigma} (NS)^T U_\Gamma d\Gamma_\sigma$
$A_V^T X = -Q_V$	$A_\Gamma^T X = Q_\Gamma$

COMPATIBILITY IN THE DOMAIN

$$\mathbf{e} = -\int_V (\mathbf{DS})^T \mathbf{u}.d\mathbf{V} + \int_\Gamma (\mathbf{NS})^T \mathbf{u}.d\Gamma$$

$$\int_V \mathbf{S}^T \left(\boldsymbol{\varepsilon} - \mathbf{D}^* \mathbf{u}\right) d\mathbf{V} = 0$$

$$\mathbf{e} = -\mathbf{A}_V \mathbf{q}_V + \mathbf{A}_\Gamma \mathbf{q}_\Gamma + \bar{\mathbf{e}}$$

ELASTICITY CONDITION

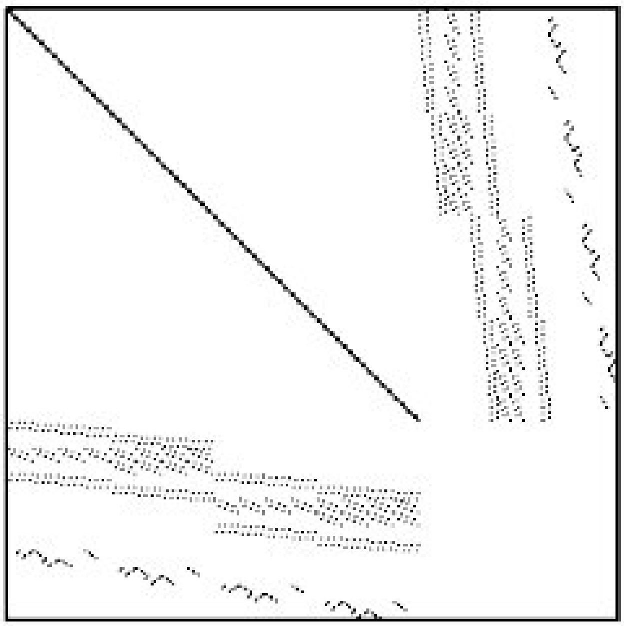
$$\int_V \mathbf{S}^t (\boldsymbol{\varepsilon} - \mathbf{f}\,\boldsymbol{\sigma}) d\mathbf{V} = 0$$

$$\mathbf{F} = \int_V \mathbf{S}^T \mathbf{f} \mathbf{S} d\mathbf{V}$$

$$\mathbf{e} = \mathbf{F}\mathbf{X}$$

GOVERNING SYSTEM

$$\begin{bmatrix} F & A_V & -A_\Gamma \\ A_V^T & \cdot & \cdot \\ -A_\Gamma^T & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x \\ q_V \\ q_\Gamma \end{bmatrix} = \begin{bmatrix} \bar{e} \\ -Q_V \\ -Q_\Gamma \end{bmatrix}$$



2. Why wavelets?

Localization & Adaptability

- Modeling singularities in tension (cracks, damage, …)

What we look for in functions…

- Hierarchical
- Orthogonal
- Fast computation and numerical analysis

2.1 What work was previously done?

- The use of only scaling functions (Daubechies orthogonal wavelets)
    - Elasticity
    - Plasticity
- Manipulation of the wavelets
    - Problems with the boundaries
    - Loss of orthogonality

2.2 Options

- Wavelets on the Interval
- Use of other wavelet systems even if not orthogonal

2.3 What we did…

- Application of Daubechies orthogonal Wavelets on the Interval based on the works of:
  - A. Cohen, I. Daubechies and P. Vial;
  - V. Perrier and P. Monasse

$$\phi_{j,k}^{Left}(x) = \sum_{l=0}^{N-1} H_{k.l}^{Left}\ \phi_{j+1,k}^{Left}(x) + \sum_{m=N}^{N+2C} h_{k,m}^{Left}\ \phi_{j+1,m}(x)$$

- $\phi_{j.k}^{Left}(x) = 2^{j/2}\ \phi_{0.k}^{Left}(2^j x)$
- $\phi_{j,m}(x) = 2^{j/2}\ \phi(2^j x - m)$
- C=2 k where k=0, …, N-1 (Cohen)
- C=N-1 (Perrier)

- Using numerical integration

3. Some results

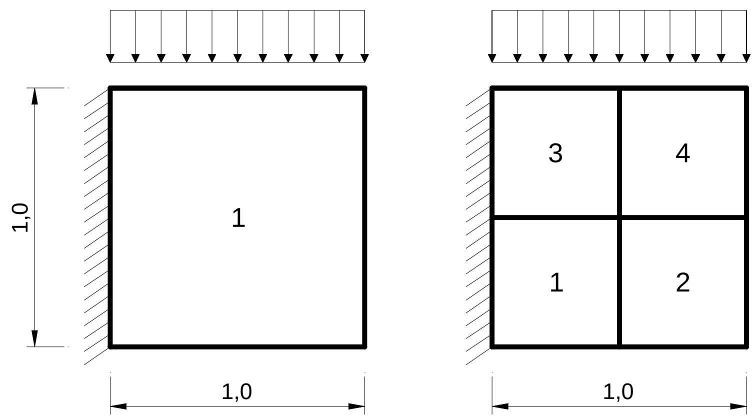
Problems

- Stretching plates
  - Short Cantilever
  - Stressed stretching plate with central crack
  - L plate

Approximations

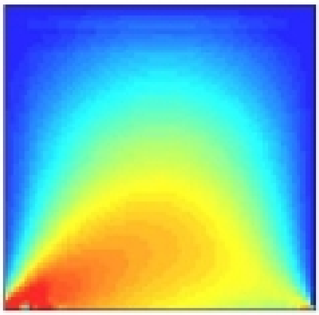
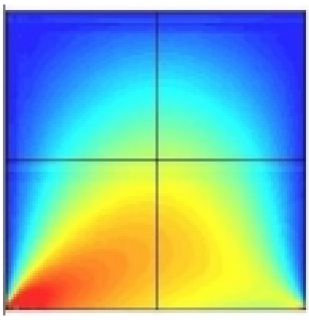
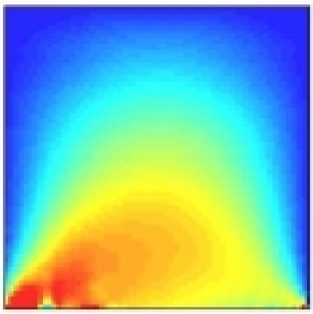
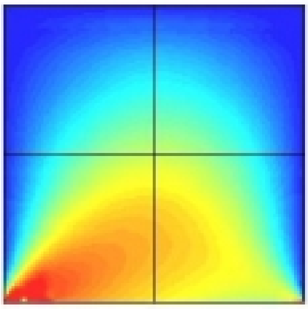
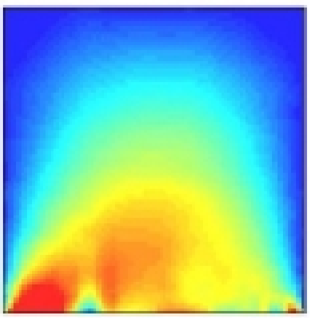
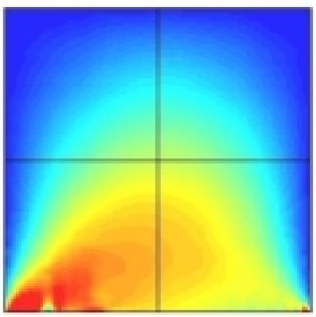
- Type 1: Scaling functions + wavelets
- Type 2: Only scaling functions

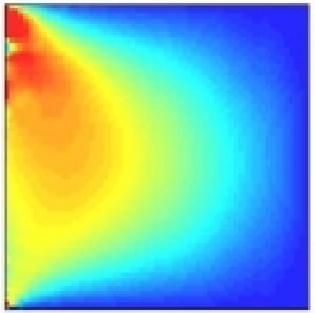
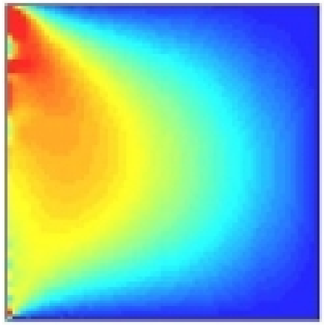
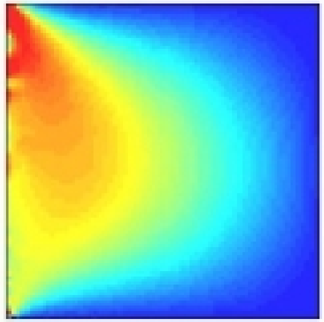
- Short Cantilever (**E = 1.0, ν = 0.3)**

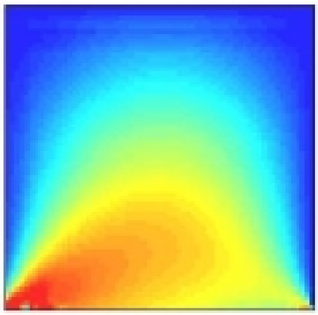
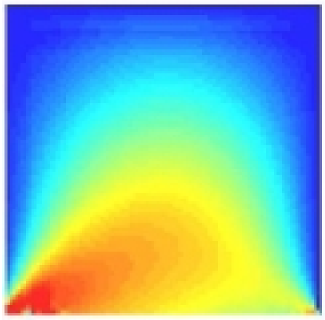
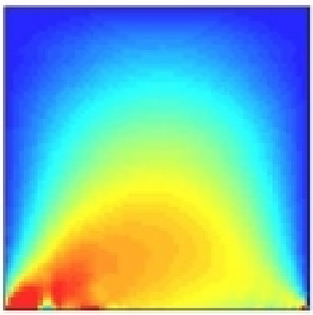
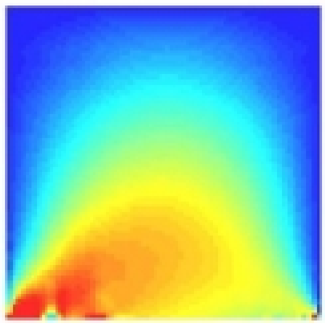
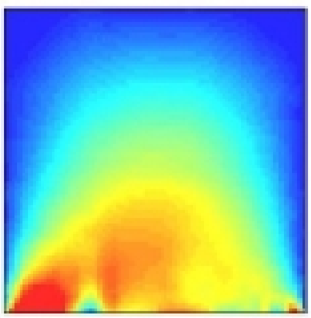


Mesh A Mesh B

Problem	Model		nele	N	jo	jx	jv	jg	α_v	β	**ndf**	nnz	spar
C	T 2	Cdis	1	4	-	4	3	3	768	176	**944**	48196	0.8927
C	T 2	Cdis1	1	4	-	5	4	4	3072	608	**3680**	186096	0.9727
C	T 2	Cdis4	1	5	-	5	4	4	3072	608	**3680**	286054	0.9579
C	T 2	Cdis7	1	6	-	5	4	4	3072	608	**3680**	415740	0.9388
C	T 1	Cdis	1	4	3	4	3	3	3072	608	**3680**	38016	0.9946
C	T 2	Cdis10	4	4	-	4	3	3	3072	672	**3744**	194176	0.9725
C	T 1	Cdis1	1	4	3	5	4	4	12288	2240	**14528**	246272	0.9977
C	T 2	Cdis11	4	4	-	5	4	4	12288	2368	**14656**	741232	0.9931
C	T 1	Cdis3	1	4	3	5	5	4	12288	8384	**20672**	783360	0.9963
C	T 2	Cdis12	4	4	-	6	5	5	49152	8832	**57984**	2812224	0.9983

		T2_CDIS 2		T2_CDIS 12	
N = 4	JX=6, JV=5, JG=5		NNZ=710 084, NDF=14 528		NNZ=2 812 224, NDF=57 984
		T2_CDIS 1		T2_CDIS 11	
	JX=5, JV=4, JG=4		NNZ=186 096, NDF=3 680		NNZ=741 232, NDF=14 656
		T2_CDIS		T2_CDIS 10	
	JX=4, JV=3, JG=3		NNZ=48 196, NDF= 944		NNZ=194 176, NDF= 3 744
		1 ELE		4 ELE	

	N = 4		N = 5		N = 6	
	JX=5, JV=4, JG=4	T2_CDIS 1	JX=5, JV=4, JG=4	T2_CDIS 4	JX=5, JV=4, JG=4	T2_CDIS 7
1 ELE						
	NNZ=186 096, NDF=3 680		NNZ=286 054, NDF=3 680		NNZ=415 740, NDF=3 680	

		T2_CDIS 2			T1_CDIS 1	
N = 4	JX=6, JV=5, JG=5		NNZ=710 084, NDF=14 528	JX=5, JV=4, JG=4		NNZ=246 272, NDF=14 528
		T2_CDIS 1			T1_CDIS	
	JX=5, JV=4, JG=4		NNZ=186 096, NDF=3 680	JX=4, JV=3, JG=3		NNZ=38 016, NDF=13 680
		T2_CDIS				
	JX=4, JV=3, JG=3		NNZ=48 196, NDF= 944			
		T 2			T 1	J0=3

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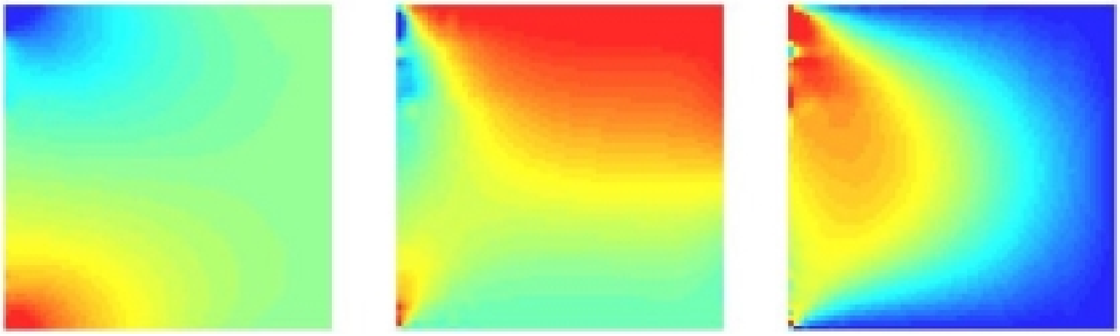
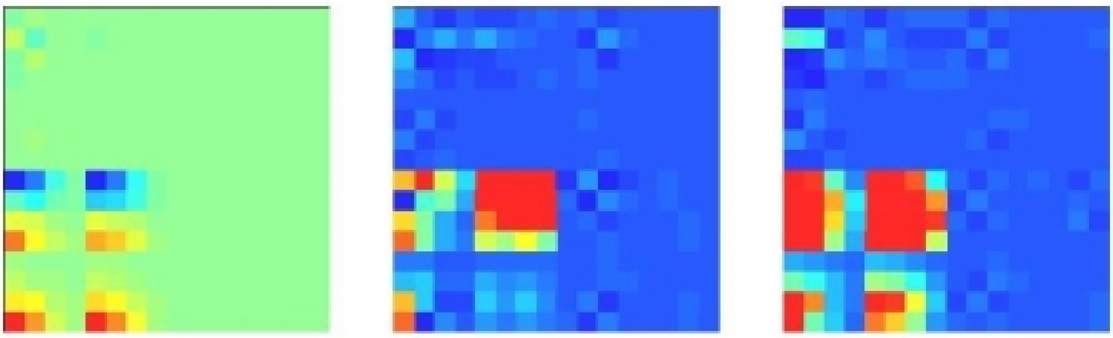
T1_CDIS

σ_{xx} σ_{yy} σ_{xy}

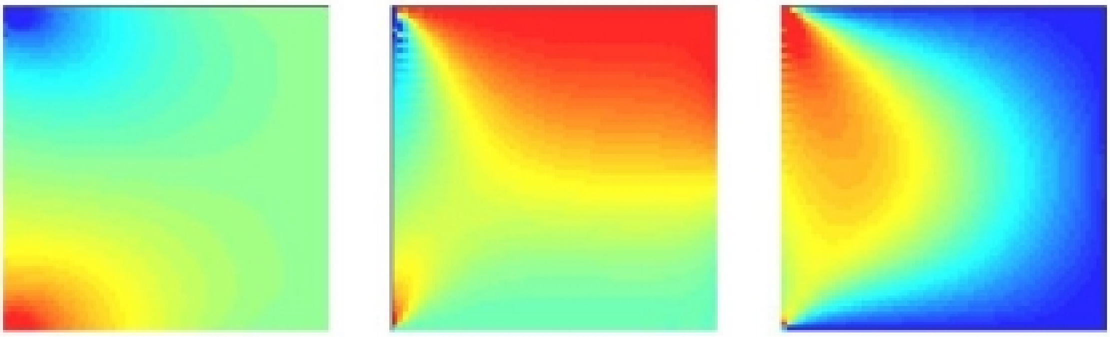


T1_CDIS - MULTIRESOLUTION

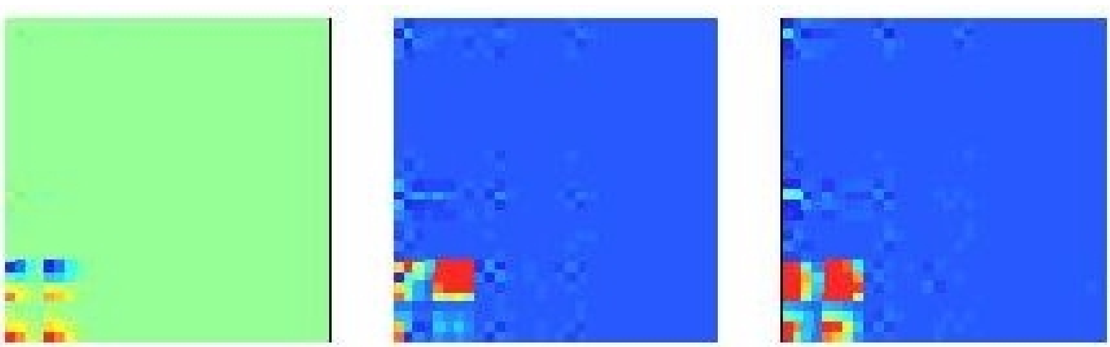


T1_CDIS3

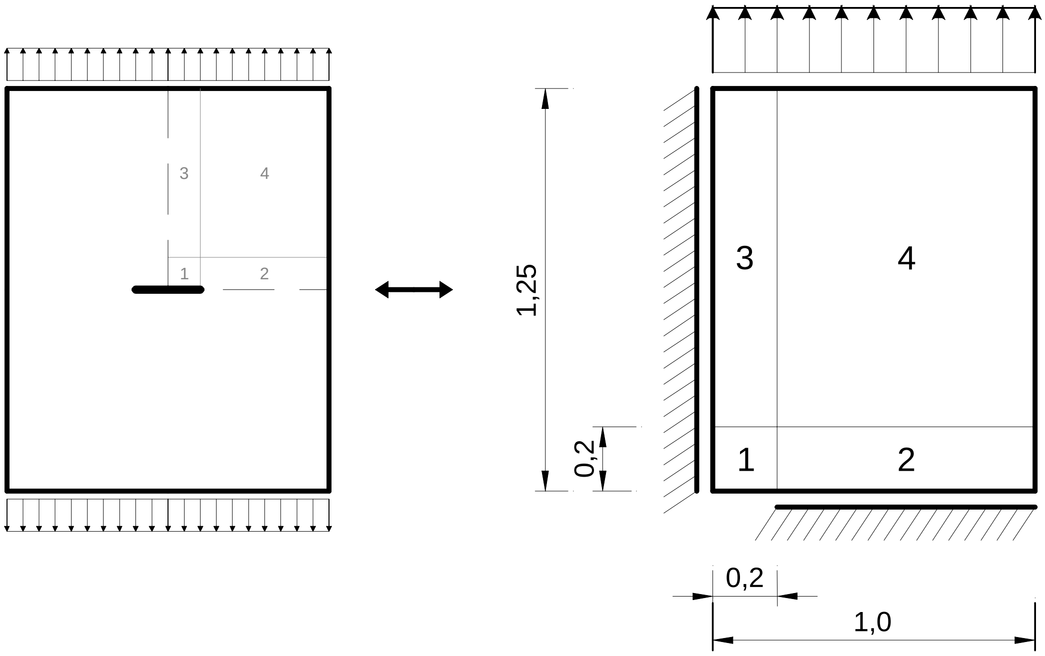
σ_{xx} σ_{yy} σ_{xy}



T1_CDIS3 - MULTIRESOLUTION



- Stressed stretching plate with central crack



Problem	Model		N	jo	jx	jv	jg	α_v	β	**ndf**	nnz	spar
Crack	T 2	T	4	-	5	4	4	12288	2384	**14672**	740596	0.9931
Crack	T 1	T	4	3	4	3	3	12288	2384	**14672**	153024	0.9986
Crack	T 1	T1	4	3	5	4	4	49152	8864	**58016**	987648	0.9994
Crack	T 1	T 2	4	3	5	5	4	49152	33440	**82592**	3136000	0.9990

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T2 T

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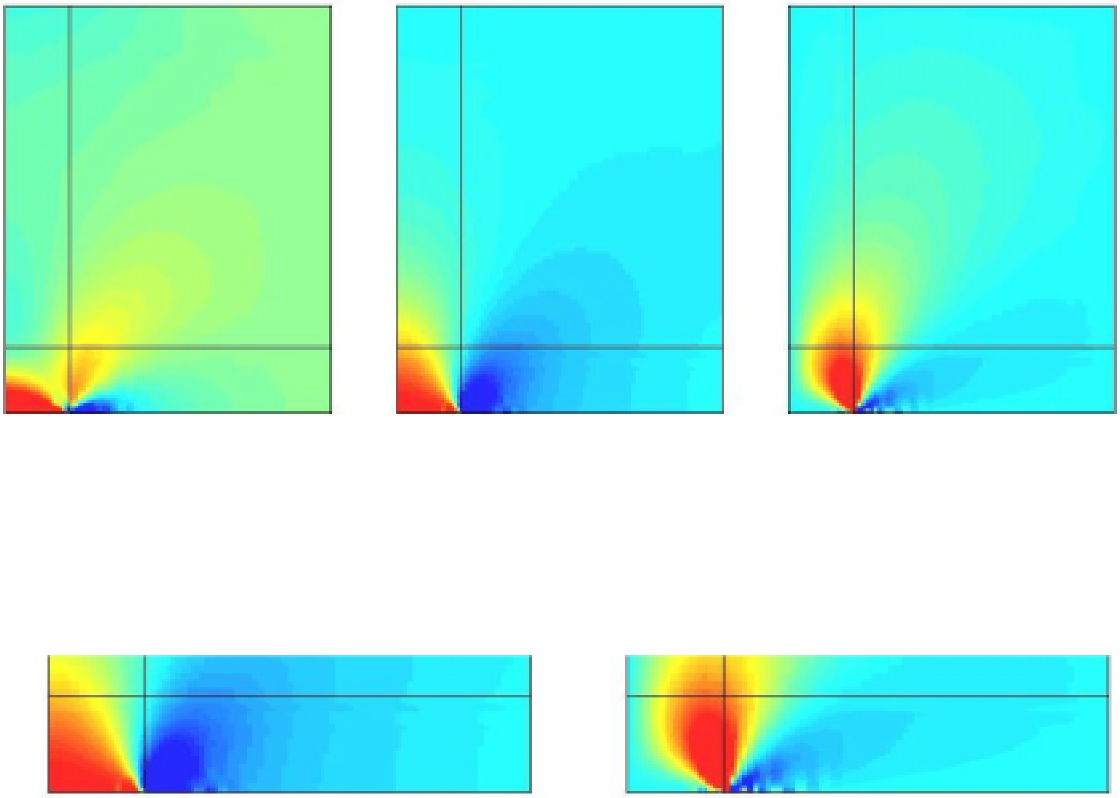
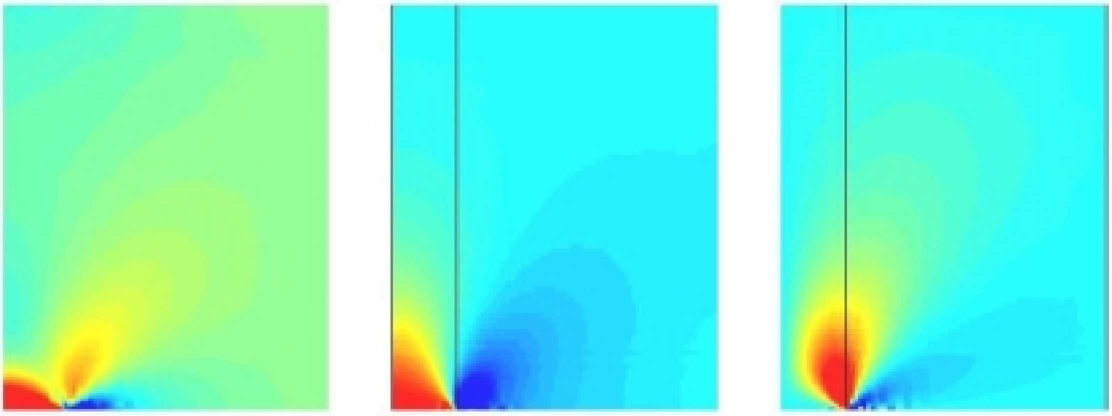


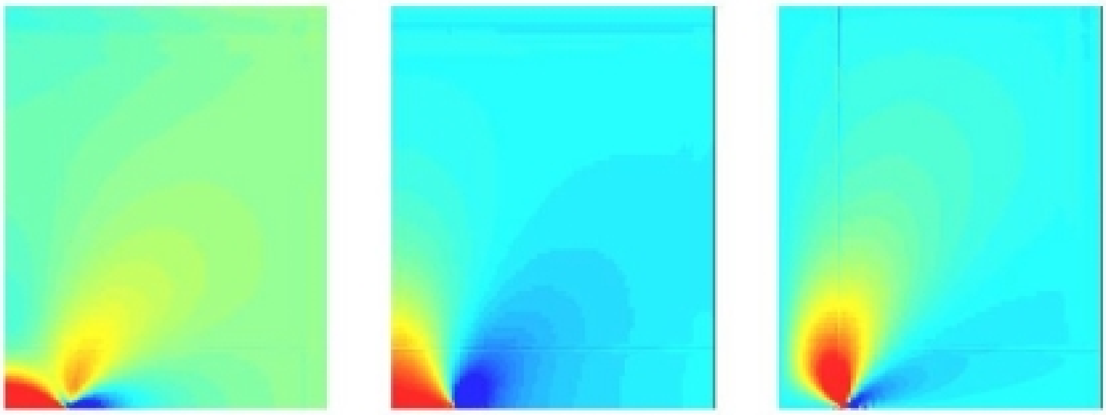
T1 T

σ_{xx} σ_{yy} σ_{xy}



T1 T1

σ_{xx} σ_{yy} σ_{xy}



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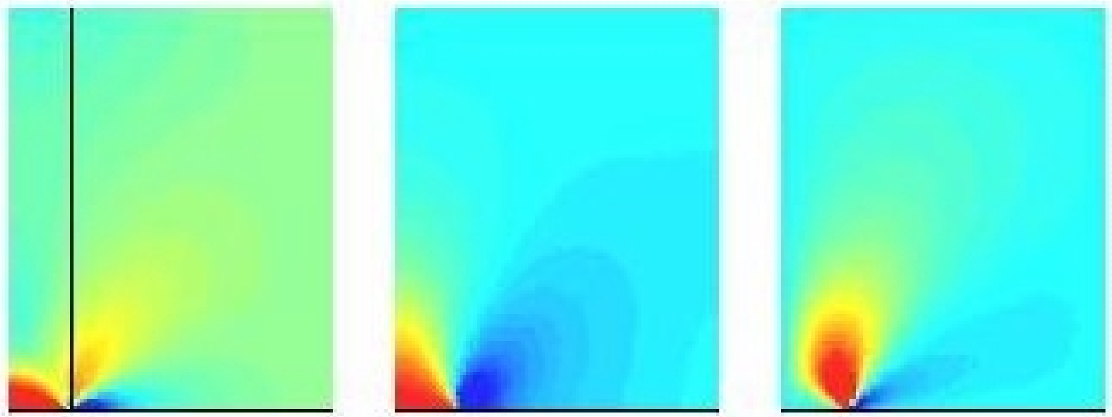
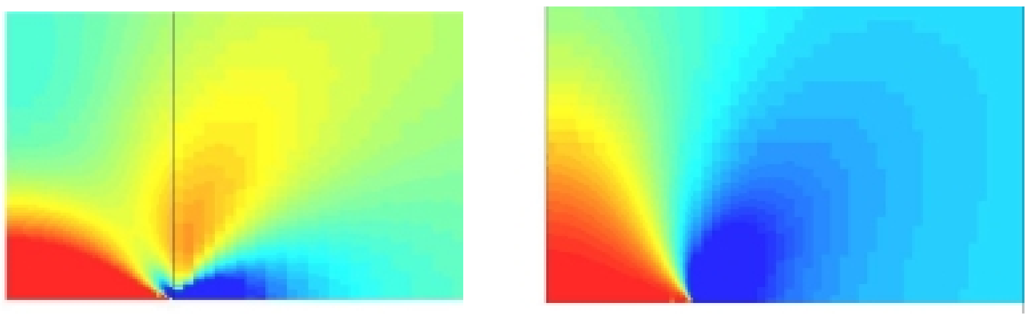
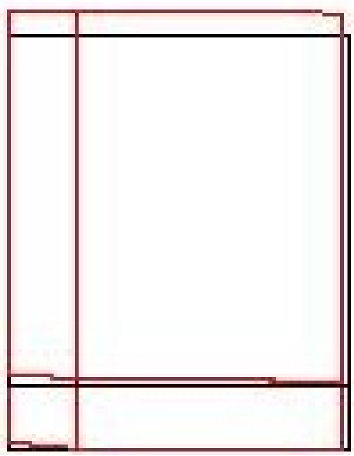
T1 T2

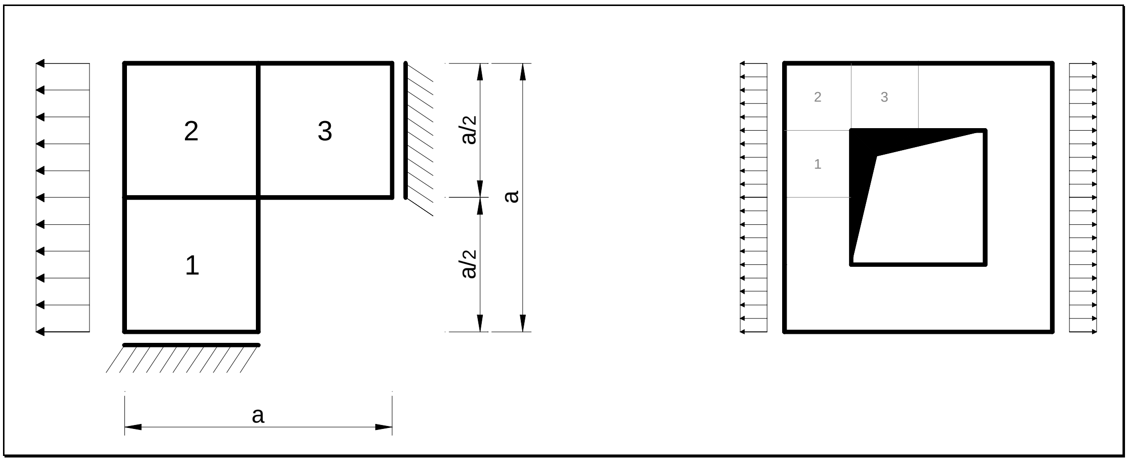
σ_{xx} σ_{yy} σ_{xy}







- L – stretching plate



Problem	Model		N	jo	jx	jv	jg	α_v	β_v	β_γ	**ndf**	nnz	Spar
L	T 1	L	4	3	4	3	3	9216	1536	288	**11040**	114812	0.9981
L	T 1	L 2	4	3	5	5	4	36864	24576	576	**62016**	2352128	0.9987

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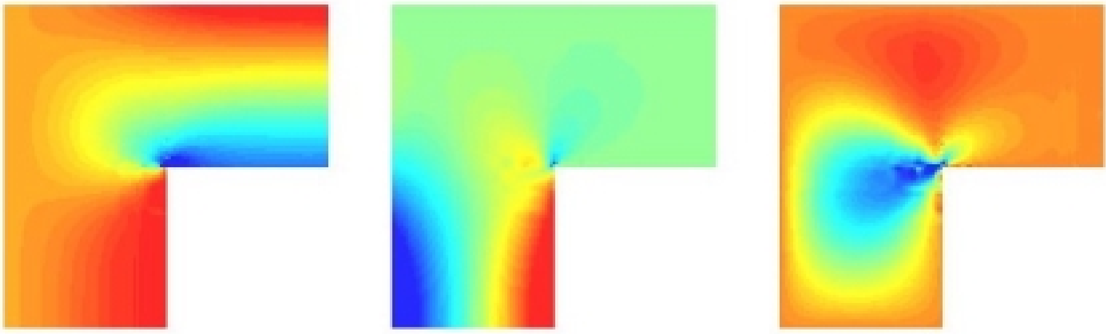
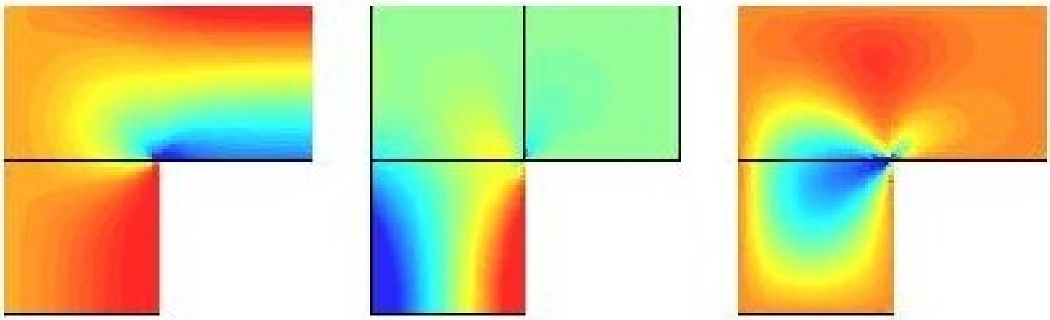
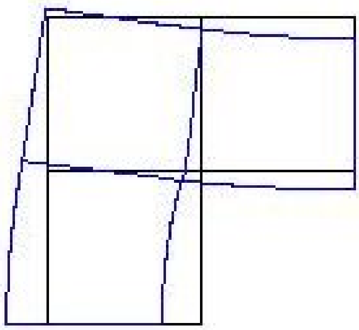
T1_L



T1_L2



DEFORMATION



4. Future work …

- Wavelet related
    - Implementation of analytical integrations based on works of Beylkin, Dahmen and Michelli, and Perrier
    - Comparison between Cohen's and Perrier's Wavelets on the Interval
    - Study of other wavelet systems on the interval
        - Orthogonal (Interacting boundary wavelet)
        - Non orthogonal (bi-orthogonal, …)
    - Implementation of adaptive schemes
    - Physical non-linear analysis (Elastoplasticity, fracture and damage mechanics)
    - 3D models