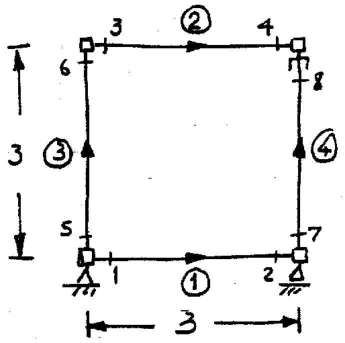
PROBLEMA PROPOSTO 3

Orientação e discretização



Esforços e deformações independentes

$$\underset{\sim}{X}_{①}^{t} = \{ M_1 \ M_2 \ N_2 \}$$

$$\underset{\sim}{X}_{②}^{t} = \{ M_3 \ M_4 \ N_4 \}$$

$$\underset{\sim}{X}_{③}^{t} = \{ M_5 \ M_6 \ N_6 \}$$

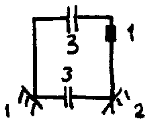
$$\underset{\sim}{X}_{④}^{t} = \{ M_7 \ M_8 \}$$

Hiperestatia

$\alpha_e = 3 - 3 = 0$

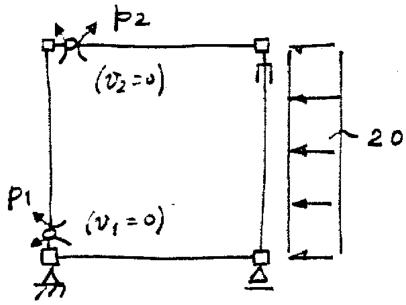
$\alpha_i = 3 - 1 = 2$

$\alpha = 2$

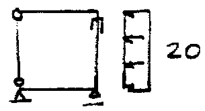


$\alpha = (3+3) - (1+2+1)$

Estrutura-base



Solução particular - Esforços $\underset{\sim}{X}_0$

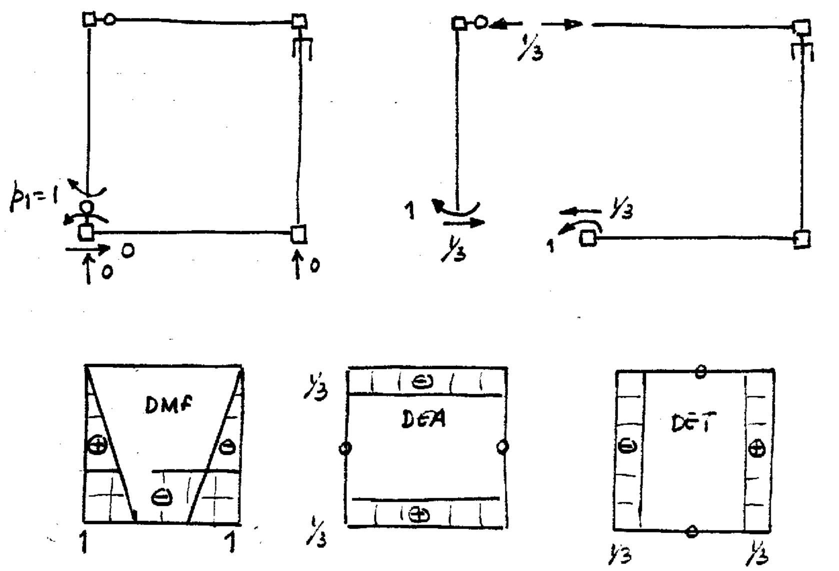


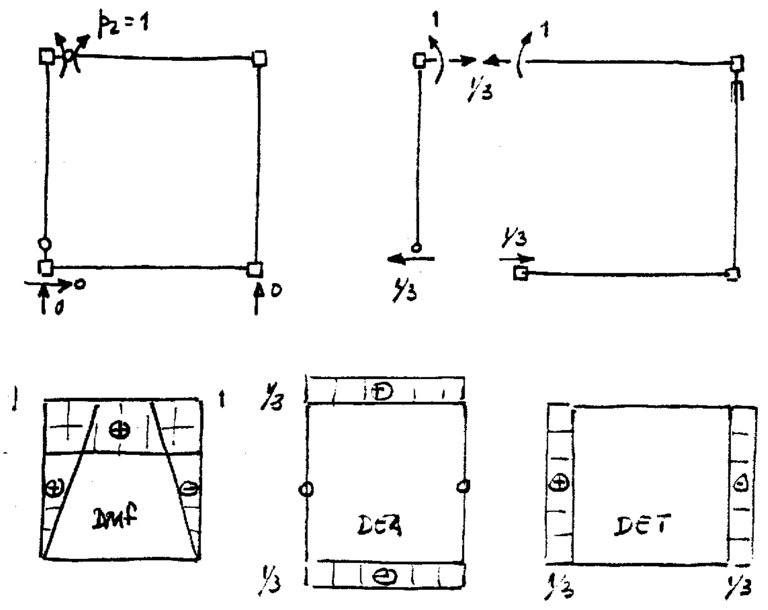
$$\underset{\sim}{X}_0^{t} = \left\{ \begin{array}{ccc|ccc|ccc|cc} 0 & 90 & -60 & 0 & 0 & 0 & 0 & 0 & 0 & 90 & 0 \\ M_1 & M_2 & N_2 & M_3 & M_4 & N_4 & M_5 & M_6 & N_6 & M_7 & M_8 \end{array} \right\}$$

Solução particular - Deformações $\underset{\sim}{u}_0$

$$\underset{\sim}{u}_0^{t} = \frac{1}{EI} \left\{ \begin{array}{ccc|ccc|ccc|cc} 45 & 90 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 67.5 & 22.5 \\ \theta_1 & \theta_2 & e_2 & \theta_3 & \theta_4 & e_4 & \theta_5 & \theta_6 & e_6 & \theta_7 & \theta_8 \end{array} \right\}$$

Solução complementar – Esforços $\underset{\sim}{X}_c = \underline{B}\,\underset{\sim}{p}$





$$\underset{\sim}{X}_c = \underline{B} \cdot \underset{\sim}{p}$$

$$\begin{bmatrix} M_1 \\ M_2 \\ N_2 \\ \hline M_3 \\ M_4 \\ N_4 \\ \hline M_5 \\ M_6 \\ N_6 \\ \hline M_7 \\ M_8 \end{bmatrix}_c = \begin{bmatrix} -1 & 0 \\ -1 & 0 \\ 1/3 & -1/3 \\ \hline 0 & 1 \\ 0 & 1 \\ -1/3 & 1/3 \\ \hline 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \hline -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$\rightarrow \underline{B}_{①}$, $\rightarrow \underline{B}_{②}$, $\rightarrow \underline{B}_{③}$, $\rightarrow \underline{B}_{④}$

Nota: $\underset{\sim}{v} = \underline{B}^t \underset{\sim}{u}$

$$\underset{\sim}{v} = \sum_{m=1}^{4} \underline{B}_m^t \underset{\sim}{u}_m$$

Solucao complementar - Deformacoes $\underset{\sim}{u}_{m_c} = \underline{f}_m \underset{\sim}{X}_{m_c} = \underline{f} \underline{B}_m \underset{\sim}{p}$

Barras 1, 2, 3 $\quad \underline{f}_m = 1/EI \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1/10 \end{bmatrix}$

Barra 4 $\quad \underline{f}_{④} = 1/EI \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$

$$\underset{\sim}{u}_{①_c} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ e_2 \end{bmatrix}_c = \frac{1}{EI} \begin{bmatrix} -3/2 & 0 \\ -3/2 & 0 \\ 1/30 & -1/30 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\underset{\sim}{u}_{②_c} = \begin{bmatrix} \theta_3 \\ \theta_4 \\ e_4 \end{bmatrix}_c = \frac{1}{EI} \begin{bmatrix} 0 & 3/2 \\ 0 & 3/2 \\ -1/30 & 1/30 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\underset{\sim}{u}_{③c} = \begin{bmatrix} \theta_5 \\ \theta_6 \\ e_6 \end{bmatrix}_c = \frac{1}{EI} \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$\underset{\sim}{u}_{④c} = \begin{bmatrix} \theta_7 \\ \theta_8 \end{bmatrix}_c = \frac{1}{EI} \begin{bmatrix} -1 & -1/2 \\ -1/2 & -1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

<u>Solução particular - Descontinuidades</u> $v_0 = B^t u_0 = \sum B_m^t u_{m_0}$

$$\underset{\sim}{v}_0 = \sum_m \underset{\sim}{v}_{m_0} \quad : \quad \underset{\sim}{v}_{m_0} = \underline{B}_m^t u_{m_0}$$

$$\underset{\sim}{v}_0 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_0 = \frac{1}{EI} \begin{bmatrix} -204.5 \\ -20.5 \end{bmatrix} \quad (\text{rad})$$

<u>Solução complementar - Descontinuidades</u> $v_c = B^t u_c = \sum B_m^t u_{mc}$

$$\underset{\sim}{v}_c = \sum \underline{B}_m^t F_m B_m p \; ; \; v_c = F p \; ; \; F = \sum F_m \; ; \; F_m = B_m^t F_m B_m$$

$$\underline{F} = \frac{1}{45EI} \begin{bmatrix} 226 & 44 \\ 44 & 226 \end{bmatrix} \quad (\text{rad}/\text{kNm})$$

<u>Equação do Método das Forças</u> $v = v_c + v_0 = F p + v_0 = 0$

$$\frac{1}{45EI} \begin{bmatrix} 226 & 44 \\ 44 & 226 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \frac{1}{EI} \begin{bmatrix} -204.5 \\ -20.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} +41.417 \\ -3.997 \end{bmatrix} \quad (\text{kNm})$$

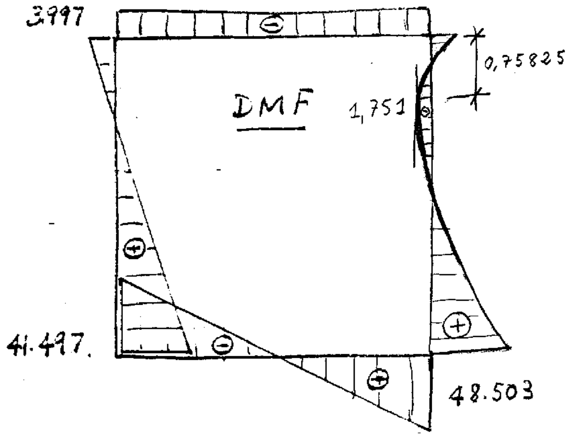
Esforços independents

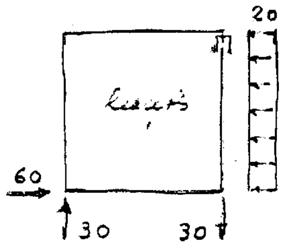
$$\underset{\sim}{X}_m = \underset{\sim}{X}_{m_c} + \underset{\sim}{X}_{m_0} = \underset{\sim}{B}_m \, \underset{\sim}{k}_r + \underset{\sim}{X}_{m_0}$$

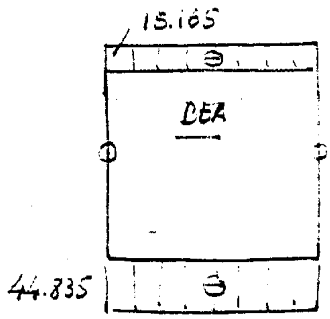
$$\underset{\sim}{X}_1 = \begin{bmatrix} M_1 \\ M_2 \\ N_2 \end{bmatrix} = \begin{bmatrix} -41.497 \\ +48.503 \\ -44.835 \end{bmatrix} \qquad \underset{\sim}{X}_2 = \begin{bmatrix} M_3 \\ M_4 \\ N_4 \end{bmatrix} = \begin{bmatrix} -3.997 \\ -3.997 \\ -15.165 \end{bmatrix}$$

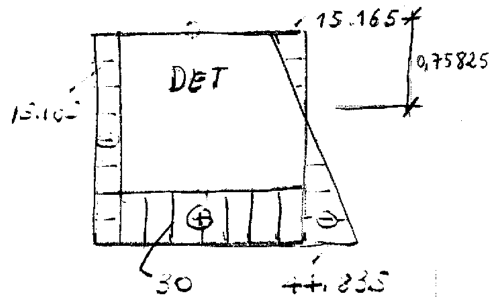
$$\underset{\sim}{X}_3 = \begin{bmatrix} M_5 \\ M_6 \\ N_6 \end{bmatrix} = \begin{bmatrix} 41.497 \\ -3.997 \\ 0 \end{bmatrix} \qquad \underset{\sim}{X}_4 = \begin{bmatrix} M_7 \\ M_8 \end{bmatrix} = \begin{bmatrix} +48.503 \\ 3.997 \end{bmatrix}$$

Diagrames de esforços







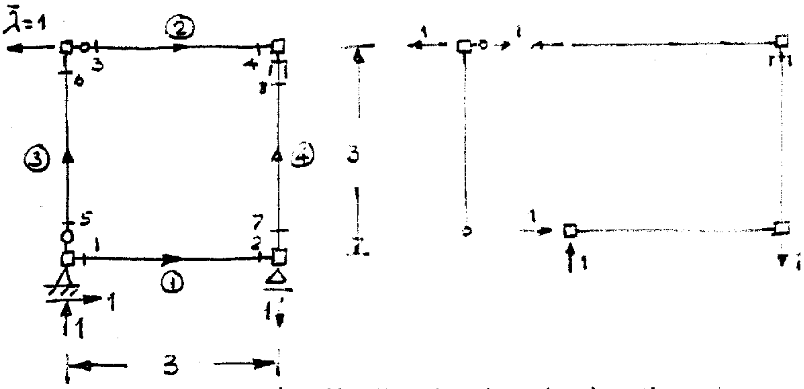


Deformações independentes $u_m = u_{m_c} + u_{m_0} = f_m B_m p_m + u_{m0}$

$$u_{1} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ e_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -17.246 \\ 27.754 \\ -4.484 \end{bmatrix} \qquad u_{2} = \begin{bmatrix} \theta_3 \\ \theta_4 \\ e_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -5.996 \\ -5.996 \\ -1.516 \end{bmatrix}$$

$$u_{3} = \begin{bmatrix} \theta_5 \\ \theta_6 \\ e_6 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +39.499 \\ +16.751 \\ 0 \end{bmatrix} \qquad u_{4} = \begin{bmatrix} \theta_7 \\ \theta_8 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} +28.001 \\ +5.749 \end{bmatrix}$$

Deslocamento horizontal do nó C $\delta = \bar{X}^t u = \sum \bar{B}_m^t u_m$



$$\bar{X}^t = \begin{array}{c} \begin{array}{ccccccccccc} M_1 & M_2 & N_2 & M_3 & M_4 & N_4 & M_5 & M_6 & N_6 & M_7 & M_8 \end{array} \\ \left[\underbrace{0 \quad 3 \quad -1}_{\bar{B}_1^t} \;\middle|\; \underbrace{0 \quad 0 \quad 1}_{\bar{B}_2^t} \;\middle|\; \underbrace{0 \quad 0 \quad 0}_{\bar{B}_3^t} \;\middle|\; \underbrace{0 \quad 0}_{\bar{B}_4^t} \right] \end{array} \bar{\lambda}$$

$$\delta_H^C = 170.233/EI$$