I.1 a) Admissibilidade cinemática

Domínio : w , $\theta_x = -\frac{\partial w}{\partial x}$, $\theta_y = -\frac{\partial w}{\partial y}$ contínuos

Bordo $y=0$: $w(x,0) = 0$

Bordo $x=4$: $w(4,y) = 0$

Bordo $y=4$: $w(x,4) = 0$ e $\left(\frac{\partial w}{\partial y}\right)_{y=4} = 0$

Solução : $w(x,y) = \frac{k}{D}(4-x)\,y\,(4-y)^2$

I.1 b) Admissibilidade estática

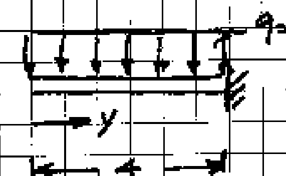
Domínio : $\frac{\partial^2 m_{xx}}{\partial x^2} + \frac{\partial^2 m_{yy}}{\partial y^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + 1 = 0$

$v_x = \frac{\partial m_{xx}}{\partial x} + \frac{\partial m_{xy}}{\partial y}$ $v_y = \frac{\partial m_{yy}}{\partial y} + \frac{\partial m_{xy}}{\partial x}$

Bordo $x=0$: $m_{xx}(0,y) = 0$ e $k_x = \left(v_x + \frac{\partial m_{xy}}{\partial y}\right)_{x=0} = 0$

Bordo $y=0$: $m_{yy}(x,0) = 0$

Bordo $x=4$: $m_{xx}(4,y) = 0$



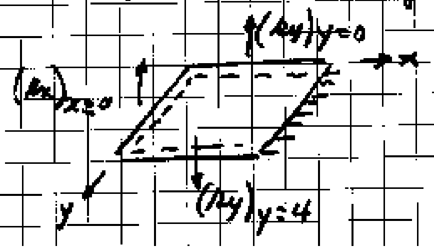
Solução : consola em y

$m_{xx} = 0$

$m_{yy} = -1 \times y^2/2$

$m_{xy} = 0$

I.1 c) As cargas nas vigas são definidas pelo esforço transverso corrigido em cada bordo:



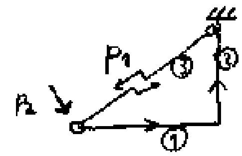
$k_x = \frac{\partial m_{xx}}{\partial x} + 2\frac{\partial m_{xy}}{\partial y} = \frac{9}{32} - x$ para $x=0$

$k_y = \frac{\partial m_{yy}}{\partial y} + 2\frac{\partial m_{xy}}{\partial x} = -\frac{1}{4}y$ para $\begin{cases} y=0 \\ y=4 \end{cases}$

Problema I.2

I.2.a) indeterminação estática: $\alpha = 2$

Sistema base:

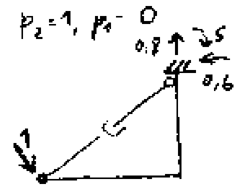


Solução complementar:

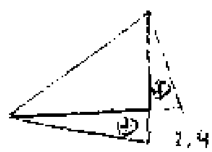
$p_1 = 1,\ p_2 = 0$

$$B = \left[\begin{array}{c|c} 0 & 0 \\ 2,4 & -3,2 \\ -0,8 & -0,6 \\ \hline 2,4 & -3,2 \\ 0 & -5 \\ -0,6 & 0,8 \\ \hline 1 & 0 \end{array}\right]$$

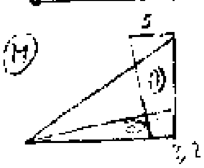
$p_2 = 1,\ p_1 = 0$



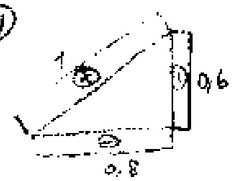
(M)



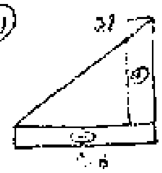
(M)



(N)



(N)



$$F_{(1)} = \frac{4}{6EI}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1,2 \end{bmatrix} \qquad F_{(1)}B_{(1)} = \frac{4}{6EI}\left[\begin{array}{c|c} 2,4 & -3,2 \\ 4,8 & -6,4 \\ -0,96 & -0,72 \end{array}\right] \qquad F_{*(1)} = \frac{4}{6EI}\left[\begin{array}{c|c} 12,288 & -14,784 \\ -14,784 & 20,912 \end{array}\right]$$

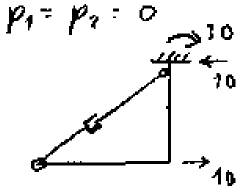
$$F_{(2)} = \frac{3}{6EI}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1,2 \end{bmatrix} \qquad F_{(2)}B_{(2)} = \frac{3}{6EI}\left[\begin{array}{c|c} 4,8 & -11,4 \\ 2,4 & -13,2 \\ -0,72 & 0,96 \end{array}\right] \qquad F_{*(2)} = \frac{3}{6EI}\left[\begin{array}{c|c} 11,952 & -27,936 \\ -27,936 & 103,248 \end{array}\right]$$

$$F_{(3)} = \frac{5}{5EI} \qquad F_{(3)}B_{(3)} = \frac{5}{5EI}\left[\begin{array}{c|c} 1 & 0 \end{array}\right] \qquad F_{*(3)} = \frac{5}{5EI}\left[\begin{array}{c|c} 1 & 0 \\ 0 & 0 \end{array}\right]$$

$$F_* = \frac{1}{EI}\left[\begin{array}{c|c} 15,168 & -23,824 \\ -23,824 & 65,565 \end{array}\right]$$

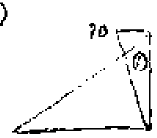
Solução particular:

$p_1 = p_2 = 0$

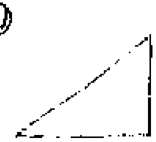


$$X_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ -30 \\ 0 \\ \hline 0 \end{bmatrix}$$

(M)



(N)



$\bar{u}_{(1)} = \underset{\sim}{0}$ $\quad u_{0(1)} = \underset{\sim}{0}$ $\quad v_{0(1)} = \underset{\sim}{0}$

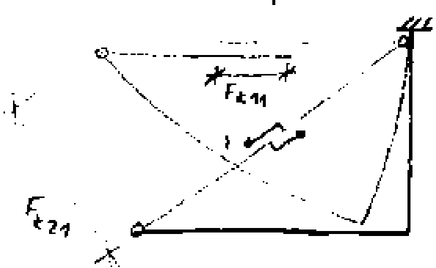
$\bar{u}_{(2)} = \underset{\sim}{0}$ $\quad u_{0(2)} = F_{(2)} X_{0(2)} = \dfrac{3}{6EI}\begin{bmatrix} -30 \\ -60 \\ 0 \end{bmatrix}$ $\quad v_{0(2)} = \dfrac{3}{6EI}\begin{bmatrix} -72 \\ 396 \end{bmatrix}$

$\bar{u}_{(3)} = 0$ $\quad u_{0(3)} = \underset{\sim}{0}$ $\quad v_{0(3)} = \underset{\sim}{0}$

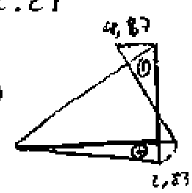
$$v_0 = \begin{bmatrix} -36 \\ 198 \end{bmatrix} \frac{1}{EI}$$

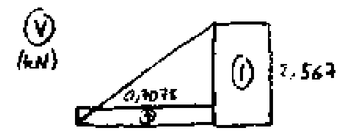
Equação do Método das Forças: $F_* \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + v_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

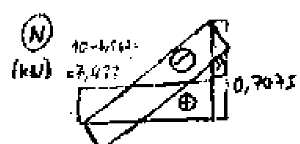
I.2.b) $p_1 = 1$



I.2.c)

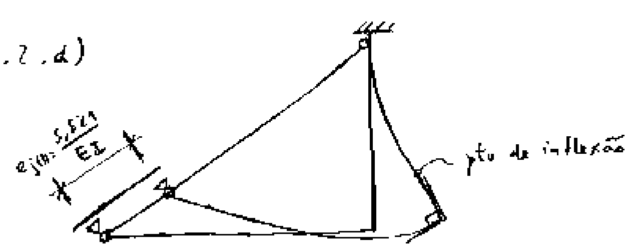






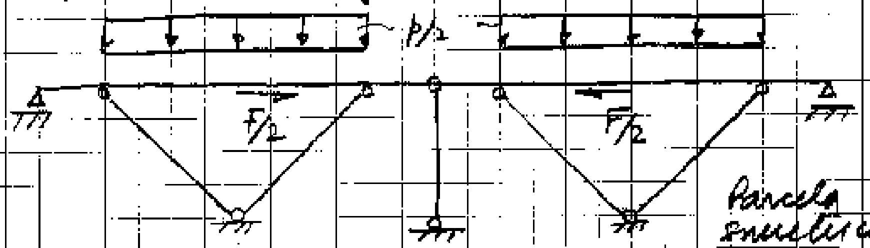
$5,521 = 7,433 \times 0,8 - 0,7075 \times 0,6$

I.2.d)

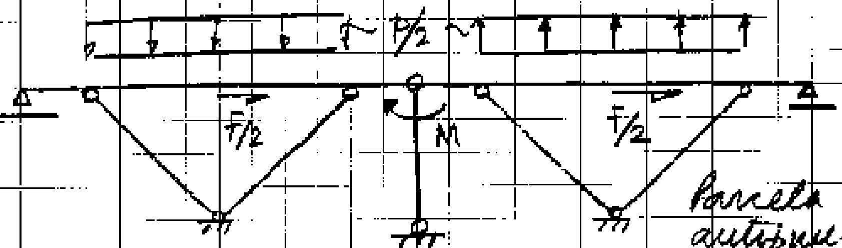


$\delta_{H1} = e_{j(1)} \times \frac{4}{5} = \frac{4,417}{EI}$ (m)

3a) Decomposição do carregamento

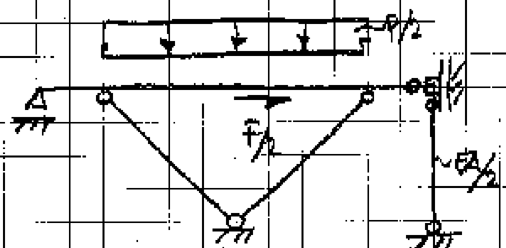


Parcela simétrica

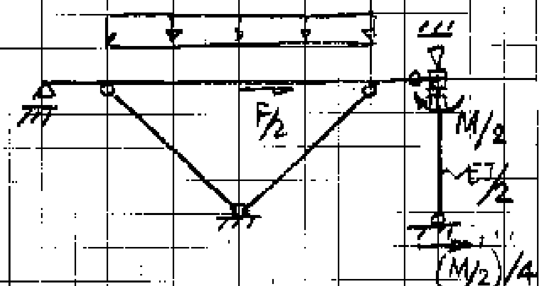


Parcela antissimétrica

3b) Simplificação de simetria



3c) Simplificação de antissimetria



3c) Estrutura dada $\alpha_e = 8-3 = 5$

$\alpha_i = 6-8 = -2$

$\alpha = 3$

Simplificação de simetria $\alpha_e = 7-3 = 4$

$\alpha_i = 3-5 = -2$

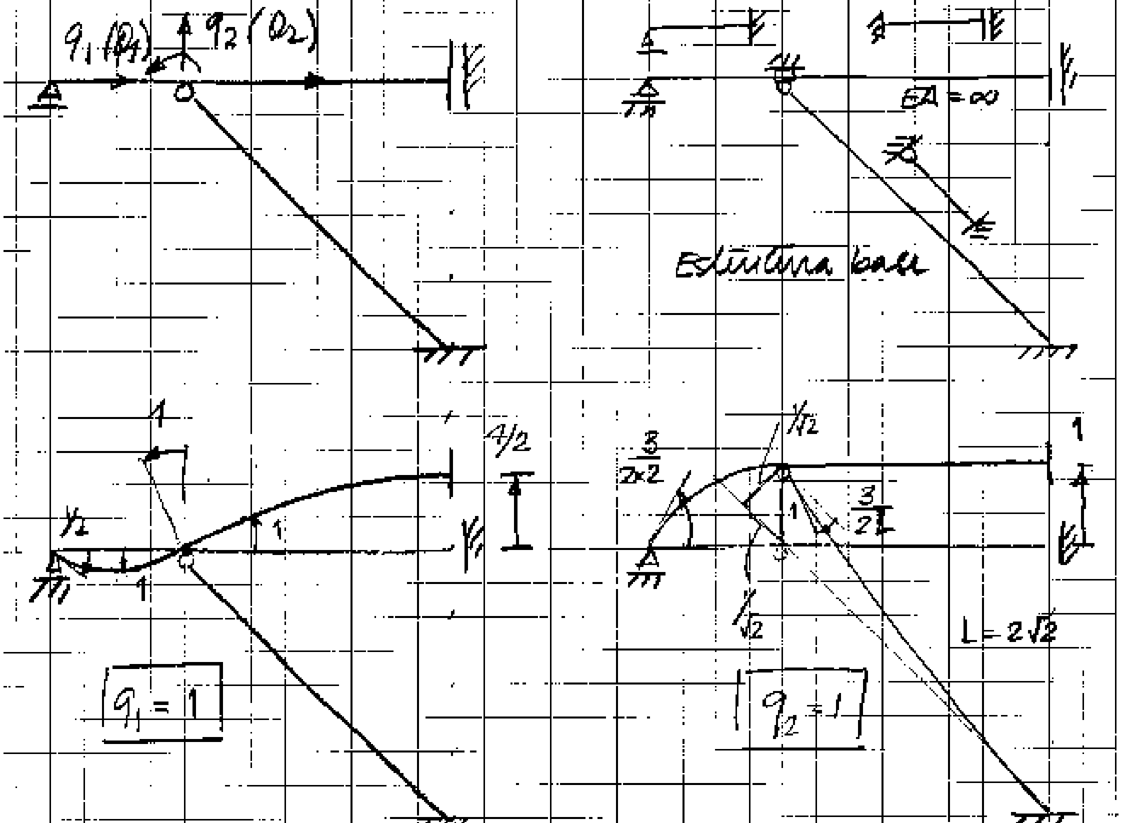
$\alpha = 2$

Simplificação de antissimetria $\alpha_e = 6-3 = 3$

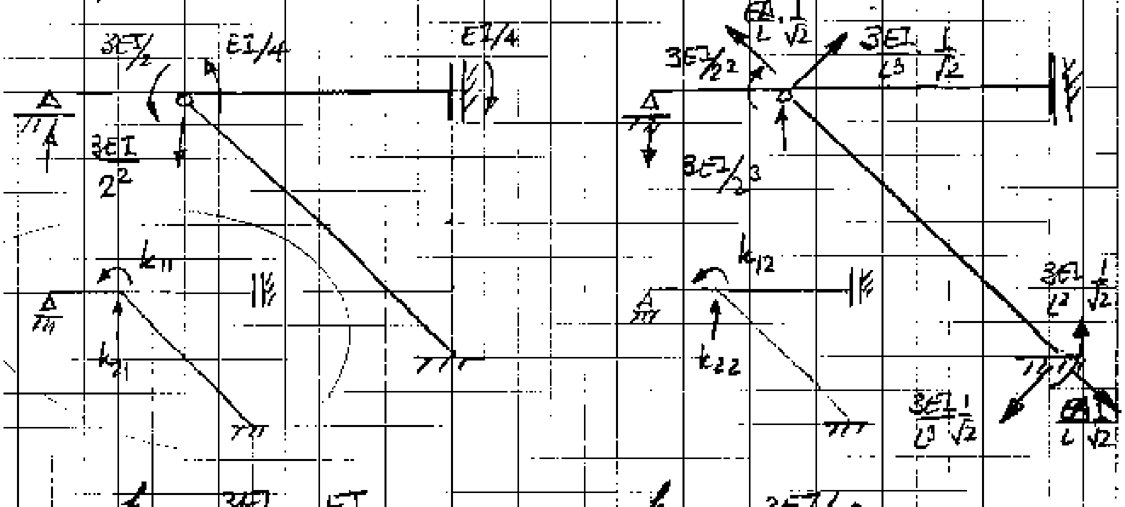
$\alpha_i = 3-5 = -2$

$\alpha = 1$

4a) Deslocamentos independentes e deformadas



4b) Equação do método dos deslocamentos $\mathbb{K}q + Q_0 = Q$



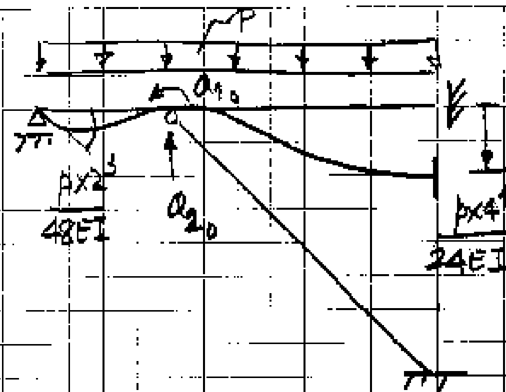
$$k_{11} = \frac{3EI}{2} + \frac{EI}{4}$$

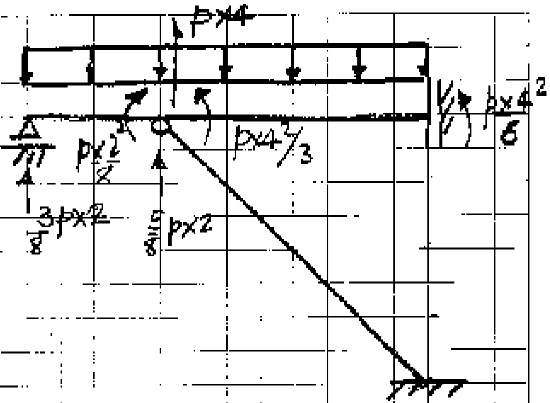
$$k_{21} = -3EI/2^2$$

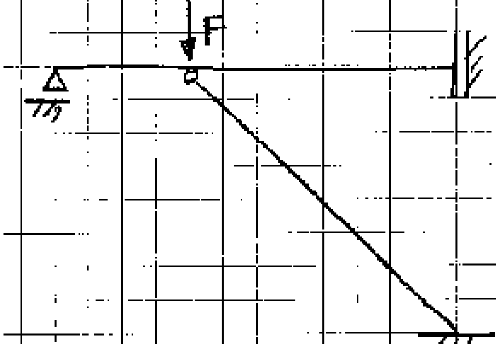
$$k_{12} = -3EI/2^2$$

$$k_{22} = \frac{3EI}{2^3} + \frac{3EI}{L^3}\cdot\frac{1}{2} + \frac{EA}{L}\cdot\frac{1}{2}$$

$EA = 4EI$; $L = 2\sqrt{2}$







$$Q_{1_0} = -p\times 2^2/8 + p\times 4^2/3$$

$$Q_{2_0} = \tfrac{5}{8}\,p\times 2 + p\times 4$$

$$Q_1 = 0 \qquad Q_2 = -F$$

4c) $$M_B = \left(+3EI/2\right) q_1 + \left(-3EI/2^2\right) q_2 + \left(-p\times 2^2/8\right)$$

$$= \left(-EI/4\right) q_1 + \left(\;0\;\right) q_2 + \left(-p\times 4^2/3\right)$$

$$\delta_C(B) = \left(-4/2\right) q_1 + \left(-1\right) q_2 + \left(p\times 4^4/24EI\right)$$

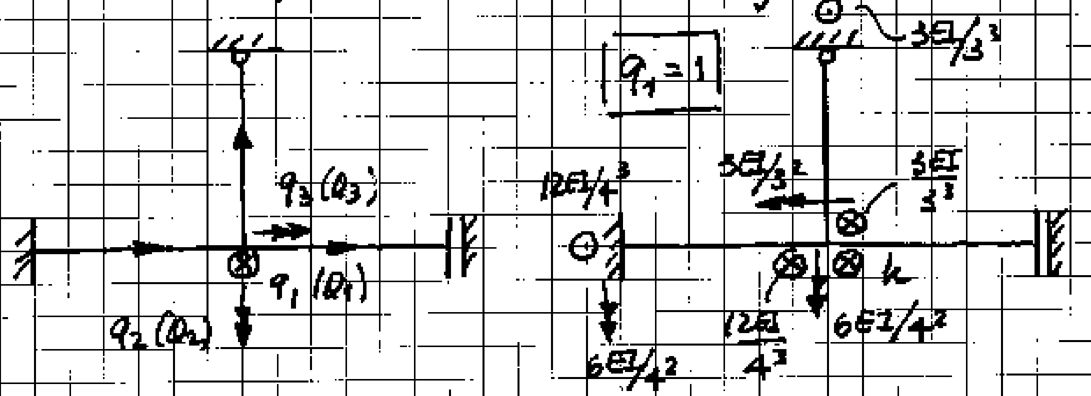
4d) Os resultados da alínea 4b) mostram que é necessário e suficiente que $q_2 = 0$ para garantir que não haja esforço na barra BD.

O valor de q_1 e a relação que deve existir entre as cargas p e F são determinados pela equação do método dos deslocamentos para $q_2 = 0$

II.3 a) Da equação do método dos deslocamentos, na ausência de cargas de vão:

$$\sum_{j=1}^{\beta} k_{ij}\, q_j = Q_i \qquad i = 1,2,\ldots,\beta$$

Se $q_j = \Delta$ e os restantes deslocamentos independentes são nulos, as forças nodais são $Q_i = k_{ij}\cdot\Delta$



$$Q_1 = \left(12\frac{EI}{4^3} + \frac{3EI}{3^3} + k\right)\Delta \;;\; Q_2 = \left(\frac{6EI}{4^2}\right)\Delta \;;\; Q_3 = \left(-\frac{3EI}{3^2}\right)\Delta$$

II.3b) O momento torsor é nulo em todas as barras