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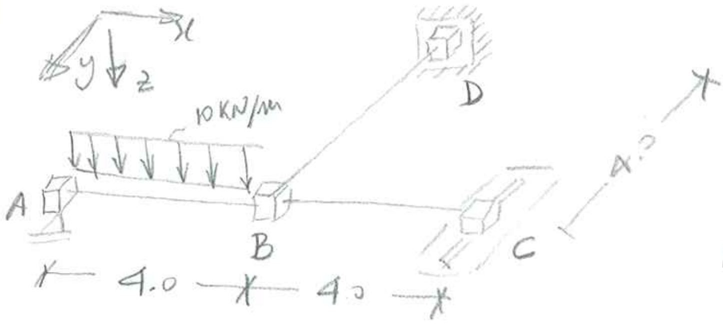
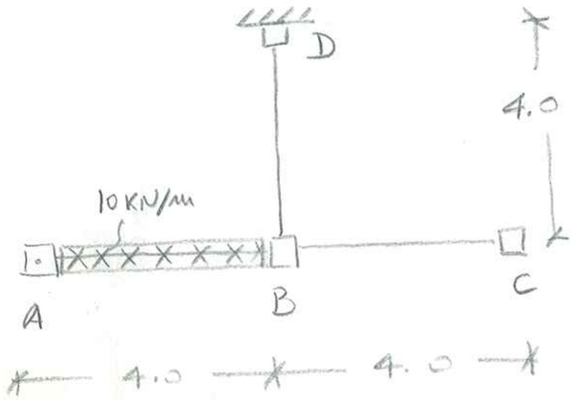
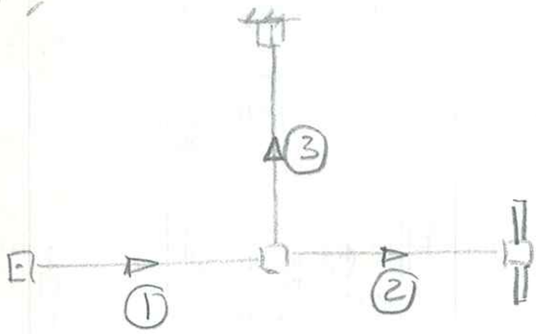


$EI =$ constante

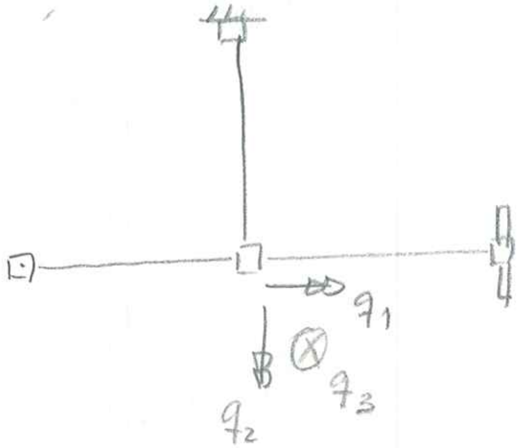
$GJ = 4EI$



1) Discretização da estrutura e orientação das barras



2) Identificação dos deslocamentos independentes



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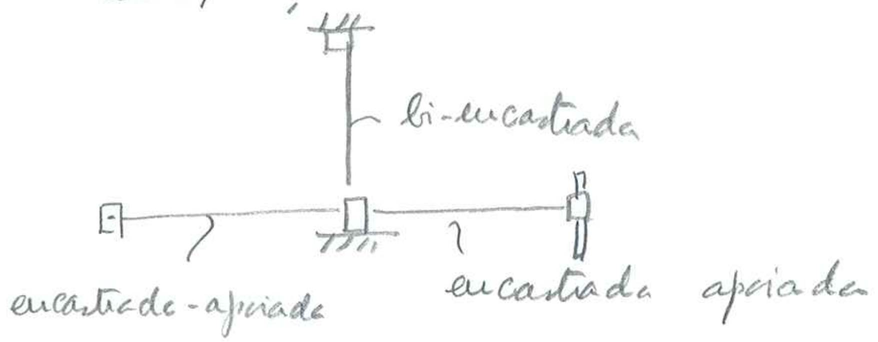
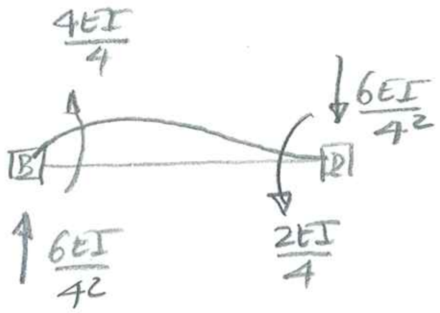
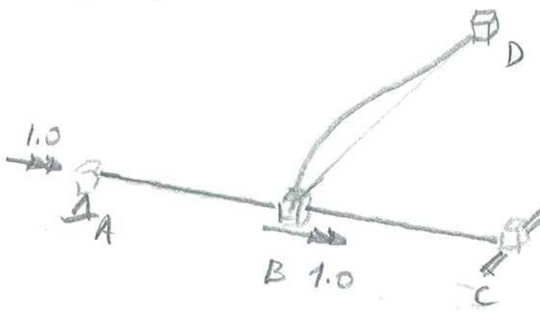
3) Identificação da estrutura-base



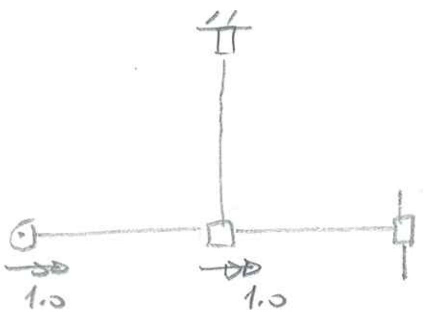
4) Solução complementar

$q_1 = 1.0 ; \ q_2 = q_3 = 0.0$

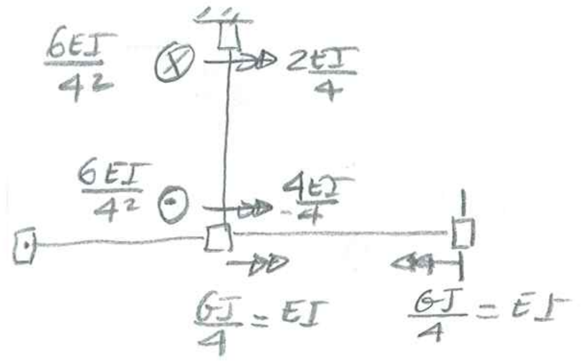




Deslocamentos



Esforços



1ª coluna de $\underline{K}_*$

$$K_{11} = EI + \frac{4EI}{4} = 2EI$$

$$K_{21} = 0$$

$$K_{31} = -\frac{6EI}{4^2}$$

1ª coluna de $\underline{X}_c$

$$\underline{X}_{c_1} = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -1 \\ \hline -1 \\ 0.5 \\ 0 \end{array} \right]$$

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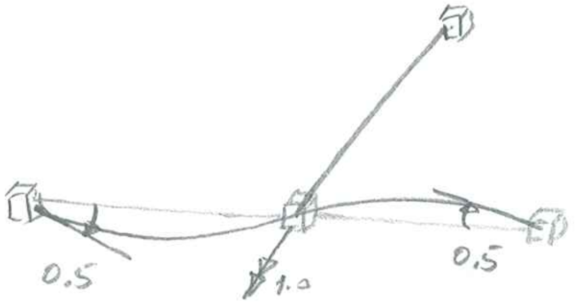
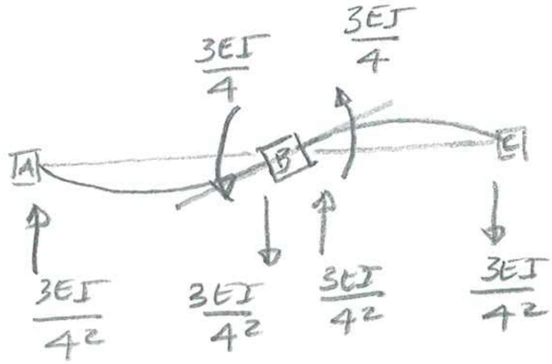
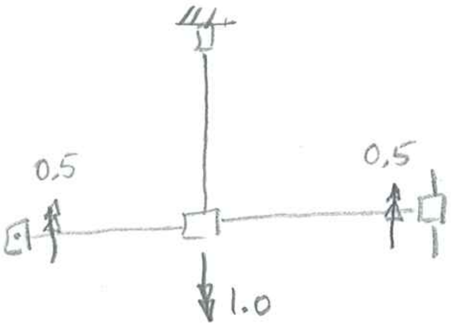
$q_2 = 1.0$; $q_1 = q_3 = 0.0$

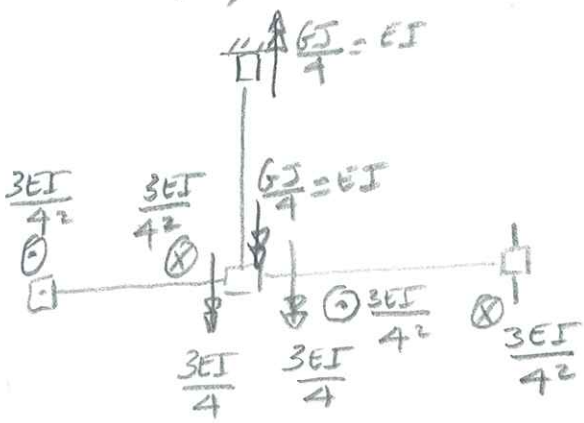




Deslocamentos



Esforços



2ª coluna de $\underline{K}_*$

$K_{12} = 0$

$$K_{22} = \frac{3EI}{4} + \frac{3EI}{4} + EI = 2.5\,EI$$

$$K_{32} = \frac{3EI}{4^2} - \frac{3EI}{4^2} = 0$$

2ª coluna de $\underline{X}_c$

$$\underline{X}_{c_2} = \begin{bmatrix} 0 \\ +3/4 \\ 0 \\ \hline -3/4 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ +1 \end{bmatrix} EI$$

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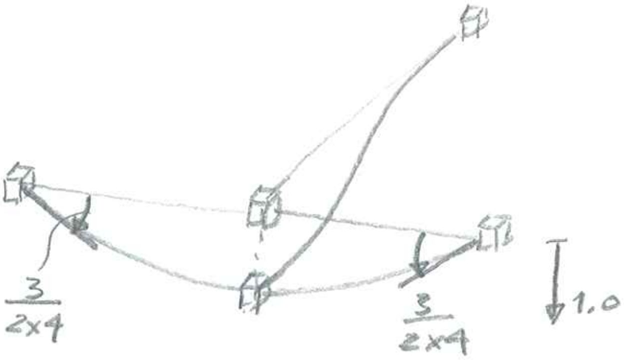
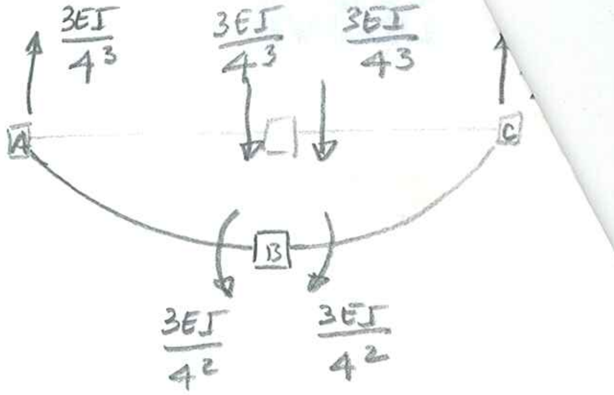
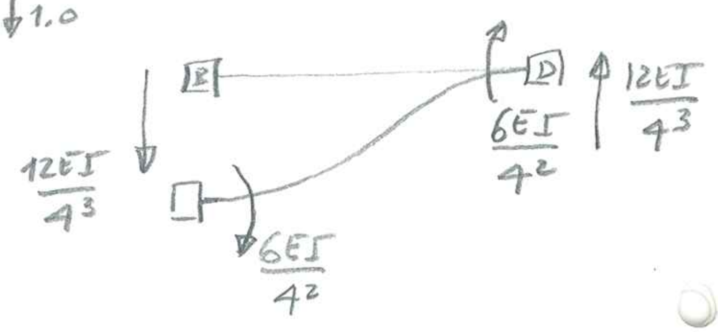
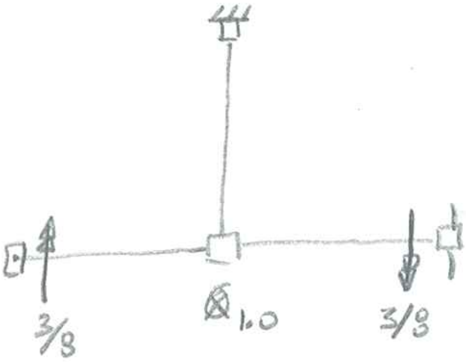
$q_3 = 1.0\ ;\ q_1 = q_2 = 0$



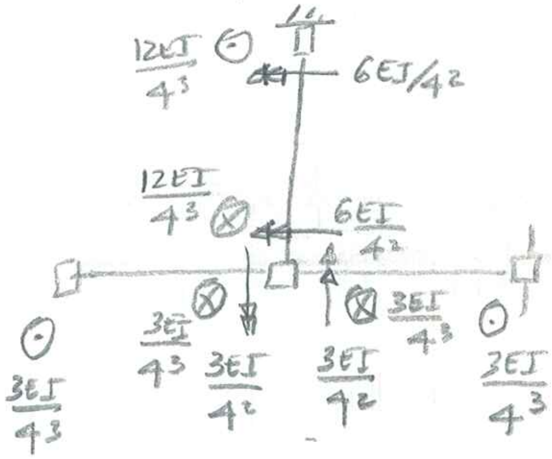




Deslocamentos



Esforços



3ª coluna de $\underline{K}_x$

$$K_{13} = -\frac{6EI}{4^2}$$

$$K_{23} = \frac{3EI}{4^2} - \frac{3EI}{4^2} = 0$$

$$K_{33} = \frac{3EI}{4^3} + \frac{3EI}{4^3} + \frac{12EI}{4^3} = \frac{9}{32}EI$$

3ª coluna de $\underline{X}_c$

$$\underline{X}_{c3} = \begin{bmatrix} 0 \\ +3/16 \\ 0 \\ \hline +3/16 \\ 0 \\ 0 \\ \hline +6/16 \\ -6/16 \\ 0 \end{bmatrix}$$

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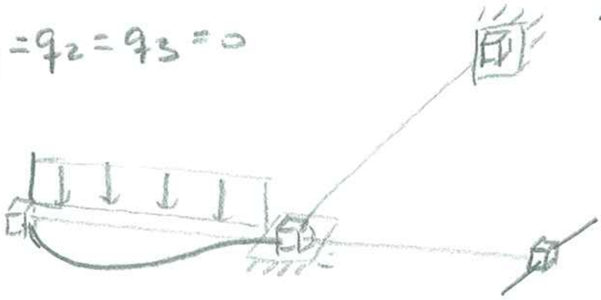
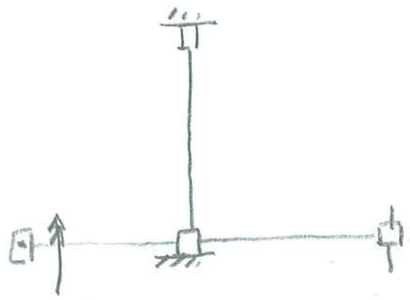
5) Solução particular

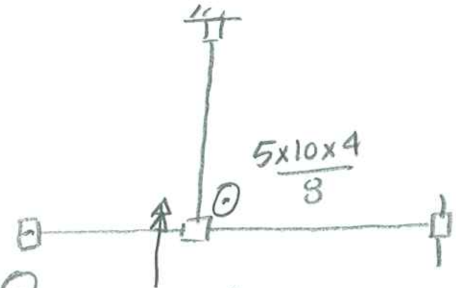
$q_1 = q_2 = q_3 = 0$



Deslocamentos



Esforços



$$\theta_{01} = 0$$

$$\theta_{02} = -\frac{10\times4^2}{8} = -20$$

$$\theta_{03} = -\frac{5\times10\times4}{8} = -25$$

$$\underline{X_0} = \begin{bmatrix} 0 \\ -20 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

6) Equações do método dos deslocamentos

$$EI \begin{bmatrix} 2 & 0 & -6/16 \\ 0 & 2.5 & 0 \\ -6/16 & 0 & 9/32 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -20 \\ -25 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 22.2222 \\ 8.0 \\ 118.5185 \end{bmatrix} \begin{matrix} (\text{rad}) \\ (\text{rad}) \\ (\text{m}) \end{matrix}$$

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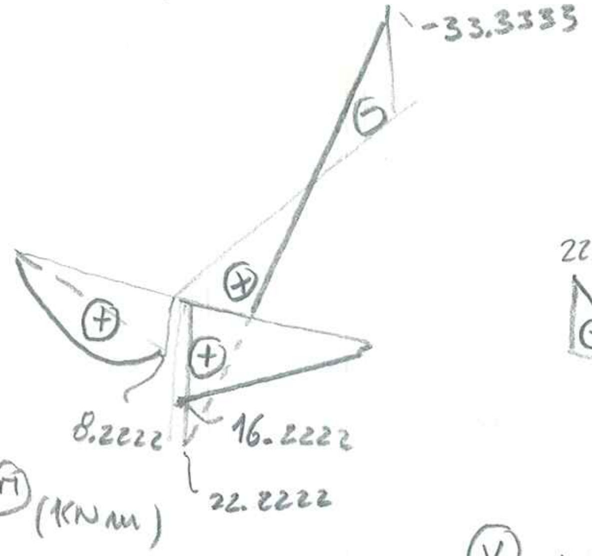
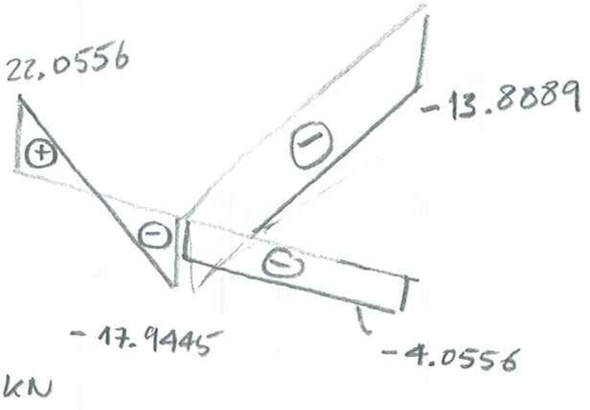
7) Determinação dos esforços independentes

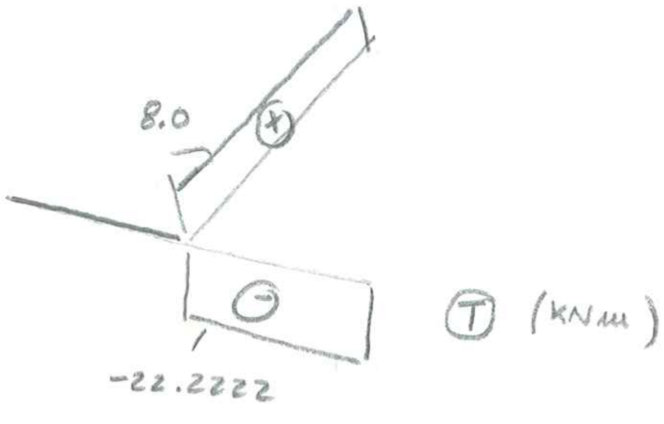
$$\underline{X} = \underline{X}_c\, q + \underline{X}_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3/4 & 3/16 \\ 0 & 0 & 0 \\ \hline 0 & -3/4 & 3/16 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ \hline -1 & 0 & 6/16 \\ 0.5 & 0 & -6/16 \\ 0 & +1 & 0 \end{bmatrix} EI \times \frac{1}{EI} \begin{bmatrix} 22.2222 \\ 8.0 \\ 118.5185 \end{bmatrix} + \begin{bmatrix} 0 \\ -20 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 0.0 \\ 8.2222 \\ 0.0 \\ \hline 16.2222 \\ 0.0 \\ -22.2222 \\ \hline 22.2222 \\ -33.3333 \\ 8.0 \end{bmatrix} \begin{array}{l} \\ \underline{X}_1 \\ \\ \\ \underline{X}_2 \\ \\ \\ \underline{X}_3 \\ \\ \end{array}$$

8) Traçado dos diagramas de esforços







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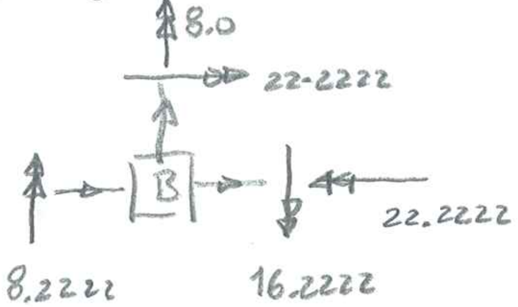
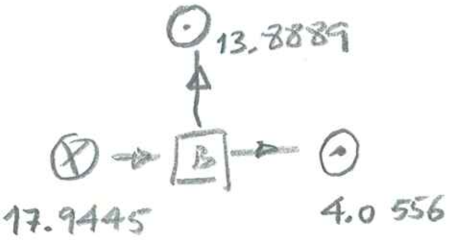
Verificação do equilíbrio no nó B

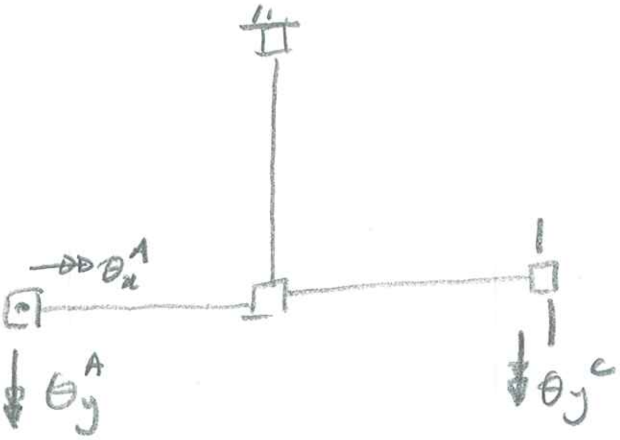


(equilíbrio de momentos)



(equilíbrio de forças verticais)

9) Cálculo dos deslocamentos dependentes



$$\theta_x^A = 1.0 * \frac{1}{EI}\left[22.2222\right] + 0.0 * \frac{1}{EI}\left[8.0\right] + 0.0 \times \frac{1}{EI}\left[118.5185\right] + 0 =$$

$$\theta_y^A = 0.0 * \frac{1}{EI}\left(22.2222\right] - 0.5 \times \frac{1}{EI}\left[8.0\right] - 3/8 \times \frac{1}{EI}\left[118.5185\right] - \frac{10 \times 4^3}{48EI} =$$

$$\theta_y^C = 0.0 \times \frac{1}{EI}\left[22.2222\right] - 0.5 * \frac{1}{EI}\left[8.0\right] + \frac{3}{8} \times \frac{1}{EI}\left[118.5185\right] + 0 =$$