# IMPULSE RESPONSE METHOD FOR SOLVING HYDRAULIC TRANSIENTS IN VISCOELASTIC PIPES

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## Abstract

The current paper focuses on the application of the Impulse Response Method (IRM) for solving hydraulic transients in pressurised viscoelastic pipes. Basic fluid equations are described in the frequency domain by transfer functions, as for steady oscillatory flows, and are used for the evaluation of response functions by the inverse Fast Fourier Transform (invFFT) technique. Non-periodic pressure perturbations are obtained by the direct convolution of the response function with the discharge change (or pressure pulse). This method is a particularly useful procedure to take into account frequency-dependent factors, such as unsteady skin friction and pipe-wall viscoelasticity, which considerably influence the dynamic behaviour of the fluid system in transient or steady oscillatory conditions. These two phenomena were incorporated into the transfer functions by using a complex-valued wave speed and propagation operators. Transient data collected from a 270 m polyethylene (PE) pipeline, with 50 mm diameter, at Imperial College (London, UK) are used for validating the developed method. Numerical results obtained are compared with experimental data. An excellent agreement is observed. The use of this method (i.e. IRM) is much faster than the typical method of characteristics and can straightforwardly include frequency-dependent factors; however, it has the disadvantage of the loss of accuracy due to the linearization of the friction term and the valve equation, and the complex application to multi-pipe systems.

*Keywords*: frequency domain, impulse response, transients, viscoelastic, Fourier analysis.

## 1. INTRODUCTION

Numerous researchers have addressed the issue of the hydraulic transients damping caused by unsteady skin friction and the viscoelastic behaviour of the pipe-wall. Basic fluid equations can be solved either, in the time domain, by the classical Method of Characteristics (MOC) or, in the frequency domain, by the Impulse Response Method (IRM).

In the time domain, additional time-dependent terms are added to basic fluid transient equations. The term related to unsteady skin friction is incorporated in momentum equation (Zielke, 1968), whereas the term that describes the retarded-viscoelastic response of the pipe wall is included in the mass-balance equation (Gally *et al.*, 1979; Rieutford and Blanchard, 1979). Gally *et al.* (1979) determined the creep-function by dynamic tests and verified the model with pressure and strain data collected in a experimental facility. Rieutford and Blanchard (1979) analysed the effect of the relaxation times of a “three Kelvin-Voigt element” model. Ghilardi and Paoletti (1986) showed that the viscoelastic dampening could be usefully used to reduce pressure surges. Rachid and Stuchenbruck (1990) modelled viscoelastic pipe behaviour coupled and uncoupled with fluid-structure interaction. Rachid *et al.* (1992) implemented several types of non-elastic rheological behaviour. Pezzinga (2002) analysed the effect of an additional PE pipe downstream of a pump to reduce induced pressure surges. Covas *et al.* (2004a; 2004b; 2005) have further analysed the effect of pipe wall viscoelasticity in hydraulic transients and used a several Kelvin-Voigt element to describe it in time domain and solve using the MOC.

In frequency domain, the concept of frequency-dependent parameters is used to describe both unsteady friction and the viscoelastic behaviour of the pipe, as both phenomena are time (i.e. frequency) dependent. The viscoelastic behaviour of the pipe-wall can be described by a frequency-dependent creep function in the complex-valued wave speed calculation. Frequency dependent friction was analysed by Brown (1962) who derived functional operators for the propagation factor and characteristic impedance for laminar flow, fast transients and rigid walls. MeiBner and Franke (1977) derived the wave speed and the damping factor formulas for an oscillating flow in a thin-walled viscoelastic pipe, and concluded that the viscoelastic attenuation of the pressure surges was much higher than the friction damping (although they did not accounted for unsteady skin friction). Rieutford (1982) analysed laminar transient flow and proposed a “one-element Kelvin-Voigt” mechanical model to describe the pipe material creep-compliance function and include it in the wave speed formula. Franke and Seyder (1983) incorporated wave speed formulas in unsteady fluid equations and solved them using the Impedance Method for steady-oscillatory flow and the Impulse Response Method for non-periodic flow. Suo and Wylie (1990a) modelled pipe-wall viscoelasticity in both oscillatory and non-periodic flows. Suo and Wylie (1990b) analysed frequency-dependent wave speed in a rock-bored tunnel due to the dynamic effect of the surrounding rock mass.

The current paper is an extension of Covas *et al.* (2004a; 2005) in what concerns the calculation of hydraulic transients in viscoelastic pipes. The difference is that the latter papers describe transients in the time domain, whereas, this paper formulates basic equations, in the frequency domain, and solves them by the Impulse Response Method (IRM). Non-periodic pressure perturbations are obtained by the direct convolution of the discharge change (or pressure pulse) with the response function. Transient data collected from a 270 m polyethylene (PE) pipeline, with 50 mm diameter, at Imperial College (London, UK) are used for validating the developed method. Numerical results obtained are compared with experimental data.

## 2. LINEARISED UNSTEADY FLOW EQUATIONS

### 2.1 Basic equations and hydraulic impedance

The simplified continuity and motion equations of fluid transients in pressurized pipes are (Wylie and Streeter, 1993):

$$\frac{dH}{dt}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{dQ}{dt}+RQ=0 \tag{2}$$

where $Q$=flow-rate; $H$=hydraulic-grade-line elevation; $a$= wave speed; $g$=gravity acceleration; $A$=pipe cross-section; $D$=pipe inner-diameter; $R$=fluid resistance per unit length is $R=32\,\nu/gAD^2$ or laminar flow and $R=nfQ^{n-1}/2gDA^n$ for turbulent flow being $f$= Darcy-Weisbach friction factor and $n$=exponent of velocity; $x$ =coordinate along the pipe axis; $t$=time.

In the frequency domain, the flow-rate and the hydraulic-grade-line elevation are assumed to have a sinusoidal variation and are divided into a steady average component, $H_o$ and $Q_o$, and an oscillatory component, $h'$ and $q'$,

$$H = H_o + h' \text{ and } Q = Q_o + q' \tag{3)-(4}$$

Terms $h'$ and $q'$ are functions of time $t$ and space $x$ and, for each pipe position $x$, have a sinusoidal variation that dampens or amplifies in time:

$$h' = H^*(x)e^{st} \text{ and } q' = Q^*(x)e^{st} \tag{5)-(6}$$

where $H^*$, $Q^*$= complex-valued head and discharge; $s$= complex frequency or Laplace variable, $s = \sigma + i\omega$; $\sigma$= real part of the complex frequency that governs the decay oscillations in time; $\omega$= angular frequency; $i$=imaginary unit number. In steady-oscillatory flow, the motion amplitude is constant in time and the term $\sigma$ is null. Introducing (3)-(4) into (1)-(2) and taking into account that the average flow-rate and piezometric-head are time-independent, and that the average flow is constant with $x$, the linearized equations for oscillatory-flow are (Wylie and Streeter, 1993):

$$C\frac{\partial h'}{\partial t} + \frac{\partial q'}{\partial x} = 0 \tag{7}$$

$$L'\frac{\partial q'}{\partial t} + \frac{\partial h'}{\partial x} + Rq^* = 0 \tag{8}$$

where $C$=capacitance, $C = gS/a^2$; $L'$=inertiance, $L' = \zeta / gS$; $\zeta$=corrective factor (Wylie and Streeter, 1993 pp. 290-291). The factor $\zeta$ is Reynolds-dependent (varies between 4/3 for laminar flow and 1.0 for turbulent flow) and is accurate for low frequency flows; for high frequency problems expressions of $R$ and $L'$ are available in literature (Brown, 1962; Zielke, 1968). Equations (7) e (8) can be solved by the separation of variables (Wylie and Streeter, 1993). Accordingly, the complex head and discharge are given by the following transfer equations:

$$H^*(x) = H_D^* \cosh(\gamma x) + Q_D^* Z_C \sinh(\gamma x) \tag{9}$$

$$Q^*(x) = \frac{H_D^*}{Z_C}\sinh(\gamma x) + Q_D^* \cosh(\gamma x) \tag{10}$$

where $H_D^*$, $Q_D^*$= complex-valued head and discharge at the upstream end; $\gamma$= complex-valued propagation constant, $\gamma^2=Cs(s/gA+R)$; $Z_C$=characteristic impedance, $Z_C=\gamma a^2/gAs$.

The Impedance Method combines the previous two equations into a single equation using the concept of hydraulic impedance $Z$ defined as the ratio between the complex head $H^*$ and the complex discharge $Q^*$, $Z = H^*/Q^*$. Accordingly, (9) and (10) are merged in a single function:

$$Z_D = \frac{Z_U - Z_C \tanh(\gamma L)}{1 - Z_U / Z_C \tanh(\gamma L)} \text{ or } Z_U = \frac{Z_D + Z_C \tanh(\gamma L)}{1 + Z_D / Z_C \tanh(\gamma L)} \tag{11}$$

where $Z_U$ and $Z_D$ =hydraulic impedance at the upstream and downstream end, respectively; and $L$=pipe length. Considering the pipe infinitely long $L \to +\infty$, (11) simplifies to $Z_D=-Z_C$ and $Z_U=+Z_C$. To obtain the pressure response, it is necessary to add to (11) adequate boundary conditions (Wylie and Streeter, 1993). For a constant-level reservoir located at the upstream end, $|H_U^*|=0$, and $Z_U = 0$ and $Z_D = -Z_C \tanh(\gamma L)$.

## 2.2 Frequency-dependent friction (laminar flow)

Given the linearization of the friction term, (7) to (11) are strictly valid for small perturbations of the system and for low frequency flows. For *frequency-dependent friction* and laminar flow, the previous equations are valid providing that terms $\gamma x$ (or $\gamma L$) and $Z_C$ are replaced by the following complex-valued propagation operators (Brown, 1962):

$$\Gamma(s)=\frac{sx}{a}\left[1-\frac{2I_1\left(ir\sqrt{s/\nu}\right)}{ir\sqrt{s/\nu}I_0\left(ir\sqrt{s/\nu}\right)}\right]^{-1/2} \tag{12}$$

$$Z(s)=\frac{a}{gA}\left[1-\frac{2I_1\left(ir\sqrt{s/\nu}\right)}{ir\sqrt{s/\nu}I_0\left(ir\sqrt{s/\nu}\right)}\right]^{-1/2} \tag{13}$$

where $r$=pipe radius; $\nu$=kinematic viscosity; and $I_0$, $I_1$=first-type Bessel function of zero and first order, respectively (Press *et al.*, 1988). For turbulent flows, there is still no accurate formulas, being commonly used empirical formulas.

### 2.3 Complex-valued frequency dependent wave speed (viscoelastic pipes)

*For viscoelastic pipes*, presented equations are valid but, wave speed, $a$, becomes a complex-valued function of frequency, $a^*$. The viscoelastic behaviour is described by a complex-valued creep function $J^*$ in the wave speed calculation:

$$a^*(s)=\left[\rho\left(\frac{1}{K}+\frac{\alpha D}{e}J^*\right)\right]^{-1/2} \tag{14}$$

where $K$=fluid bulk modulus of elasticity; $\rho$=fluid density; $e$=pipe-wall thickness; $\alpha$=dimensionless parameter that depends on the pipe cross-section and axial constraints. The frequency-dependent complex creep compliance function is described by:

$$J^*(\omega)=J'(\omega)-iJ''(\omega) \tag{15}$$

where $J'$ is the storage creep compliance and $J''$ is the loss creep compliance. The time domain creep function $J(t)$ experimentally determined can be converted into the corresponding material functions, in the frequency domain, by using Laplace transform methods (Ferry, 1970; Riande *et al.*, 2000). Should the Prony series representations of the Generalised Viscoelastic solid be used to describe the creep compliance function (Covas *et al.*, 2005)

$$J(t)=J_0+\sum_{k=1}^{N}J_k\left(1-e^{-\frac{t}{\tau_k}}\right) \tag{16}$$

the respective storage and loss creep compliance result in

$$J'(\omega)=J_0+\sum_{k=1}^{N}\frac{J_k}{\omega^2\tau_k^2+1} \tag{17}$$

$$J''(\omega)=\sum_{k=1}^{N}\frac{\omega\tau_k J_k}{\omega^2\tau_k^2+1} \tag{18}$$

where $J_0$=elastic creep-compliance; $J_k$=creep-compliance of the spring of the Kelvin-Voigt element $k$ defined; $\tau_k$=retardation time of the dashpot of $k$-element; $N$=number of Kelvin-Voigt element. Replacing (17)-(18) in (15), and in (14), it yields for $s=i\omega$ the wave speed:

$$a^*(s)=a_0\left(1+\frac{\rho a_0^2\alpha D}{e}\sum_{k=1}^{N}\frac{J_K}{1+s\tau_k}\right)^{-1/2} \tag{19}$$

The equivalent wave speed $a_e$ and the exponential attenuation factor $\phi$ are given by:

$$a_e=\left|a^*\right|^2/\operatorname{Re}(a^*) \text{ and } \phi=\omega\operatorname{Im}(a^*)/\left|a^*\right|^2 \tag{20}$$

where Re and Im stand for the real and the imaginary parts of a complex valued number.

## 3. IMPULSE RESPONSE METHOD

The Impulse Response Method (IRM) allows the calculation of both oscillatory and non-periodic transients using the linearized frequency domain transfer functions including frequency dependent friction and wave speed. This method mathematically correlates an input impulse applied to a system with the corresponding output function.

By viewing the input impulse as a series of different amplitude and linearly independent pulses, and applying the principle of superposition, the system response $f(x,t)$ is found by the direct convolution of the impulse response function $r(x,t)$ with the input pulse $s(t)$:

$$f(x,t)=r(x,t)*s(t)=\int_0^t r(x,t-\tau)s(\tau)d\tau \tag{21}$$

In the discrete case, $s(t)$ is represented by its sampled values $s_k$ at equal time intervals, $r(x,t)$ by a discrete set of numbers $r_k$ of finite duration $M$ and the discrete convolution by:

$$f_j=(r*s)_j=\sum_{k=-M/2+1}^{M/2} s_{j-k}\,r_k \tag{22}$$

The impulse response function describes the behaviour of the system and is an intrinsic characteristic of the system, independent of the input function. This function is obtained using the Fourier integral of the Dirac-delta function (unit impulse function) which considers an impulse the sum of an infinite number of exciters $d\omega/\pi$ :

$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}d\omega \tag{23}$$

Applied to hydraulic transients in a reservoir-pipe-valve system, the input pulse can be the flow variation at the valve $\Delta q_D(t)$, the output the physical piezometric head at the valve section $H_D(t)$ and the head response function at a section the valve is

$$r_{H_D}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} Z_D e^{i\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}(-Z_C\tanh(\gamma L))e^{i\omega t}d\omega \tag{24}$$

where the transfer function $H_D^*=Z_D d\omega/\pi$, $Z_D$ is given by (11) and $Z_U$=0. Numerically, the above equation can be calculated by the Inverse Fourier Transform (IFT) of the transfer function $Z_D$, as by definition, IFT is described by (Press *et al.*, 1988):

$$f(t)=\int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}d\omega \tag{25}$$

In summary, first, transfer functions of the system are determined; then, the response function is evaluated by the Inverse Fast Fourier Transform of the discrete transfer function; and finally the transient pressure response is calculated by the discrete convolution of the response function with the flow variation at the valve.

## 4. REVIEW OF EXPERIMENTAL DATA

Transient data collected from a polyethylene pipe rig at Imperial College (London, UK) are used for testing the IRM. The pipeline is made of high-density polyethylene SDR11 PE100 NP16, with 63 mm nominal diameter (DN) and 6.2 mm wall thickness (Fig. 1). The total length of the pipeline is 271.5 m (length between the vessel and the downstream globe valve). Pipe sections are electrofused and rigidly fixed to a vertical wall with plastic brackets, 1 m spaced along its length, and with metal frames at the elbows, to restrain the pipe from any axial movement. The pipe-rig includes a centrifugal pump (Q=2.5 l/s; H=35 m) and a pressurised tank with 750 l volume, at the upstream end, and a globe valve to control the flow and to generate transient events, at the downstream end. The globe valve discharges directly to a free surface flow drainage pipe. Transient events were generated by the fast closure of this valve. Data acquisition system is composed of an acquisition board, strain-gauge type pressure transducers and a notebook computer. The acquisition board has eight analogue inputs channels and a maximum sampling rate of 9600 Hz per channel. Pressure transducers have pressure ranges of 0 to 10 bar (absolute pressure) and an accuracy of 0.3% of the full range. The steady-state flow is measured with an electromagnetic flow meter located immediately after the pressure vessel at the upstream end. A complete description of the experimental facility can be found in (Covas *et al.*, 2004a; 2004b).

Various data sets were collected with a sampling rate of 600 Hz. These tests covered a wide range of flows from laminar ($Q$=0.056 l/s; $Re$=1400) to smooth turbulent ($Q$=2.0 l/s; $Re$=50000). A significant pressure dampening in the consecutive pressure waves was observed (Covas *et al.*, 2004a; 2004b). This could not be thoroughly justified by unsteady-friction effects, as this phenomenon has never been observed with such intensity in metal pipes under the same flow conditions. In fact, this is typical of polymer pipe-wall viscoelasticity. This behaviour significantly influences the pressure response during transients by attenuating the maximum pressure fluctuations in the pipeline and by increasing the dispersion of the pressure wave.

## 5. CASE STUDIES

### 5.1 Frequency-dependent friction

The IRM was used for evaluation hydraulic transients in Imperial College system considering the pipe rheological behaviour linear elastic. Considered elastic wave speed was 390 m/s. The IRM can be summarised in a three-step procedure, as it is depicted in Fig. 2 to Fig. 4: (i) evaluation of the system transfer function (Fig. 2); (ii) calculation of the response function by the Inverse Fast Fourier Transform of the discrete transfer function (Fig. 3); and (iii) the transient pressure response calculation by the discrete convolution of the response function with the flow variation at the valve (Fig. 4). Two formulations for the friction calculation are presented: the first refers to the linearised constant friction and the second the frequency-dependent friction developed by Brown (1962) for laminar conditions.

Numerical results obtained for the initial flow of 1.008 l/s are compared with the collected piezometric-head data at the downstream end of the pipe (Fig. 4). The classic linearised solution shows large discrepancies in both the pressure amplitude and phase with experimental data. These increase substantially with time. On the other hand, the solution obtained for frequency dependent friction approaches the shape of collected pressure wave, still showing large discrepancies. None of the solutions can describe the observed damping of transient pressures. Additionally, the flow is smooth-wall turbulent and the formulation for laminar flows, which is even more conservative as unsteady friction is less important for turbulent conditions than for laminar flows.

The general conclusion is that unsteady friction cannot thoroughly describe the attenuation and dispersion of transient pressures in polyethylene pipes

### 5.2 Frequency-dependent wave speed

The IRM was used for evaluation hydraulic transients considering the pipe rheological behavior linear viscoelastic. This behaviour is described by a creep compliance function *J(t)* that depends on the molecular structure of the material, temperature and pipe constraints. This function can be measured experimentally by a simple creep or a dynamic test, or it can be estimated by adjusting transient data to the numerical results. The creep function was represented by a three-element Kelvin-Voigt model. Parameters $J_k$ and $\tau_k$ were calibrated minimising the difference between measured and calculated heads for laminar flow calculating unsteady friction by Trika's formula (Covas *et al.*, 2005): $a_0$=395 m/s (i.e. $J_0$=0.70E-9 Pa$^{-}$1), $\tau_1$=0.05 s, $\tau_2$= 0.5 s, $\tau_3$=10 s, $J_1$=0.0805E-9 Pa$^{-1}$ s, $J_2$= 0.1083E-9 Pa$^{-1}$ s, $J_3$=0.57635E-9 Pa$^{-1}$. The creep data and the viscoelastic behaviour of the pipes are described by the complex-valued wave speed as explained in Section 2.3. The three-step of IRM are presented in Fig. 5 and Fig. 6.Some remarks should be made after the analysis of these figures.

First the transfer function and impulse response function both considering or not frequency dependent friction are almost the same and can hardly be distinguished in Fig. 5; this is because the viscoelastic nature of the pipe is the major energy dissipater and determines the response of the pipe. Second, both solutions agree fairly well with collected data both in amplitude, phase and shape, still the solution with frequency dependent friction has a slightly better agreement. The minor discrepancies observed between numerical results and collected data are, among other things, due to the time variation of the piezometric head at the upstream boundary, as the pressurized vessel is supplied by a pump and is not a constant head reservoir as assumed in the transfer functions. In this context, data have been corrected by removing the head variation observed at upstream from the downstream measurements. After this, as excellent agreement was observed. Still, numerical results obtained in the frequency domain using IRM are not as accurate as the ones obtained in the time domain (Covas *et al.*, 2005). This is mainly due to the linearizations of fluid friction and of the valve manoeuvre, reason why this approach is left behind many times.

## 6. SUMMARY AND CONCLUSIONS

The current paper presented the calculation of waterhammer in polyethylene pipes taking into account unsteady friction effects and the viscoelastic behaviour of pipe-walls by means of the Impulse Response Method. Equations were linearised and solved in the frequency domain. The method was tested with experimental data. Pipe wall viscoelasticity was described by a complex-valued wave speed, whereas unsteady friction effects were calculated by complex-valued propagation operators. The numerical results obtained considering the pipe linear elastic and linear viscoelastic models were compared with experimental data, neglecting and taking into account unsteady friction. Transient head obtained for the viscoelastic case (complex-valued wave speed) showed a very good agreement with the experimental data. Conversely, the pressure obtained for the elastic case with no unsteady friction showed a large discrepancy with the observed data. The major challenge of the current and the future work is the distinction between frictional and mechanical dampening, as the viscoelastic behaviour of pipe-walls has a dissipative and dispersive effect on the pressure wave, similar to unsteady friction losses. The use of this method (i.e. IRM) is much faster than the typical method of characteristics and can straightforwardly include frequency-dependent factors; however, it has the disadvantage of the loss of accuracy due to the linearization of the friction term and the valve equation, and the complex application to multi-pipe systems.

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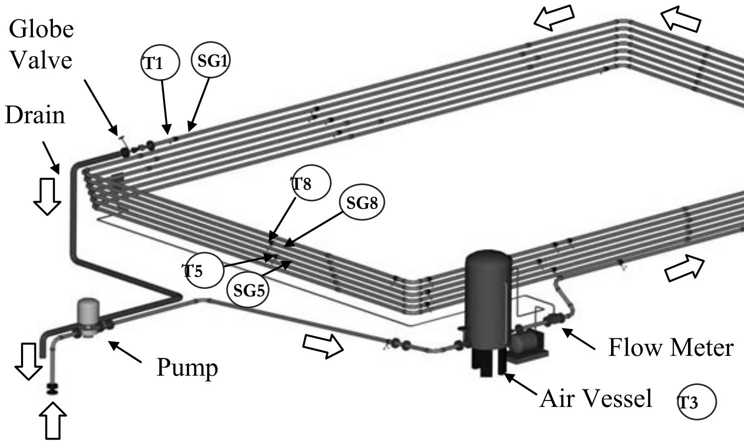

Fig. 1 – Imperial College experimental facility

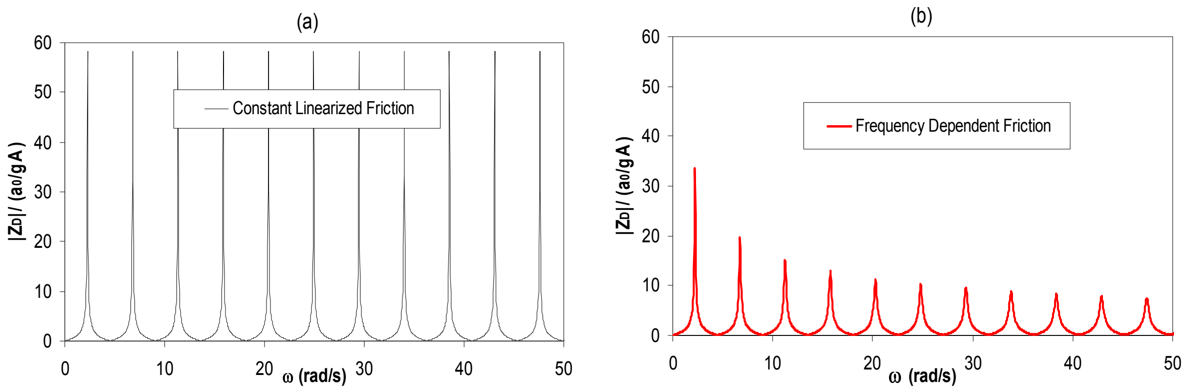

Fig. 2 – Transfer function $Z_D(\omega)$ at the downstream end for the linear elastic reservoir-pipe-valve system with (a) constant friction and (b) frequency-dependent friction (approximation for laminar flow)

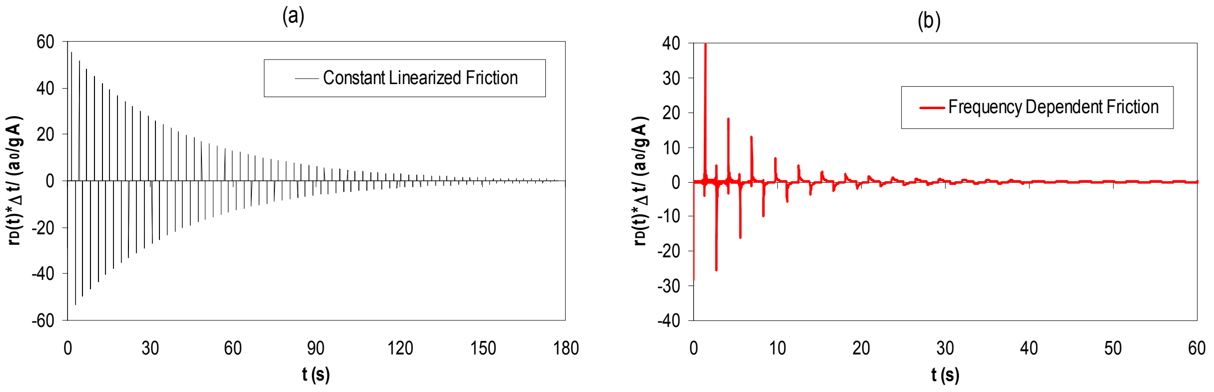

Fig. 3 – Impulse response function $r_{H_D}(t)$ at the downstream end for the linear elastic the reservoir-pipe-valve system with (a) constant friction and (b) frequency-dependent friction (approx. for laminar flow)

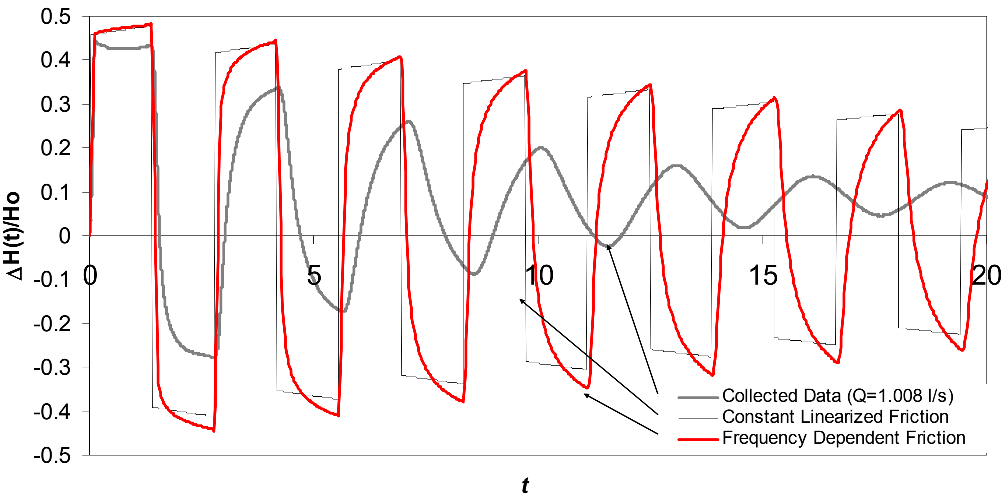

Fig. 4 – Piezometric head $H_D$ at the downstream end for the linear elastic reservoir-pipe-valve system versus collected transient data.

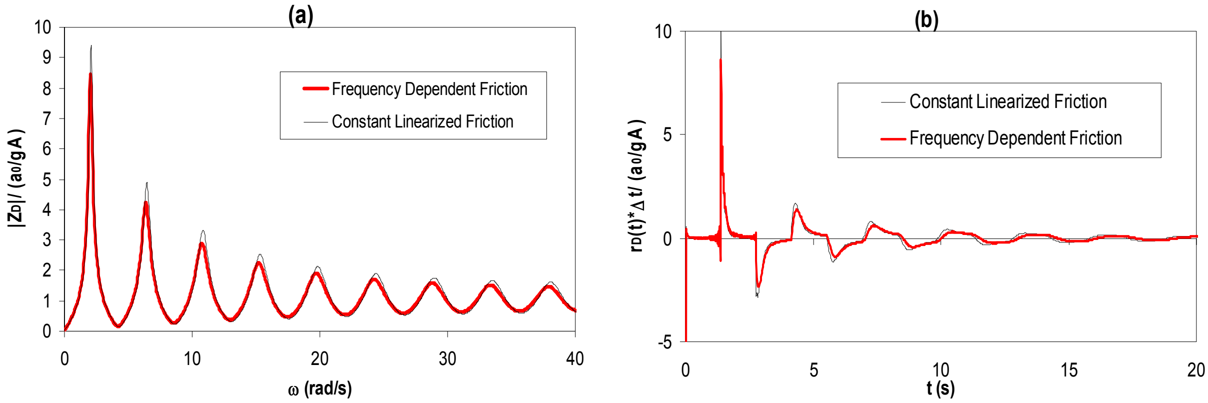

Fig. 5 – (a) Transfer function $Z_D(\omega)$ and Impulse response function $r_{H_D}(t)$ at the downstream end for the linear viscoelastic reservoir-pipe-valve system with constant friction and frequency-dependent friction

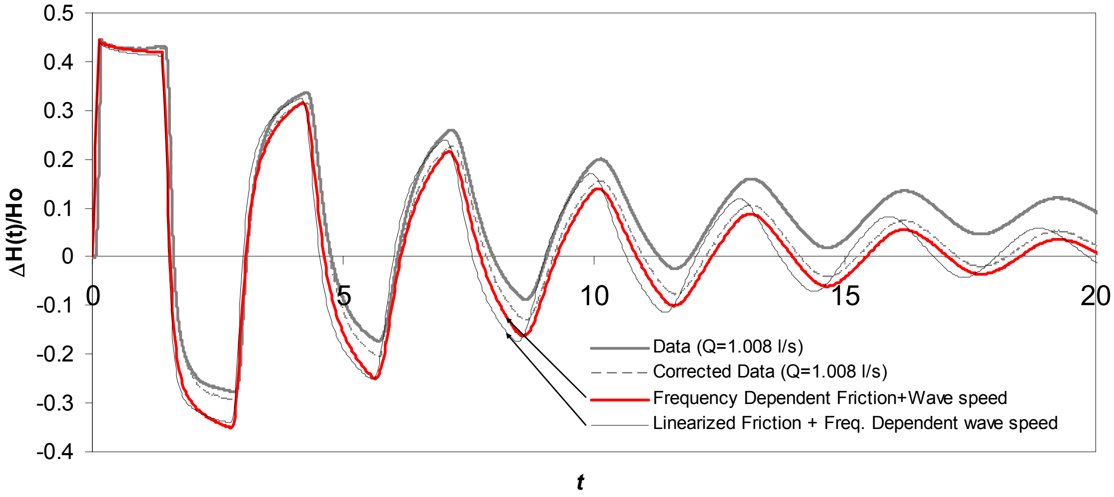

Fig. 6 – Piezometric head $H_D$ at the downstream end for the linear viscoelastic reservoir-pipe-valve system versus collected transient data.