ANALYSIS OF SOME EFFICIENCY INDICATORS[1]

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1. Introduction

The analysis of the higher education system efficiency, as well as the analysis of the higher education system quality, can properly be carried out through the use of indicators that objectively quantify their factors. In present times, the discussion of matters like the Bologna Declaration implementation, the comparative advantages and disadvantages of the various national systems and the financial difficulties felt by the institutions is under its way, it is then important to understand the system behavior and the relationship between some of its state indicators.

An analogy between the behavior of the higher education system and the behavior of a cascade of linear reservoirs is presented. Residence times, graduations, abandons and success indicators are used to establish the analogy.

The analogy allows the estimation of the amount of time that the system requires to attain a steady state and, once there, provides an easy way to estimate the values of the indicators that define or will define the state of the system.

2. Formalism

The flow of the students through the curricular years of a given course of studies is envisaged as a cascade of linear reservoirs, each one of them eventually experiencing escaping leakage or loss (Figure 1).

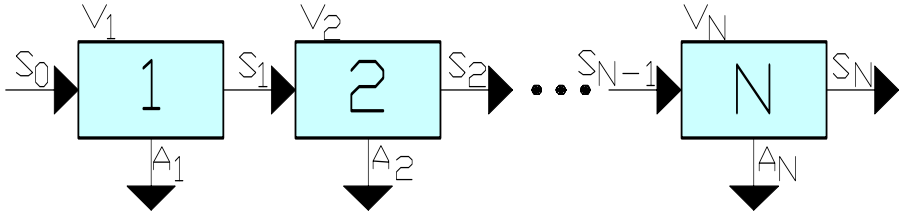

Figure 1 – Model course with N curricular years

Thus, consider that in each curricular year, out of the secondary education system, S_0 students are admitted to the first year of the course, V_i students are enrolled in year i, S_i students pass from year i to the next year, or they finish the course after N years, and A_i students abandon the course at year i, either voluntarily or compulsively.

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The following indicators per curricular year can be established:

- **Residence time indicator** ($\mathbf{RI_i}$): ratio of the number of students enrolled in year i (V_i) to the number of students that entered that year (S_{i-1}),

$$RI_i = \frac{V_i}{S_{i-1}} \tag{1}$$

- **Graduations indicator** ($\mathbf{GI_i}$): ratio of the number of students that passed from year i to the next year (S_i) to the number of students that entered year i (S_{i-1}),

$$GI_i = \frac{S_i}{S_{i-1}} \tag{2}$$

- **Abandons indicator** ($\mathbf{AI_i}$): ratio of the number of students that abandon the course at year i (A_i) to the number of students enrolled in year i (V_i),

$$AI_i = \frac{A_i}{V_i} \tag{3}$$

- **Success indicator** ($\mathbf{SI_i}$): ratio of the number of students that passed from year i to the next year (S_i) to the number of students enrolled in year i (V_i),

$$SI_i = \frac{S_i}{V_i} \tag{4}$$

Similar indicators can be built when one considers the years aggregated in a full course that is just one reservoir (Figure 2):

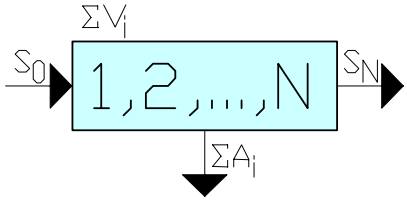

Figure 2 - Model aggregate course with N curricular years

- **Residence time indicator** (**RI**): ratio of the number of students enrolled in the course (ΣV_i) to the number of students that entered the course (S_0),

$$RI = \frac{\Sigma V_i}{S_0} \tag{5}$$

- **Relative residence time indicator** (**RRI**): ratio of the residence time indicator to the course number of curricular years (N),

$$RRI = \frac{RI}{N} \tag{6}$$

- **Graduations indicator** (**GI**): ratio of the number of students that finish the course (S_N) to the number of students that enter the course (S_0),

$$GI = \frac{S_N}{S_0} \tag{7}$$

- **Abandons indicator** (**AI**): ratio of the number of students that abandon the course (ΣA_i) to the number of students enrolled in the course (ΣV_i),

$$AI = \frac{\Sigma A_i}{\Sigma V_i} \tag{8}$$

- **Success indicator** (**SI**): ratio of the number of students that finish the course (S_N) to the average number of students per year enrolled in the course (ΣV_i / N),

$$SI = N\frac{S_N}{\Sigma V_i} \tag{9}$$

Seeking simplicity, consider that every curricular year as the same characteristics: $RI_i=RI_*$, $GI_i=GI_*$, $AI_i=AI_*$, $SI_i=SI_*$, which remain constant over time. Such a system is designated a homogeneous system ($*$).

In a steady state, the following relations hold for a homogeneous system:

$$V_i = S_0\frac{SI_*^{i-1}}{(SI_* + AI_*)^i} = S_0\frac{GI_*^i}{SI_*} \tag{10}$$

$$RI_* = \frac{GI_*}{SI_*} \tag{11}$$

$$GI_* = \frac{V_i}{V_{i-1}} = \frac{SI_*}{SI_* + AI_*} = 1 - AI_*\ RI_* \tag{12}$$

$$AI_* = \frac{SI_*}{GI_*}(1 - GI_*) = \frac{1 - GI_*}{RI_*} \tag{13}$$

$$\Sigma V_i = S_0\ RI_*\frac{1 - GI_*^N}{1 - GI_*} \tag{14}$$

$$RI = RI_*\frac{1 - GI_*^N}{1 - GI_*} = \frac{1 - GI}{AI} \tag{15}$$

$$GI = GI_*^N = 1 - AI\ RI \tag{16}$$

$$AI = AI_* \tag{17}$$

$$SI = N\ SI_*\frac{GI_*^{N-1}\ (1 - GI_*)}{1 - GI_*^N} = N\frac{GI}{RI} = \frac{GI}{RRI} \tag{18}$$

$$\lim_{GI_*=1}\frac{1 - GI_*^N}{1 - GI_*} = 1 \tag{19}$$

3. Behavioral analysis

When a given course starts for the first time and the entrance of the students is made exclusively through the first year of the course, there is an elapsed time of N years before some of the students finish their studies. In a homogeneous system, in which the number of students that enter the course remains constant (S_0=constant), a steady state is achieved and all the indicators become constants after some time.

Figure 3 presents the time evolution of a homogeneous course graduations indicator (GI) as a function of the abandons indicator (AI). The course has a curricular duration of 5 years (N=5) and a success indicator in each year of 80 % ($SI_* = 0.80$). Six abandons indicators are shown (AI=0.00, 0.02, 0.04, 0.06, 0.08 and 0.10). The steady state is attained for each abandon indicator after about 10 years or after about a time equal to about twice the curricular duration of the course. In steady state, the graduations indicator is equal to 1.00 when the abandons indicator is equal to 0.00. The graduations indicator decreases markedly as the abandons indicator increases.

Figure 4 presents the number of students in each year per unit of entering students (V_i / S_0) for a homogeneous system in a steady state with AI=0% and AI=10%. The course with 5 years duration (N=5) has a per year success indicator of 80 % (SI_*=0.80).

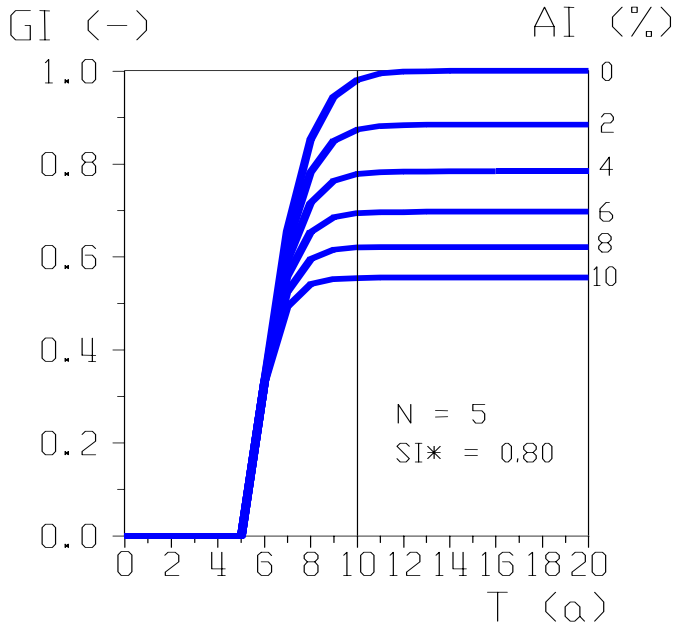

Figure 3 - Time evolution of a homogeneous course

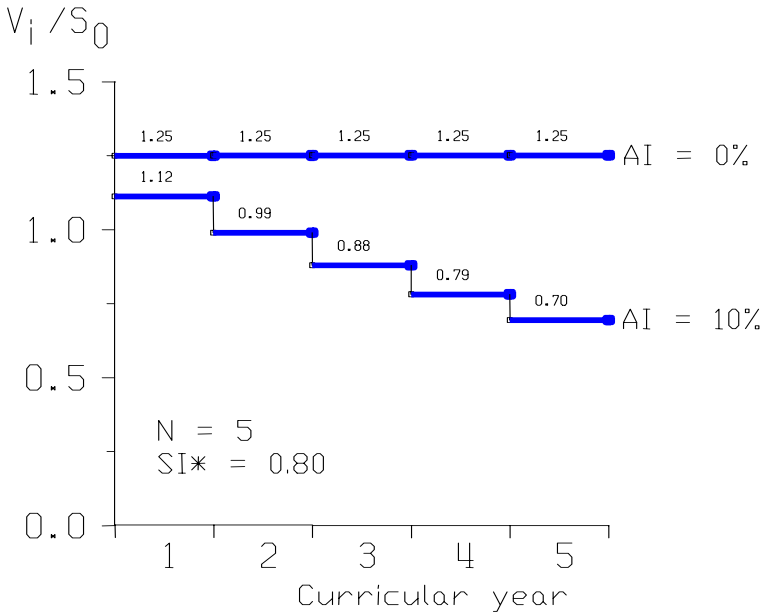

Figure 4 - Number of students in each year per admitted student

Figure 3 and Figure 4 show the striking effect of the abandon indicator on the graduations indicator and on the number of students enrolled in the course. As a matter of fact, considering the assumed conditions, when the abandons indicator is 10 %, the graduations indicator is about 56 % and the residence time indicator is 4.48 years. This value of the residence time indicator is about 72 % of the value 6.25 years that corresponds to the abandons indicator of 0 %.

Table 1 - Relationship between aggregate indicators and per year indicators

GI*	GI/GI*	SI/SI*
1.0	1.00	1.00
0.9	0.66	0.80
0.8	0.41	0.61
0.7	0.24	0.43
0.6	0.13	0.28
0.5	0.06	0.16
0.4	0.03	0.08

Table 1 presents the ratios of the aggregate to per year graduations indicators and success indicators as a function of the per year graduations indicator. As expected, the referred to ratios decrease markedly faster than the per year graduations indicator.

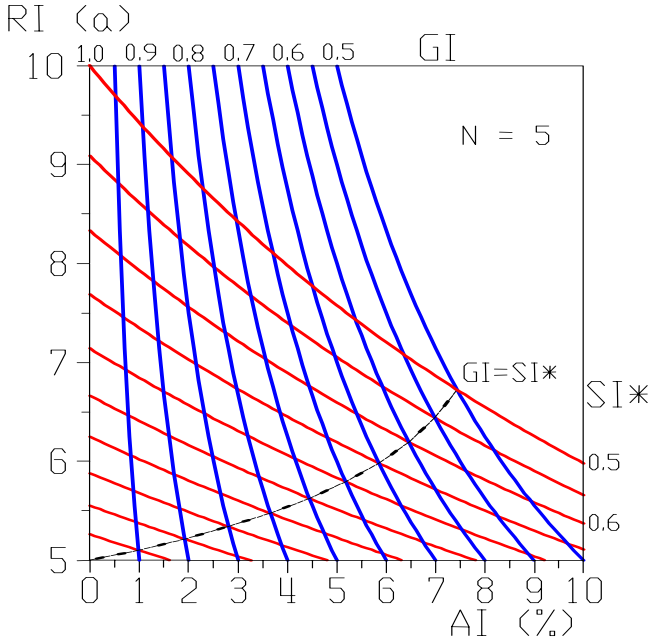

Figure 5 - Relationship between indicators of a steady state homogeneous course

Figure 5 illustrates pictorially some of the mathematical relations that link the indicators of a steady state homogeneous course. These relations were referred to in section 2.

One can observe that a sufficient condition for a high value of the graduations indicator is a low value of the abandons indicator. Also, a necessary condition for both a high value of the graduations indicator and a low value of the residence time indicator is a high value of the success indicator, which per se guarantees a high value of the graduations indicator, a low value of the residence time indicator and a low value of the abandons indicator.

The analysis carried out above is clear when the focus is on the gradients of the graduations indicator and of the success indicator surfaces. In fact, the gradient of the former surface is roughly parallel to the abandons indicator axis whereas the gradient of the latter surface points to the origin of both axes. Thus, the success indicator can be elected as a good numeric system efficiency indicator.

When there are no abandons (AI=0), the graduations indicator equals 1 (GI=1.00) and the residence time indicator is maximum (RI=N/SI$_*$=N/SI).

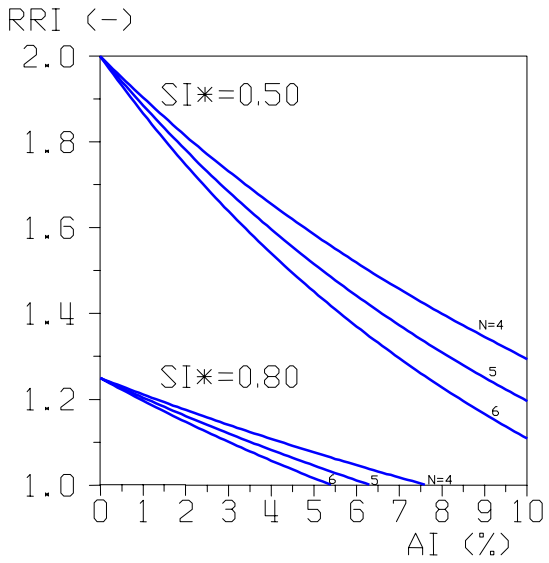

Figure 6 - Relative residence time indicator for homogeneous courses with different durations and success indicators

Figure 6 presents the relative residence time indicator as a function of the abandons indicator for two values of the success indicator (SI$_*$=0.50, 0.80) and for three course durations (N=4, 5 and 6). For a given abandons indicator, the relative residence time indicator increases with the reduction of the course duration. This increase is even greater when the abandons indicator increases.

4. Conclusion

The concepts built upon the behavior of higher education systems or of one of its courses of studies are often subjective and imprecise. The analysis that was carried out in this paper, based on simplified versions of the real system, which is more complex and variant than the models that were adopted, allows one to clarify and to quantify some of those concepts through the use of indicators.

The motivation behind the study included the concern for the efficiency of the system, the impact of the Bologna Declaration, the impact of a demographic reduction in potential candidates that can be expected in some countries and the impact of an eventual change in the financing rules.

It seems consensual that the system will be all the more efficient the more graduates it produces with the less residence time. Leaving the financial aspects aside, the analysis shows that this corresponds to a high success indicator (IS_*). In fact, the higher the success indicator ($IS_* \rightarrow 1$) the lesser the possible limits of variation of the residence time indicator and of the abandons indicator and the higher the graduations indicator.

High success indicators can be achieved by attracting the best students, teachers and staff. Providing them, or having them to provide for, the equipments and financial resources that would allow the fulfillment of the institutional objectives, without creating the need for prohibitive fees or public expenditure, is also important. Beyond these factors, the attraction of the best could be eased by the history of the employability of the graduates and of the quality of their professional carriers.

Reference:

Hipolito, J. R.; 2002: "Estudo sobre Alguns Indicadores de Eficiência do Ensino Superior" (in Portuguese), http://www.civil.ist.utl.pt/~jh/ordeng2002.pdf .