Dynamic orifice model on waterhammer analysis of high or medium heads of small hydropower schemes

Modèle d’orifice dynamique dans l’analyse du coup de bélier des petites centrales de haute et moyenne chutes

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ABSTRACT
The most severe hydropower transients are induced in long hydraulic circuits due to extreme operating conditions. A computational model was developed in order to on one hand ensure waterhammer system control and, on the other hand, provide a more reliable and easier analysis for different specific speed turbines and alternative solutions of the system as a whole, through interaction between different hydraulic components. In reaction turbines, runaway conditions and guide vane closure cause significant discharge variations and pressure fluctuations that can affect the design of conveyance systems. A new approach for groups modelling as dynamic orifices concept was developed enabling the characterisation of the integrated system. The simulation results were compared with laboratory tests. This model can be used in the initial stages of civil works design as an efficient way to better characterise the hydrodynamic behaviour of the system when equipped with reaction turbines.

RÉSUMÉ
Les transitoires hydrauliques les plus sévères sont provoqués par des conditions extrêmes de fonctionnement des systèmes hydroélectriques, en particulier dans le cas de conduites longues et conduites forcées. Un logiciel a été développé de façon à simuler et assurer le contrôle du coup de bélier et, aussi, à permettre une analyse plus simple et sûre pour différents types de turbines et des solutions alternatives pour le système hydraulique. Le programme développé permet l’interaction de plusieurs éléments. Pour des turbines à réaction les conditions, d’emballage et de fermeture du distributeur peuvent provoquer des variations significatives du débit et des fluctuations de pression dont il faut tenir compte lors de la conception des systèmes d’adduction. Une nouvelle approche, basée sur le concept d’un orifice dynamique équivalent, a été développé pour modéliser les groupes, ce qui permet la caractérisation du système intégré. Les résultats obtenus ont été comparés avec des essais en laboratoire. Cette technique de calcul peut être utilisée lors de l’analyse préliminaire des travaux de génie civil, comme un moyen efficace de mieux caractériser le comportement hydrodynamique du système lorsqu’il est équipé de turbines à réaction.

Introduction
Small hydropower schemes, with long conduits, are the subject of safety and economic concerns due to the occurrence of severe hydraulic transients. In waterhammer analysis, abnormal operation under extreme conditions is the base of any design study. When equipped with reaction turbines of low inertia and low specific speed, during abnormal conditions, runaway can easily occur, which can induce dangerous overpressures, depending on the type of turbine runner. A serious safety problem can occur after a full-load rejection when the overspeed effect, causes maximum overpressure, which can be attained very rapidly (in as little as three seconds). The total duration of guide vane closure will be an important parameter, but not the most important in this case.

An efficient model for transient analysis will enable the operational behaviour of the hydro-system to be evaluated. This analysis may lead to unconventional solutions by eliminating conservative protection devices such as a surge tank. Actually, at the initial design, the stage of civil works conception, the characteristics of the groups are not yet well defined and not completely available. In order to overcome such lack of knowledge, a computational model based on a dynamic orifice concept was developed. The specific speed allows characterisation of the turbine behaviour that will influence hydrotransients along the conveyance system.

State-of-the-art
The prediction of overpressures induced by hydraulic turbines has always been an important objective in the development of pressure transient analysis. Early studies focused on the calculation of the maximum penstock overpressure due to a more or less rapid flow stoppage induced by turbine nozzles or guide vanes. Simple formulas were proposed based on the fundamental penstock and flow parameters or time constants and on the time of gate manoeuvre. An example is Michaud’s formula (1878), or Vensano’s formula in the U.S.A., for slow manoeuvres. Allievi (1903) was closely involved in unsteady flow behaviour and safety in hydroelectric schemes when developing his mathematical formulation of the waterhammer phenomenon. However, as a boundary condition of a hydraulic system, a reaction turbine will induce a compound effect on the flow during a full load rejection: the guide vane effect and the speed or runner effect. Since the sixties, improved computer methods have enabled more detailed analysis of pressure transients in pipe systems with different components and operating conditions. The complete characteristic curves of hydraulic machines, including pumps and turbines, together with rotating mass equation and by the compatibility equations of the method of characteristics, applied along the pipelines, can now be solved with high accuracy using numerical techniques.
One of the ways to represent reaction type turbine characteristics is based on unit parameters combining discharge, speed, torque, power, head, and machine diameter. For transient analysis it is convenient to make use of dimensionless homologous relationships such as the Suter parameters (Chaudhry, 1987; Wylie and Streeter, 1993). Two families of curves can then be developed for each guide vane opening or position. Numerical interpolation or transformation techniques can be applied to these non-linear curves to solve the coupled machine rotating masses equation and penstock or pipeline equations (Boldy, 1976; Chaudhry, 1980). For a full-load rejection condition, design constraints are placed on both turbine overspeed and penstock overpressure. The gate closure law needs to be specified to control both these effects. Optimisation techniques were developed by different authors to deal with this situation (Ruus, 1966; Wozniak and Fett, 1973; Nai-Xiang and Zu-Yan, 1989).

For design of hydraulic conveyance structures, the general specifications of the equipment and the overpressures along the penstocks or galleries must be determined. Most approximate formulas are based on the start-up (or inertia) time of the water mass at full load, \( T_W \) (e.g. Lein, 1965):

\[
\frac{\Delta H_M}{H_o} = K_C \frac{T_W}{T_C}
\]

in which \( \Delta H_M \) = maximum head variation, \( H_o \) = turbine gross head, \( T_C \) = closing time and \( K_C \) = a factor that depends upon the turbine specific speed \( N_s \).

Typically, \( K_C \) values vary between 1.3 and 1.5. Equation (1) coincides with Michaud’s formula for \( K_C = 2 \). In fact, according to the theoretical studies of De Sparre (Remenieras, 1961) the maximum overpressure due to a full gate closure will be less than the value obtained by Michaud’s formula. However, Gariel (1918) showed that the “maximum maximorum” overpressure induced by a critical partial gate closure, with a linear discharge variation, will be Michaud’s value (Remenieras, 1961). Due to the runner overspeed effect in reaction turbines, \( \Delta H_M \) will be a function of the inertia of the rotating masses or a function of the unit starting time, \( T_W \). More recently, Bahamonde (1991) presented an approximate analysis of the combined variation of pressure and runner speed rise during closure of the turbine gate. However, this method, based on Allievi’s methodology for a uniform rate of gate opening variation, does not take into consideration the rapid discharge variation due to overspeed.

The turbine behaviour as well as the dynamic response of the hydraulic system will depend on the specific speed. Among the effects induced by a particular type of turbine are pressure surges due to runner overspeed, and the peculiar “S” shape effect of the characteristic curves of low specific turbines, notably in pump-turbines and in double runner Francis turbines (Taulan, 1983). Brekke (1976) also showed the influence of turbine characteristics on turbine governing.

Two levels of tools are available for hydraulic transient analysis of hydroelectric schemes: complete models based on the full set of machine equations obtained for each unit type, and classical methods based on the gate effect or on empirical data. The dynamic orifice model developed in this paper sets out to fill the gap between these two approaches.

### The dynamic orifice model - new hydraulic approach for turbine analysis

The dynamic orifice model offers a flexible general model based on a small number of parameters, the objective of which consists in analysis of the extreme condition generated by runaway and/or guide vane closure as the most unfavourable condition, with regard to hydraulic circuit design (overpressures). It includes prediction of overspeed effect under runaway conditions on hydrot transient response all the way along conveyance system. The model can be seen as a useful tool for an integrated computer analysis of any multi-component systems (reservoir, total pressurised penstock, or a mixed circuit composed by a canal and a forebay with free-surface flow and a penstock with pressure flow, and typical response of any type of turbo-generator groups).

The dynamic orifice model is based on the concept of the turbine acting as a hydraulic resistive component where the head lost by the flow is characterised by a dimensionless orifice formula that has a dynamic discharge coefficient. This coefficient is composed of two terms: a gate factor and a runner overspeed factor. The first is the gate-opening coefficient, which defines the maximum turbine discharge for a given head and speed as a function of the gate opening. The second factor modifies the discharge coefficient as a function of the runner speed because, for reaction type turbines at constant head and gate position, the discharge is a function of the runner speed. The coupled response will depend on the specific turbine speed \( N_s \) (Büchi, 1957; Mataix, 1975; Raabe, 1985) as well as the rated turbine speed \( N_R \), discharge \( Q_R \) and head \( H_R \).

With low specific speed turbines the discharge decreases with runner speed. Conversely, for high specific speed turbines the discharge may increase with speed. This turbine behaviour has a significant effect on the transient response of the conveyance system after a full load rejection and must be taken into account in the simulation of extreme operational conditions. Both the overspeed factor of the turbine discharge coefficient and the turbine hydraulic torque are based on dimensionless relationships and on a few simple parameters, and are characterised by heuristic equations that are approximations of the real characteristic curves.

In impulse turbines the transient discharge is decoupled from the runner speed and the turbine discharge does not change with wheel speed so long as the gate or nozzle opening and the head remain constant. For this type of turbine, the overspeed factor is equal to unity and the model contains only the pure orifice model. When the dynamic orifice is coupled to flow modelling along the conveyance system, the model has the capacity to simulate:

- pressure and discharge variations and the head envelopes along the pipelines as well as, for mixed circuits, also the head variation along the canal and forebay;
- transient pressure and discharge variations in the powerhouse for full-load rejection and/or guide vane closure (or nozzle closure);
- the rotational speed variation of the groups.
Dynamic orifice equations

The dynamic orifice model is based on two fundamental dimensionless parameters: \( \alpha_R = \frac{Q_{RW}}{Q_R} \) and \( \beta_R = \frac{N_{RW}}{N_R} \), with \( Q_{RW} \) = turbine discharge at runaway speed, \( Q_R = \) rated turbine discharge, \( N_{RW} = \) turbine runaway speed and \( N_R = \) rated turbine speed, all for the rated turbine head. In this paper the subscripts R and RW indicate the rated and the runaway conditions (at rated head), respectively.

These parameters depend on the turbine type: a low specific speed Francis turbine (Figure 1) will have \( \alpha_R \leq 1 \) and a high specific speed, Propeller and Kaplan turbines will have \( \alpha_R > 1 \). Empirical relationships can be found in the literature allowing an approximate valuation of \( \alpha_R \) and \( \beta_R \) as a function of \( N_s \) (Fazalare, 1991; Ramos, 1995 and Ramos and Almeida, 1996). In practice the turbine manufacturers know the values of these parameters.

![Diagram](image-url)

Fig. 1. Analogy between pump characteristic curves based on Suter parameters for turbine zone under runaway conditions, with turbine simulation through Dynamic Orifice modelling (for a low specific speed turbine \( \alpha_R=0.65 \))
through turbine tests. Based on published information and on data furnished by turbine manufacturers, the following equation was obtained for \( \alpha_R \):

\[
\alpha_R = 0.3 + 0.0024 N_s
\]

(2)

and for \( \beta_R \), another equation was proposed by Fazalare, 1991:

\[
\beta_R = 1.6 + 0.002 N_s
\]

(3)

both equations with \( N_s \) in (m, kW).

According to data obtained from real case studies, for low inertia units in small hydroelectric powerplants equipped with reaction turbines, \( \beta_R \) is about 2 \pm 20%.

Figure 1 shows the analogy between the turbine operation with the pump when operating in turbine zone, as well as the discharge reduction, in low specific speed machines, due to runaway conditions.

The dynamic orifice equation, being a modified orifice flow, is based on the turbine head-discharge equation:

\[
\frac{Q}{Q_h} = C_g C_s \sqrt{\frac{H}{H_R}}
\]

(4)

where \( Q \) stands for the turbine discharge; \( H \) is the turbine head; \( C_g \) is the gate coefficient varying between one and zero according to gate opening, \( y \); and \( C_s \) is the runner speed coefficient, which equation can be written as

\[
C_s = \left[ 1 + \frac{\alpha_R - 1}{\beta_R - 1} \left( \frac{N}{N_R} \sqrt{\frac{H_R}{H}} - 1 \right) \right]
\]

(5)

where \( N \) is the runner speed. In the model it is assumed that \( C_g = f(y) \).

As presented in Figure 1, it was assumed a linear variation of the discharge with rotating speed under runaway conditions. This approach was verified through laboratory tests applied to a low and high specific speed Francis turbine runner.

In this way, the hydraulic torque (\( T_{th} \)) is obtained through the following equation:

\[
T_{th} = \left( \frac{H}{H_R} \right)^{1/2} \left( \frac{C_g \eta N_R}{N R} \right) \left[ 1 - \left( \frac{N}{N_R} \sqrt{\frac{H_R}{H}} - 1 \right) (\beta_R - 1) \right]
\]

(6)

where \( \eta \) is the unit efficiency and

\[
T_{th} = \left( \frac{60}{2\pi} \right) \eta \frac{Q_R H_R}{N_R}
\]

(7)

The unit efficiency for a load rejection condition can be considered to vary according to the following approximate equations:

- For \( y > 0.5 \) and \( N \geq N_R \)

\[
\eta = \eta_o \left( \frac{N_{RW} - N}{N_{RW} - N_R} \right)
\]

(8)

- For \( y \leq 0.5 \) and \( N \geq N_R \)

\[
\eta = 2\eta_o \left( \frac{N_{RW} - N}{N_{RW} - N_R} \right)
\]

(9)

with \( \eta_o \) the unit efficiency for initial condition.

Additional model conditions are introduced in order to consider the turbine speed-no-load condition and the runner speed decay after the guide vane closure.

**Rotating mass equation**

The unbalanced torque between turbine and generator changes according to the angular momentum equation for the rotating mass according to the following equation

\[
T_{th} - T_G = I \frac{d\omega}{dt}
\]

(10)

in which \( T_{th} = \) net hydraulic turbine torque, \( T_G = \) electromagnetic resistant torque, \( I = \) total polar moment of rotational mass inertia \( (I = WR^2/\gamma) \) and \( \omega = \) angular velocity of the rotating mass.

After a full load rejection the electromagnetic resistance torque, \( T_G \), can be set equal to zero. According to equation (10), the polar moment has a significant influence on the speed variation of the rotating mass of the turbo-generators. For low inertia units the runner speed increases rapidly after a full-load rejection, and can attain runaway conditions.

**Canal and pipeline equations**

Transient regimes in hydraulic conveyance circuits of hydro-power plants can be modelled by 1-D flow models when the reaches are straight and uniform and the cross-section is much less than its length.

In canals it is important to know the unsteady flow behaviour caused by rapid strong discharge variations, with possible formation of shock waves, in order to define the canal geometry. In this case there is a discontinuity in the water surface or bore, with rapid varied flow, and a gradual varied flow upstream and downstream of the bore.

Explicit methods with second order accuracy have proved to be suitable for flow modelling with shocks and bores. The following general assumptions are accepted for free-surface equations (Chaudhry, 1987, Almeida and Koelle, 1992 and Ramos, 1995):

- the transient flow is 1-D with the horizontal water surface and uniform velocity in each cross-section;
- the streamline curvature is small and vertical accelerations are neglected, hence the pressure is hydrostatic;
• transient friction losses and turbulence are modelled by empirical resistance laws (quasi-stationary assumption).

The elastic model of pressurised flow for small hydroelectric penstock systems is compatible with the following assumptions (Chaudhry, 1987, Pejovic, Boldy and Obradovic, 1987, Almeida and Koelle, 1992 and Wylie and Streeter, 1993):
• the flow is slightly compressible;
• the velocity and pressure follow a uniform distribution in each cross-section (α = α' = 1);
• the rheological behaviour is elastic and linear;
• the convective terms in the basic equations are neglected comparing with the other terms.

Any disturbance induced in the flow is propagated with a celerity that will strongly influence the dynamic response in the hydraulic circuit. For pressure flows the celerity of the elastic waves corresponds to the storage capacity of the fluid compressibility and pipe deformation:

\[ c = \sqrt{\frac{K}{\rho [K/E] \psi}} \]  \hspace{1cm} (11)

and in free-surface flows corresponds to the kinetic energy of the flow:

\[ c = \sqrt{\frac{gA_2 (A_2h_{G2} - A_1 h_{G1})}{A_1 (A_2 - A_1)}} \]  \hspace{1cm} (12)

where A stands for the flow cross-section; E is Young’s modulus of elasticity of the conduit walls; K the bulk modulus of elasticity; \( h_0 \) the depth of the centred area A; \( \psi \) the dimensionless parameter that depends on the elastic properties of the conduit; and 1 and 2 are upstream and downstream of wave front. For free-surface flow the complete dynamic model based on the Saint-Venant equations must be written in a conservative form in order to simulate the propagation of bores (Franco, 1996):

\[ \frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = D(U) \]  \hspace{1cm} (13)

where U, F(U) and D(U) are the following vectors:

\[ U = \begin{bmatrix} A \\ Q \end{bmatrix}, \quad F(U) = \begin{bmatrix} Q \\ \frac{Q^2}{A} + gAh \end{bmatrix}, \quad D(U) = \begin{bmatrix} 0 \\ \frac{gA|\psi| - J}{Q^2} \end{bmatrix} \]  \hspace{1cm} (14)

The pressure transients in pipes are modelled by the well-known waterhammer equations (Chaudhry, 1987 and Wylie and Streeter, 1993). The basic differential equations of unsteady pressurised flows can also be written in matrix form, yielding the following vectors:

\[ U = \begin{bmatrix} H \\ Q \end{bmatrix}, \quad F(U) = \begin{bmatrix} c^2 \\ gAQ \end{bmatrix}, \quad D(U) = \begin{bmatrix} 0 \\ \frac{JgA}{Q^2} - |\psi| \end{bmatrix} \]  \hspace{1cm} (15)

in which x = distance along the canal bottom or the pipe axis; t = time; A = cross-section flow area; Q = turbine discharge; h = water depth (canal); H = piezometric head (for penstock); i = channel bottom slope; J = slope of the energy grade line; g = gravitational acceleration; c = wave celerity in open channel/pressure pipe. The interior points will be solved using the MacCormack method. Following MacCormack’s recommendation, the predictor and corrector steps are used alternately with the finite forward and backward differences. The method of characteristics transforms these equations into a pair of ordinary differential equations valid along the characteristic lines in the (x, t) plane. These equations can be replaced by algebraic equations that are solved together with the other basic equations of the computational model. No approximate model based on the rigid column theory can be applied due to the potential rapid flow velocity variation caused by the overspeed effect. All the basic equations are solved together in order to obtain the variations of head (or pressure), discharge (or flow velocity) along the pipe sections, and turbine runner speed for a given guide vane motion.

The dynamic orifice equations, rotating mass equation and pipeline (and canal) equations compose the complete computational model.

Operational advantages and validation

The dynamic orifice model has the advantage that the dynamic runaway conditions induced by any type of reaction turbine units can be easily evaluated. This technique, together with computer modelling of the other components of the system, enables an integrated hydrotransient analysis to be developed that has proved sufficiently accurate for conveyance system design purposes, especially for the feasibility and general design stages. Figure 1 shows the turbine discharge (Q) variations for different values of head (H), guide vane opening (y) and rotational speed (N), based on the dynamic orifice model (equations (4) and (5)) applied to the turbine overspeed zone for a full-load rejection. The dynamic behaviour of a low and a high specific speed turbine was compared with predictions using the dynamic orifice model and lab tests. Examples of the transient variations of several quantities are presented in Figures 2 and 3, respectively for low and high Ns runners placed downstream of the pipeline.

Waterhammer effects

Prediction of overpressures along the penstock and interaction with forebay and canal response (e.g. for mixed hydraulic circuits), after a full load rejection, including guide vane and overspeed effects for any specific speed of reaction turbine, is
possible with the complete set of equations (4) to (15), presented here in a simplified way.

As an example of the results obtained through systematic computer simulations, Figure 4 shows the dimensionless maximum upsurge or overpressure values induced by full load rejection at the downstream end of a single uniform penstock, $\Delta H_M/H_0$, for $\beta_R = 2.0$, as a function of $\alpha_R$.

The symbols are defined as $H_0 =$ gross head (m), $T_C =$ guide vane full closure time (s), $T_E =$ pipeline elastic time constant (s), $T_W =$ pipeline hydraulic inertia time constant (s) and $T_m =$ unit starting time (s).

This figure shows that for low specific speed Francis turbines or for $\alpha_R < 1$, the overspeed effect will potentially induce greater overpressures compared to the gate effect. In fact, for $\alpha_R \rightarrow 1$ the flow reduction induced by turbine overspeed does not modify the maximum overpressure, because in this case the waterhammer depends only on the gate effect, including the time of gate closure. For this particular situation ($\alpha_R = 1$), the computational re-
sults show that ∆HM/Ho become proportional to Tw/Tc as in equation (1) with KC = 1.4.

When runaway speed is attained in a very short time interval of order TE, the overpressure due to overspeed can be evaluated by the following modified Joukowsky formula:

\[
\frac{\Delta H_M}{H_o} = 2 h_w \left( \frac{1 - \alpha_R}{20} \right) \beta_R \frac{T_E}{T_m}
\]  \hspace{1cm} (16)

with hw the Allievi parameter (typically hw < 1 for high-head systems).

Equation (16) will give the maximum overpressure induced by the overspeed effect, especially for low-inertia units. The dynamic orifice model is able to show the influence of the group’s inertia on overspeed and pipe-fluid elasticity through TE/Tm. It can be observed that, for each Tw/Tc value, the maximum overpressure due to overspeed can even (for low αR) exceed the maximum Michaud value corresponding to the critical partial gate closure, as deduced by Gariel (equation (1) with KC=2.0).

Conclusion

Empirical formulae or simplified models for hydrotransient analysis of a long hydraulic circuit, based only on the closure time of the guide vane, can not be accurate enough for design purposes. However, it can be very difficult to obtain the complete set of the turbine characteristic curves in the early stages of the design, depending on the available manufacturer’s data.

From the practical operational point of view, the application of the dynamic orifice technique, in real case studies, has enabled more reliable and economical layout solutions for long hydraulic conveyance systems, in most cases of small hydropower plants avoiding costly protection systems against waterhammer. Computer simulations based on DO model take few seconds or minutes, depending on the type of the hydraulic system (e.g. a total pressurised pipe or a mixed system, with part in open canal followed by the penstock), using a PC computer, including the interaction between different elements of the hydropower scheme. Thus, the dynamic orifice conception is a powerful aid for hydrotransient analysis and for understanding the conveyance system response (e.g. canal and penstock) and powerhouse design dependence of hydraulic phenomena parameterisation. The allowable penstock pressure, the guide vane manoeuvre time and the polar inertia moment can be specified before the full machine characteristics are known, in order to control excessive overpressures, especially for the critical full load rejection of low specific turbine speed.

The feasibility of a small hydropower project depends heavily on civil construction costs and on environmental impacts, requiring accurate computational simulations and cost-benefit studies in order to mitigate these factors. With this integrated model more efficient solutions can be selected from among different alternatives at an early but very important design stage.

References


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Nomenclature

The following symbols are used in this paper:

- A area of pipe cross-section (m²);
- Cg gate coefficient (-);
- Cs runner speed coefficient (-);
- c wave celerity (m/s);
- g gravitational acceleration (m/s²);
- H net head (m);
- Hs gross head (m);
- HR turbine rated head (m);
- hw Allievi parameter \( h_w = \frac{cV}{2g H_s} \) (-);
- I polar moment of inertia of rotating masses (kg m²);
- L pipeline length (m);
- N runner speed (rev/min);
- NR turbine rated speed (rev/min);
- NRW turbine runaway speed at rated head (rev/min);
- Ns specific speed \( N_s = \frac{N_r P_r^{1/2}}{H_r^{3/2}} \) (rev/min);
- Pr rated turbine power (kW);
- Pmax maximum power for rated head (kW);
- Q turbine discharge (m³/s);
- QR turbine rated discharge (m³/s);
- QRW turbine discharge at runaway speed (m³/s);
- R friction loss coefficient \( R = \frac{1}{Q^2} \) (m⁻²s⁻²);
- TC guide vane complete closure time (s);
- TE elastic time constant \( T_e = \frac{2L}{c} \) (s);
- TG electromagnetic torque (N m);
- TH hydraulic motor torque (N m);
- Tm machine starting time \( T_m = \frac{WD^2 N_s^2}{3575 P_r} \times 10^{-3} \) (s), with \( WD^2 = 4gL \);
- Tw inertia time constant \( T_w = \frac{LV}{gH_o} \) (s);
- t time (s);
- V flow velocity (m/s);
- WH head turbomachine characteristics (Suter parameters) (-);
- WT torque turbomachine characteristics (Suter parameters) (-);
- x distance along the pipe (m);
- y dimensionless gate opening (-);
- ΔHM maximum head variation (m);
- η unit efficiency (-);
- ηR unit rated efficiency (-);
- η0 unit efficiency for initial flow condition (-);
- ρ water mass density (kg/m³);
- ω angular velocity \( \omega = \frac{2\pi N}{60} \) (rad/s).