Water hammer in pressurized polyethylene pipes: conceptual model and experimental analysis

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This paper analyzes the dynamic effects of pipe wall viscoelasticity on hydraulic transients. These effects have been observed in transient data collected from two polyethylene (PE) pipe systems. The first is a 270 m pipeline, 50 mm diameter, at Imperial College London, and the second is the world’s longest experimental PE pipeline, 1.3 km long, 110 mm diameter, buried underground at Thames Water Utilities (London, UK). A mathematical model has been developed to calculate hydraulic transients in polyethylene pipe systems based on the assumption that the viscoelastic behaviour of pipe walls is linear. An additional term has been added to the continuity equation to describe the retarded deformation of the pipe wall and the resulting governing equations are solved by the Method of Characteristics. The numerical results are compared with both the classic elastic solution and with collected transient data. Good agreement between numerical results for the viscoelastic solution and observed data was obtained by fitting the creep function J(t). Unlike classic water hammer analysis, the developed mathematical model is capable of accurately predicting transient pressures in polyethylene pipes and the circumferential strains in the pipe walls.

Keywords: Water hammer; Viscoelasticity; Polyethylene; Pipe wall; Stress; Strain

1. Introduction

Water hammer analysis is important in the design of water pipeline systems to select pipe materials, to select the wall thickness (pressure ratings) and to specify surge protection devices. Classic water hammer analysis assuming linear elastic behaviour of pipe walls and quasi-steady state friction losses is commonly used for design purposes (Fox 1977, Chaudhry 1987, Almeida and Koelle 1992, Wylie and Streeter, 1993). These simple assumptions, however, sometimes break down when hydraulic transients are generated by rapid changes in flow conditions. While unsteady friction losses are prevalent in metal and concrete pipes, nonlinear elastic behaviour of pipe walls is significant in plastic pipes and it cannot be ignored.

In recent years, a new generation of low cost and high resistant polymeric materials, like polyethylene, has been extensively used in pressurized pipe systems, due to their temperature, chemical and abrasion resistance, high-pressure ratings, light weight, easy and fast installation, and low-price. These materials do not exhibit linear-elastic behaviour and, since the rheological behaviour of a pipe determines the pressure response of a pipe system, classic water hammer theory cannot accurately describe fast transients in these pipes, not even for extreme values.

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The viscoelastic behaviour of polymers has been extensively studied and is well-known (Ferry 1970, Aklonis et al. 1972, Riande et al. 2000). This behaviour significantly influences the pressure response during transient events by attenuating the maximum or minimum pressure variations in the pipeline and by increasing the dispersion (i.e. the phase shift) of the pressure wave. Several attempts have been made to simulate the rheological behaviour of plastic pipes both in steady-oscillatory flows and in non-periodic transient events. Two main approaches that have been followed use either a frequency-dependent wave-speed or a retarded viscoelastic term in the unsteady fluid equations.

The first approach assumes that the effect of the viscoelastic behaviour of the pipe wall (dampening and dispersion of the pressure wave) can be described by a frequency-dependent wave speed: instead of a constant elastic modulus, a frequency-dependent creep function, $J(\omega)$, is used to calculate the wave speed. This formulation is particularly useful when solving fluid equations in the frequency domain. Meissner and Franke (1977) derived the wave speed and the damping factor formulae for an oscillating flow in a thin-walled viscoelastic pipe. Rieutford (1982) proposed a “one-element Kelvin – Voigt” mechanical model to describe the pipe material creep-compliance function and include it in the wave speed formula. Franke and Seyder (1982) incorporated wave speed formulae in unsteady fluid equations and solved them using the Impedance Method for steady-oscillatory flow and the Impulse Response Method for non-periodic flow. Suo and Wylie (1990) modelled pipe viscoelasticity in periodic and non-periodic flows and compared the results with data collected by other authors.

The second approach is based on the mechanical principle associated with viscoelasticity in which strain can be decomposed into instantaneous-elastic strain and retarded-viscoelastic strain. The instantaneous elastic strain is incorporated into the pipe elastic wave speed in the traditional water hammer equations, and the retarded viscoelastic strain is introduced as an additional term added to the mass-balance equation (Gally et al. 1979, Rieutford and Blanchard 1979). Rieutford and Blanchard (1979) presented a complete conceptual model of hydraulic transients in viscoelastic pipes, and a sensitivity analysis of the effect of relaxation times of a “three-element Kelvin – Voigt” model in pressure dampening and dispersion. Gally et al. (1979) presented a complete mathematical model and obtained time and temperature dependent creep-compliance functions from dynamic tests. The mathematical model developed by these authors was verified with pressure and circumferential-strain data collected in a polyethylene pipeline. Ghilardi and Paolletti (1986), Rachid and Stuchenbruck (1990) and Rachid et al. (1992) later pursued this research in other non-elastic pipes. Pezzinga (2002) experimentally analyzed the effect of an additional PE pipe inserted downstream of a pump to reduce pressure surges.

The objective of the current paper is to show the effect of the viscoelastic behaviour of a PE pipe in transient pressures using physical data collected from two PE pipelines. In addition, the paper presents a state-of-the-art hydraulic transient solver incorporating additional terms to take into account both unsteady friction and pipe wall viscoelasticity. Several experimental tests were carried out in two PE pipe systems at Imperial College (IC) and Thames Water (TW). The collected data are compared with the elastic water hammer solution and with the viscoelastic solution. Sources of uncertainties associated with this function and the mathematical model are also discussed.

2. Transient data collection

Two experimental programmes were carried out at Imperial College (IC) and in cooperation with Thames Water (TW), aimed at the collection of reliable data sets for the investigation of transient-based leak detection and location techniques (Covas et al. 2001, Covas 2003, Covas et al. 2003a). Given the viscoelastic nature of the PE pipe used in these facilities, these programmes have been used to collect transient data for a better understanding of the rheological behaviour of PE pipe material during hydraulic transients.

A brief description of both pipe systems is first presented.

2.1 Imperial College laboratory data

2.1.1 Experimental apparatus. An experimental facility with a single pipeline was assembled at the Department of Civil and Environmental Engineering, Imperial College, London (Covas et al. 2004a, 2004b) for leak detection purposes. This facility was also used to collect physical data for calibration and validation of a novel hydraulic transient solver. The pipe was made of high-density polyethylene (SDR11 PE100 NP16) with 50.6 mm inner diameter (63 mm outside diameter) and a total length of 277 m. The pipe sections are electrofused and rigidly fixed to the wall in order to avoid pipe movement. The pipe-rig includes a centrifugal pump ($Q_0 = 2.5 \text{ l/s}; H_0 = 35 \text{ m}$) and a pressurized tank with 750 l volume at the upstream end, and a downstream globe valve to control the flow and to generate the transient event. The steady-state flow is measured with an electromagnetic flow meter located immediately downstream of the pressurized tank. Several modifications were carried out in the rig (Covas et al. 2001, Covas et al. 2004b) and the latest configuration is presented in figure 1.

The data acquisition system (DAS) was composed of an acquisition board (with 8 channels and a maximum sampling frequency of 9600 Hz per channel), four strain-gauge type pressure transducers (T), three strain gauges...
(SG) and a notebook computer. The pressure transducers have pressure ranges of 0 to 10 bar and accuracy of 0.3% of the full range. The strain gauges have one single grid, 1 cm length and an electric resistance of 350 Ohms with a class of accuracy ± 0.2%.

### 2.1.2 Data collection and analysis.

Various data sets were collected for different flow conditions with a sampling rate of 600 Hz. Since the acquisition board only allowed eight simultaneous measurements, pressure and circumferential strain data were collected at three different locations (locations 5, 8 and 1) and pressure was also monitored at the upstream end (location 3). Strain gauges were installed in the circumferential direction of the pipe to measure the radial displacement of the cross-section. Transient tests were run for a wide range of steady-state flows, from laminar \(Q_0 = 0.05 \text{l/s; } Re = 1400\) to smooth-wall turbulent regimes \(Q_0 = 2.0 \text{l/s; } Re = 50 000\). Transient events were generated by the closure of the globe valve. Transient data collected for \(Q_0 = 1.0 \text{l/s and } Re = 12700\) are presented in figure 2.

When the valve is closed, the maximum overpressure at transducer T1 (the nearest to the downstream end) does not remain constant, but decreases slightly and rapidly within the first 0.5 s (figure 2). This is followed by a slight pressure increase until the wave inverts 1.6 s after the valve closure. The pressure drop is accompanied by a strain increase. This phenomenon is characteristic of the viscoelastic behaviour of pipe walls in which there is an immediate elastic response of the material. The material stiffens when instantaneously loaded, followed by a retarded stress release with a strain increase and, consequently, a pressure drop. The pressure
increase afterwards is due to the line packing. This is particularly evident at the valve section, with 10% increase of maximum pressure.

A significant pressure dampening in the consecutive pressure waves is observed. This could not be thoroughly justified by unsteady-friction effects, as this phenomenon has never been observed with such intensity in metal pipes under the same flow conditions. In addition, pressure and strain curves have a particular convex shape during the loading phase and, the inverse, a concave shape, during the unloading (pressure release) phase. Once more, this is a characteristic of polymer viscoelastic behaviour, given the similarities in the strain curve during a loading and an unloading phase for a general polymer, and the first pressure and strain waves during the transient event (figure 2).

It was initially thought that the viscoelastic behaviour of pipe walls during fast transients was dependent on the loading rate (pressure-increase rate) that is a function of the valve closure time. Several tests were run for 1 l/s flow for three fast manoeuvres with closing-times, $T_C$, of 0.09 s, 0.12 s and 0.15 s and with loading rates of 215 m/s, 160 m/s and 130 m/s, respectively. Slow manoeuvres ($T_C \geq 2L/a \approx 1.6$ s) were not performed as the valve was closed manually. Collected data are depicted in figure 3. The maximum pressure increase and the consequent pressure drop vary with the valve closure time; but the overall pressure response remains the same. The calibrated creep function must be either determined experimentally or calibrated with collected data.

Maximum overpressures calculated by the classic Joukowsky formula ($\Delta H = a_0Q_0/gS$) using a wave speed $a_0$ estimated based on the static pipe modulus of elasticity, are 10 to 30% lower than observed overpressures (Covas 2003, Covas et al. 2004a, 2004b). This is because the static modulus of elasticity of a HDPE pipe varies between 0.7 and 1.0 GPa and, for $Q_0 = 1.0$ l/s, the corresponding theoretical overpressures of 14.2 m and 16.7 m, and the observed overpressure is 19.4 m.

### 2.2 Thames Water quasi-field data

#### 2.2.1 Pipeline description.
The second set of tests were carried out in the world’s longest experimental PE pipeline with 1.3 km length, located at Kempton Park, Thames Water, London (figure 4). The system consists of two main pipes: (i) an inlet pipe starting at an upstream reservoir with two submersible pumps, 90 m long and made of medium-density PE (MDPE) with 70 mm ID; and (ii) the main pipe, 1.2 km long, buried underground, made of MDPE with 108 mm ID. The main pipe rises above ground at three points 500 m, 900 m and 1300 m (from the upstream end) and passes through a 25 m testing station. A gate valve was used to control the flow and one of the three butterfly valves (i.e. BV1, BV2 and BV3, located at 475, 875 and 1300 m from the upstream end) to generate the transient event. Flow was measured in an electromagnetic flow meter. The DAS was the same as used in the IC tests. Transient tests were run for several steady state flows ($Q_0 = 0.9, 2.0, 2.3, 2.5$ and $3.1$ l/s) corresponding to a smooth-wall turbulent regime. Tests were run with and without leaks at two locations L1 and L2.

#### 2.2.2 Data collection and analysis.
Experimental tests were carried out generating the transient event by the closure of one of the butterfly valves (BV1, BV2 and BV3), located at the pipe loops passing through the portakabin (6th, 4th and 2nd loop, respectively). Pressure data were collected at four different locations of the pipeline, at

![Figure 3. Piezometric head at T1 for three valve closures, $Q_0 = 1.0$ l/s (Imperial College).](image)
transducers T2, T8, T5 and T1, located at ca. 468.0 m, 473.5 m, 876.5 m and 1279.0 m from the upstream end, respectively. The sampling rate was 200 Hz. Transient tests were run for several steady state flows ($Q_0 = 0.9, 2.0, 2.3, 2.5$ and $3.1$ l/s) corresponding to the smooth-wall turbulent regime ($Re = 10 \, 500$ to $36 \, 500$ in the main pipe). Figure 5 depicts transient data collected at four pressure transducers. Several comments are in order:

(i) The pressure signal is significantly different from that obtained in the IC pipeline. This is likely because, at the TW upstream end, there is a check valve that closes when the flow inverts, generating a secondary transient wave, whereas, at the IC pipe, there is pressurized air.

(ii) The change in diameter from the main pipe of 108 mm to the inlet pipe of 70 mm causes a reflection in the pressure signal; this reflection corresponds to a pressure peak with an increase of 30% of the maximum overpressure generated at the valve section (figure 5).

(iii) When the transient pressure wave reaches the upstream end at $t \sim 20$ s, the flow inverts and the check valve closes quasi-instantaneously, inducing the
partial reflection of the incident wave; the flow, then, starts oscillating between the two closed valves.

(iv) Immediately after the BV1 closure, an overpressure is induced into the system. This overpressure has a sudden pressure drop (typical of PE pipes) and multiple reflections generated by small air pockets (Covas 2003, Covas et al. 2003b). These reflections are similar to background noise in the field, which is why these tests are run in quasi-field conditions.

3. Mathematical model

3.1 Linear-elastic model

The theoretical fundamentals of transient analysis are common to pressurized and open-channel unsteady-state flows. The flow movement, when temperature changes are negligible, is described by the mass-balance and the momentum-conservation principles. In general, (i) flow is considered to be one-dimensional and with a pseudo-uniform velocity profile in each cross-section; (ii) the fluid is assumed to be one-phase, homogeneous, compressible and with temperature and density changes negligible compared with pressure-flow fluctuations; (iii) the rheological behaviour of the pipe material is linear-elastic; (iv) the conduit is with pressure-flow fluctuations; (iii) the rheological behaviour of the pipe material is linear-elastic; (iv) the conduit is a straight uniform element without lateral in/outflows (unless modelled accordingly) and is completely constrained from any axial or lateral movement. Taking into account these assumptions, the classical water hammer equations that describe one-dimensional transient flow in pressurized pipes are:

\[
\frac{dH}{dt} + \frac{a_0}{gS} \frac{\partial Q}{\partial x} = 0 \tag{1}
\]

\[
\frac{\partial H}{\partial x} + \frac{1}{gS} \frac{dQ}{dt} + hf = 0 \tag{2}
\]

where \( Q \) = flow rate (m/s); \( H \) = piezometric head (m); \( a_0 \) = elastic wave speed (m/s); \( g \) = gravitational constant (m/s); \( S \) = pipe cross-section (m); \( x \) = coordinate along the pipeline axis (m); \( t \) = time (s); \( hf \) = head loss per unit length (m/m).

Elastic wave speed, \( a_0 \), represents the compressibility of the fluid and the distensibility of the conduit. This parameter can be estimated by the following theoretical formula that assumes a linear elastic behaviour of pipe walls (Chaudhry 1987, Wylie and Streeter 1993):

\[
a_0 = \sqrt{\frac{K/\rho}{1 + \frac{n^2 \frac{\zeta}{\zeta_0}}{\varepsilon}}} \tag{3}
\]

in which \( E_0 \) = Young’s modulus of elasticity of pipe walls; \( K \) = bulk modulus of elasticity of the fluid; \( \rho \) = fluid density; \( D \) = internal pipe diameter; \( e \) = pipe wall thickness; \( \alpha \) = dimensionless parameter that depends on the cross-section dimensions and pipe axial constraints. For a thick walled pipe \((D/e < 25)\) anchored along its length, \( \alpha \) is given by (Wylie and Streeter 1993):

\[
\alpha = \frac{2e}{D} (1 + \nu) + \frac{D}{D + e} (1 - \nu^2) \tag{4}
\]

where \( \nu \) = Poisson’s ratio (ratio between axial and circumferential strain). Equation (3) presupposes a linear-elastic behaviour of the pipe wall expressed by Hooke’s law, \( \sigma = E_0 \epsilon \), and negligible fluid-density changes given by fluid bulk modulus, \( K = d\rho/(d\rho/\rho) \). This equation is generally applied to estimate elastic wave speed when there is no transient data available. Otherwise, more accurate techniques (e.g. inverse transient analysis) should be used to estimate this parameter.

3.2 Unsteady-friction model

For slow transients, the slope of the energy line \( hf \) can be estimated by steady-state formulations (Chaudhry 1987); however, rapid transient events require a more accurate representation of friction losses. The friction term \( hf \) is decomposed into two terms: a steady-state component \( hf_s \) and an unsteady component \( hf_u \), as \( hf = hf_s + hf_u \). The steady-state term is calculated by typical steady-state formulae. The unsteady component is usually neglected in the classic analysis. However, rapid transient events require a more appropriate representation of this effect.

Unsteady friction losses are a result of frictional and inertial forces in the fluid due to the inversion of the velocity profile during fast transient events or high-oscillating frequencies. These losses have been widely studied for the last 50 years and are usually described by an additional term that is added to the momentum equation. While unsteady friction can be reasonably well described for laminar flow (Zielke 1968, Trikha 1975), no universally accepted formula has been developed yet for turbulent conditions. Several formulations for unsteady friction calculation have been presented in the literature, assuming that these losses are dependent on: (i) instantaneous mean velocity (Hino et al. 1977); (ii) instantaneous acceleration (Daily et al. 1956, Carstens and Roller 1959); (iii) weights of past time local accelerations (Zielke 1968, Trikha 1975, Vardy et al. 1993, Vardy and Brown 1996); (iv) local and convective acceleration (Brunone et al. 1991, 1995, Vitkovsky et al. 2000); and (v) velocity profiles (Bratland 1986, Vardy and Hwang 1991).

Concerning the steady-state friction \( hf_s \), an implicit second-order accuracy scheme was used to integrate it for turbulent and laminar flow. For the unsteady friction term \( hf_u \), several unsteady friction formulations were used to
compute this component (Covas 2003, Covas et al. 2004c); (i) Zielke’s (1968), (ii) Trikha’s (1975), (iii) Vardy et al.’s (1993) and (iv) Vitkovsky et al.’s (2000) formulations.

3.3 Linear-viscoelastic model

3.3.1 Constitutive model. Polyethylene pipes have different rheological behaviour compared to elastic pipes. When subjected to a certain instantaneous stress, $\sigma_0$, polymers do not respond according to Hooke’s law: plastics have an immediate elastic response and a retarded viscous response. In this way strain can be decomposed into an instantaneous elastic strain, $e_e$, and a retarded strain, $e_r$ (figure 6a):

$$e(t) = e_e + e_r(t)$$

(5)

According to the Boltzmann superposition principle, for small strains, a combination of stresses that act independently in a system result in strains that can be added linearly (figure 6b). The total strain generated by a continuous application of a stress, $s(t)$, is (Aklonis et al. 1972):

$$e(t) = J_0 \sigma(t) + \int_0^t \sigma(t - \tau) \frac{\partial J(t')}{\partial \tau'} d\tau'$$

(6)

in which $J_0 = \text{instantaneous creep-compliance}$ and $J(t') = \text{creep function at time } t'$. For linear-elastic materials, the creep compliance $J_0$ is equal to the inverse modulus of elasticity, $J_0 = 1/E_0$.

Assuming that the pipe material (i) is homogeneous and isotropic, (ii) has linear viscoelastic behaviour for small strains, and (iii) has a constant Poisson’s ratio $\nu$ so that the mechanical behaviour is only dependent on creep compliance, the circumferential strain $e = (D - D_0)/D_0$ is given by:

$$e(t) = \frac{\sigma_0 D_0}{2e_0^2} [p(t) - p_0]J_0 + \frac{2\nu(t - t')D(t - t')}{2\sigma(t - t')} \int_0^t [p(t - t') - p_0] \frac{\partial J(t')}{\partial \tau'} d\tau'$$

(7)

in which $p(t) = \text{pressure at time } t$; $p_0 = \text{initial steady-state pressure}$; $J_0 = \text{instantaneous creep compliance}$; $J(t) = \text{creep compliance function defined by } J(t) = e(t)/\sigma$ for a constant circumferential stress $\sigma$, $\sigma = 2pD/2e$; $D(t)$ and $D_0 = \text{inner diameter at time } t$ and $t = 0$, respectively; $e(t)$ and $e_0 = \text{wall thickness at time } t$ and $t = 0$, respectively; $\sigma(t)$ and $\sigma_0 = \text{pipe wall constraints coefficient at time } t$ and $t = 0$, respectively. The first term of this equation corresponds to the elastic strain $e_e$ and the integral part to the retarded strain $e_r$.

In order to evaluate the strain and incorporate this formulation into transient-flow equations, it is necessary to obtain experimentally the creep compliance function $J(t)$ of the material and to represent this function by a mathematical expression that can be implemented numerically. The creep compliance function $J(t)$ can be determined by a simple creep test or by dynamic testing for sinusoidally varying stress. The mathematical representation of the creep function is presented in the following section.

3.3.2 Equivalent mechanical model. There are two broad classes of models that can be used to simulate viscoelastic behaviour, namely, mechanical models and molecular theories. The mechanical models comprise a combination of elements, springs and dashpots (figure 7), that represent the viscoelastic response of a real system. The molecular

![Figure 6](image-url)  
Figure 6. (a) Stress and strain for an instantaneous constant load (left); (b) Boltzmann superposition principle for two stresses applied sequentially (right).

![Figure 7](image-url)  
Figure 7. Several mechanical models.
approach is based on the representation of polymer molecules and their movement in the viscous medium. The current analysis uses mechanical models to describe pipe wall behaviour.

The simplest mechanical model is a Hookean spring that represents pure elastic behaviour without any inertial effects: when subjected to an instantaneous stress, \( \sigma \), the spring responds instantaneously with a strain \( \varepsilon \) that is related to the stress by Hooke’s law: \( \sigma = E \varepsilon \). The proportionality constant, \( E \), is the Young’s modulus of elasticity. In reality, no materials obey Hooke’s law, but some materials like steel and iron behave according to this law over a wide range of stresses and strains; however, none responds without ‘any inertial effects’ (Aklonis et al. 1972). Another basic mechanical model is the pure dashpot, which is a piston in a cylinder filled with a liquid with viscosity \( \mu \) (figure 7). This model represents a linear viscous behaviour, typical of fluids, given by Newton’s law: \( \sigma = \mu \partial \varepsilon / \partial t \), which integrated for a constant stress \( \sigma \) yields \( \varepsilon(t) = \sigma / \mu \).

Several mechanical models can be used that combine these two elements: the spring and the dashpot connected in series form the Maxwell model and, in parallel, the Voigt model (figure 7). However, these models describe the behaviour of simple systems and do not accurately approximate the viscoelastic behaviour of polymers. Thus, generalized models have been proposed to represent more closely the behaviour of these materials. Generalizations can be carried out by rearranging Maxwell elements in parallel or, alternatively, Kelvin–Voigt elements in series. Using one of these models depends on the system having a predominant behaviour similar to a viscoelastic fluid (Maxwell model) or to a viscoelastic solid (Voigt model).

The polyethylene pipe rig is described by a generalized Kelvin–Voigt model, which is particularly appropriate for viscoelastic solids (figure 7). A spring has been added to the model to account for the elastic-instantaneous response of the pipe material. Using this model, the creep compliance function can be approximated by the following expression:

\[
J(t) = J_0 + \sum_{k=1}^{N} J_k \left(1 - e^{-\frac{t}{\tau_k}}\right)
\]

(8)

in which \( J_0 \) = creep-compliance of the first spring defined by \( J_0 = 1/E_0 \); \( J_k \) = creep compliance of the spring of the Kelvin–Voigt element \( k \) defined by \( J_k = 1/E_k \); \( E_k \) = modulus of elasticity of the spring of \( k \)-element; \( \tau_k \) = retardation time of the dashpot of \( k \)-element, \( \tau_k = \mu_k / E_k \); \( \mu_k \) = the viscosity of the dashpots of \( k \)-element.

In general, there are an infinite number of mechanical models representing one system, either using the same elementary model or combining different models in series and in parallel. In the current generalized Kelvin–Voigt model used to describe the polyethylene behaviour, several Voigt elements in series were used. The number of these elements is based on the balance between solution accuracy and computational effort.

3.3.3 Water hammer equations in viscoelastic pipes. In order to take into account the viscoelastic behaviour of the pipe wall in plastic pipes, the continuity equation (1) has to be derived from the Reynolds transport theorem. Taking into account the relationship between cross-sectional area, \( S \), and total hoop strain, \( \varepsilon (dS/dt = 2S\varepsilon /dt) \), and the two components of strain [equation (5)], the continuity equation yields:

\[
\frac{dH}{dt} + \frac{a_0^2}{gS} \frac{\partial Q}{\partial x} + \frac{2a_0^2 d_0}{g} \frac{\partial \varepsilon}{\partial x} = 0
\]

(9)

The first two terms are similar to the classic continuity equation (1) and the third term represents the retarded effect of pipe wall. The elastic strain is included in the piezometric-head time derivative and in the elastic wave speed. The elastic wave speed \( a_0 \) is calculated by equation (3), in which the Young’s modulus of elasticity \( E_0 \) is replaced by the inverse of elastic creep function, \( E_0 = 1/J_0 \).

Equation (9), solved simultaneously with equation (2) and with the retarded strain given by the second term of equation (7), describes the pressure-flow fluctuations along a pressurized pipeline as well as the circumferential strain in the pipe wall during both fast and slow hydraulic transients.

For viscoelastic pipes, a new concept of “effective” wave speed, \( a \), can be defined in terms of the creep compliance function, \( J(t) \), as follows:

\[
a(t) \approx \sqrt{\frac{K}{\rho \left[1 + \frac{2D}{e K J(t)}\right]}}
\]

(10)

Although the wave speed, \( a \), defined by equation (10) is not used in the fluid equations, this is a time-dependent function that decreases during the transient as a result of the increase of the creep function. This accounts for the pressure wave dampening and dispersion during a transient event. Equation (10) is particularly useful for solving problems in the frequency domain.

3.3.4 Complete Characteristic Equations and Finite-Difference Schemes. Equations (10) and (2) consist of a set of differential equations that can be solved by several numerical methods. The Method of Characteristics (MOC) has been extensively used for this type of problem given its computational efficiency in handling complex hydraulic systems and manipulating different boundary
conditions. The stability of this method requires the verification of a numerical restriction for the time and space steps, given by the Courant–Friedrich–Lewy stability condition: \( \frac{dx}{dt} = u \pm a_0 \) which corresponds to the propagation of flow features along the characteristic lines with a numerical wave speed, \( \frac{dx}{dt} \), equal to the real wave speed, \( u \pm a_0 \). The set of all characteristic lines in time and space constitutes the computational grid represented in figure 8.

Courant–Friedrich–Lewy condition allows the transformation of the set of partial differential equations into a system of ordinary differential equations (Gally et al. 1979):

\[
C^-: \frac{dH}{dt} \pm \frac{a_0}{gS} \frac{dQ}{dt} + \frac{2a_0^2}{g} \left( \frac{d^2 \varepsilon}{dt^2} \pm a_0 \frac{\partial \varepsilon}{\partial t} \right) \pm a_0 h_f = 0
\]

valid along the characteristic lines \( dx/dt = V \pm a_0 \) in which the operator ‘total derivative’ is described by \( d/dt = \partial/\partial t + (u \pm a_0)\partial/\partial x \). Accordingly, equations (11) are solved numerically by the Method of Characteristics using the following numerical scheme:

\[
C^-: \left[ H_{ij} - H_{ij,j-1} \right] \pm \frac{a_0}{gS} \left[ Q_{ij} - Q_{ij,j-1} \right] \pm a_0 \Delta t_{i,j} h_f + \\
+ \frac{2a_0^2/g}{Q_{ij,j-1}} \pm a_0 \left( \frac{\partial \varepsilon}{\partial t} \right)_{i,j} \Delta t_{i,j} + 0 \quad \text{valid along } \Delta x/\Delta t = \pm a_0.
\]

The use of this characteristic grid has several disadvantages: (i) additional equations have to be added to equations (12) to be consistent with steady state flow conditions (Wylie and Streeter 1993); (ii) compatibility schemes are quite complex in multi-pipe systems; and (iii) the accuracy gained by including the convective terms is minimal.

### 3.3.5 Simplified characteristic equations and finite-difference schemes

In the current applications, the fluid velocity is negligible compared with the elastic wave speed, and characteristic equations \( C \) and \( C \) can be further simplified (Covas 2003, Covas et al. 2004c):

\[
C^-: \frac{dH}{dt} \pm \frac{a_0}{gS} \frac{dQ}{dt} + \frac{2a_0^2}{g} \left( \frac{\partial \varepsilon}{\partial t} \right) \pm a_0 h_f = 0
\]

which is valid along straight characteristic lines, \( dx/dt = \pm a_0 \). Using a rectangular computational grid (figure 9), the following finite difference scheme has been used (Covas 2003, Covas et al. 2004c):

\[
C^-: \left( H_{ij} - H_{ij,j-1} \right) \pm \frac{a_0}{gS} \left( Q_{ij} - Q_{ij,j-1} \right) + \\
+ \frac{2a_0^2\Delta t}{g} \left( \frac{\partial \varepsilon}{\partial t} \right)_{i,j} \pm a_0 \Delta t h_f = 0
\]

valid along \( \Delta x/\Delta t = \pm a_0 \).

#### 3.3.6 Retarded strain

The retarded strain time-derivative in equation (14) is calculated by deriving the second term of equation (7) considering the creep-function defined by the mechanical model of the generalized viscoelastic solid (equation (8)) as follows:

\[
\varepsilon_r(x, t) = \sum_{k=1}^{N} \hat{\varepsilon}_k(x, t) = \\
\sum_{k=1}^{N} \frac{\hat{J}_k}{2e} \int_{0}^{t} \left[ H(x, t - t') - H_0(x) \right] \frac{J_k}{\tau_k} \varepsilon_r(x, t') \, dt'
\]

\[
\frac{\partial \varepsilon_r(x, t)}{\partial t} = \sum_{k=1}^{N} \frac{\partial \hat{\varepsilon}_k(x, t)}{\partial t} = \\
\sum_{k=1}^{N} \frac{\hat{J}_k}{2e} \frac{\gamma}{\tau_k} \varepsilon_r(x, t) - e \hat{\varepsilon}_k(x, t) \frac{\gamma}{\tau_k}
\]

where \( \gamma = \rho g \). Considering the creep function defined by equation (8) and introducing the time-derivative of this function \( J(t) \) into equation (7), yields the following relation for each Kelvin–Voigt element \( k \):
The strain time-derivative derivative is calculated by the analytical differentiation of equation (17). After mathematical manipulations, for each element \( k \), it yields the following numerical approximations (Covas et al. 2004):

\[
\varepsilon_{rk}(x,t) = \int_0^t F(x,t-t') \frac{J_k}{\tau_k} e^{-\frac{t'}{\tau_k}} dt'
\]

(17)

with

\[
F(x,t) = \frac{gD}{2e} \gamma [H(x,t) - H_0(x)]
\]

(18)

The strain time-derivative derivative is calculated by the analytical differentiation of equation (17). After mathematical manipulations, for each element \( k \), it yields the following numerical approximations (Covas et al. 2004):

\[
\frac{\partial \varepsilon_{rk}(x,t)}{\partial t} = \frac{J_k}{\tau_k} F(x,t) - \frac{\varepsilon_{rk}(x,t)}{\tau_k}
\]

(19)

\[
\varepsilon_{rk}(x,t) = J_k F(x,t) - J_k e^{-\frac{t}{\tau_k}} F(x,t-\Delta t) - J_k \tau_k (1 - e^{-\frac{\tau_k}{\Delta t}})
\]

(20)

\[
F(x,t) = F(x,t-\Delta t) + e^{-\frac{t}{\tau_k}} (x,t-\Delta t)
\]

Parameters \( J_k \) and \( \tau_k \) of the viscoelastic mechanical model should be adjusted to the creep-compliance experimental data. Pipe diameter \( D \), the wall-thickness \( e \) and the pipe wall constraint coefficient \( z \), which are typically time-dependent parameters, were assumed constant and equal to steady-state values.

4. Imperial College case study

A major challenge in the analysis is the distinction between the dynamic effect of unsteady friction and pipe wall viscoelasticity, as both dissipate and disperse the transient pressure wave and both have parameters to calibrate. Several solutions for this problem have been analyzed in (Covas 2003, Covas et al. 2004b) that are briefly presented herein: (ii) the model is applied with only one of the dynamic effects; or (ii) the creep function is calibrated for laminar flow and then verified for turbulent flow.

4.1 Linear-elastic model with unsteady friction

Initially, it was assumed that the pressure damping was only generated by unsteady skin friction and fluid inertial effects (pipe wall viscoelasticity was neglected). This is the normal procedure when using linear-elastic solvers with unsteady friction. Three formulations for unsteady friction calculation were used: Trikha’s (1975), Vardy et al.’s (1993) and Vitkovsky et al.’s (2000).

The transient event was simulated for smooth-wall turbulent flows (\( Q_0 = 1.008 \text{ l/s}; Re = 25\,500; f.Re = 648 \)) using Trikha’s (1975), Vardy et al.’s (1993) and Vitkovsky et al.’s (2000) formulations. Numerical results obtained using these three formulations, the classic water hammer solution, collected data and respective LSE are presented in figure 10. The classic water hammer solution has a major discrepancy with collected data (LSE = 100.43 m) that significantly increases in time. The inclusion of unsteady friction improves the solution slightly, though neither Trikha’s (1975) nor Vardy et al.’s (1993) formulations could describe the observed transient pressure damping in this smooth-wall turbulent flow and have high LSE (i.e. 45.78 and 83.23 m). With regard to Vitkovsky et al.’s (2000), an extremely high decay coefficient was calibrated to fit the numerical results with the observed pressures (i.e. \( k' = 0.14 \)). Although the maximum and minimum pressures obtained in numerical simulations considering unsteady friction fairly agree with the collected data, the shape of the pressure wave is not well represented. The unsteady friction could not simulate the attenuation and dispersion of the pressure wave during the transient.

4.2 Linear-viscoelastic model neglecting unsteady friction

4.2.1 Creep function calibration. The estimation of the creep-compliance \( J(t) \) function associated with a particular
rate of loading is important for the numerical simulation of the viscoelastic behaviour of polymers. This time-dependent function depends on the molecular structure of the material, on the temperature in which the test is run and on the load-rate, having to be determined experimentally for each particular case. The creep function \( J(t) \) can be estimated by a simple creep test measuring the time strain increase for a constant-instantaneous stress \( \sigma \), \( J(t) = \varepsilon(t)/\sigma \). Alternatively, this function can be obtained from dynamic testing for a sinusoidally varying stress in which the transfer function of the material would be determined for a continuous range of loading frequencies (measuring stress and strain amplitude and phase shift, for the range of frequencies, and relating these with \( J(t) \)). In this work, the creep compliance \( J(t) \) was estimated by fitting numerical results of simulations to transient data.

The calibrated creep \( J(t) \) was represented by the generalized Kelvin–Voigt model, in which the number of Voigt elements depended on the sample length, \( \Delta T \). If only 5 s of the transient data were used for the calibration, a three-element Kelvin–Voigt model was sufficient to represent it; on the other hand, for a 20 s sample, a six-element Kelvin–Voigt model was necessary in order to achieve a reasonable level of accuracy. Thus, a six-element model was used for all numerical simulations.

The parameters \( J_k \) and \( \tau_k \) of the model were calibrated using an optimization algorithm that minimises the difference between the measured and calculated piezometric-head at location 1 (transducer T1). Genetic algorithms (Goldberg 1989) were used to carry out the search. Parameters \( \tau_k \) and \( J_0 \) were fixed before the optimization was carried out, thus the only optimized parameters were \( J_k \). The best-fitted parameters \( J_k \) for several sample sizes (i.e. \( \Delta T = 1.6, 2, 5 \) and \( 10 \) s) and for a steady-state flow \( Q_0 \) of 0.75 l/s are listed in table 1. These combinations of parameters are possible mechanical representations of the pipe material creep function. These functions agree well, except the one associated with the smallest sample size (i.e. \( \Delta T = 1.6 \) s), which is slightly higher than the others (figure 11).

Results of creep calibration for \( \Delta T = 5 \) s are presented in figures 12 and 13. Figure 12 shows the effect on the creep function because of increasing the number of Kelvin–Voigt elements. The more elements are added to the mechanical model, the more the creep function increases. The effect on pressure response of increasing the number of Kelvin–Voigt elements in the mechanical model is shown in figure 13.

![Figure 11. Creep compliance functions (\( N = 5 \)) for polyethylene at 20°C. Calibration results using genetic algorithms for several sample sizes.](image)

![Figure 12. Creep functions for several \( k \)-elements Kelvin–Voigt model (with \( k = N + 1 \) and \( N = 1\ldots5 \)).](image)

<table>
<thead>
<tr>
<th>( \Delta T ) (s)</th>
<th>( J_0 ) (10 Pa)</th>
<th>Creep coefficients ( J_k ) (10 Pa) for the retardation times indicated below:</th>
<th>Least square error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau_1 = 0.05 ) s</td>
<td>( \tau_2 = 0.5 ) s</td>
<td>( \tau_3 = 1.5 ) s</td>
</tr>
<tr>
<td>1.6</td>
<td>0.740</td>
<td>0.047</td>
<td>0.061</td>
</tr>
<tr>
<td>2</td>
<td>0.740</td>
<td>0.060</td>
<td>0.094</td>
</tr>
<tr>
<td>5</td>
<td>0.740</td>
<td>0.065</td>
<td>0.100</td>
</tr>
<tr>
<td>10</td>
<td>0.740</td>
<td>0.063</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Fixed parameters before calibration. Corresponding wave speed \( a_0 \) of 380 m/s.
13. The more elements added to the model, the higher the attenuation and the dispersion of the pressure wave. This is characteristic of the viscoelastic behaviour of the pipe wall to dissipate rapidly flow energy and to create delay in the pressure wave.

The creep function for six Kelvin–Voigt elements \( N = 5 \) and the respective “effective” wave-speed calculated by equation (10) are presented in figure 14. The creep-function time-increase is accompanied by the wave-speed time-decay from an instantaneous value of 380 m/s (for \( J = 0.74 \times 10 \) Pa) until a limiting value that corresponds to the static wave speed of 300 m/s (for \( J = 1.21 \times 10 \) Pa). This pressure wave-speed time-dependency during the transient event has been observed in steady-oscillatory flow (Meissner and Franke 1977). It is for this reason that the wave speed in viscoelastic pipes can be described as a frequency-dependent function \( \omega = 2\pi/t \) and included in the fluid equations in the frequency domain.

4.2.2 Transient pressures. In figures 15, 16 and 17, numerical results obtained for a flow of 1.0 l/s are compared with the collected piezometric-head time variation at locations 5, 8 and 1 (transducers T5, T8 and T1, respectively). Two solutions are presented: the first, represented by a dashed-line, was calculated using the classical water hammer equations in which pipe walls are assumed to be elastic-linear and unsteady-friction effects are neglected; and the second, represented by a continuous-
Figure 15. Piezometric head at location 5: data vs. numerical results ($Q_0 = 1.0 \text{ l/s}; T = 20^\circ\text{C}$).

Figure 16. Piezometric head at location 8: data vs. numerical results ($Q_0 = 1.0 \text{ l/s}; T = 20^\circ\text{C}$).

Figure 17. Piezometric head at location 1: data vs. numerical results ($Q_0 = 1.0 \text{ l/s}; T = 20^\circ\text{C}$).
thin-line, was calculated with the six-element Kelvin–Voigt model incorporated into the hydraulic transient model. The analysis of these figures allows the following conclusions to be drawn.

First of all, the classic water hammer solution (‘elastic model’) results show large discrepancies in both the pressure amplitude and phase with experimental data. These discrepancies increase substantially with time and distance from the valve. The reason is that the pressure wave calculated by the elastic model is not sufficiently damped nor dispersed. Even when an unsteady-friction formulation (Vitkovsky et al. 2000) is implemented, the numerical results cannot fit with the observed data.

Secondly, the viscoelastic water hammer solution fits perfectly with the collected data in any of the three locations of the pipe. The creep function used in this viscoelastic model was the one obtained in the calibration of ‘$\Delta t = 5$ s’ sample (table 1). This function shows a good agreement for the first 20 s of the transient although it was adjusted only for 5 s of data. A detail of the first peak of the pressure wave is presented in a larger scale in figures 15, 16 and 17 in order to show that the agreement of the model with the experimental data is very good at this ‘zoomed’ scale.

These results demonstrate that the classical elastic solution cannot accurately simulate pressure transients in polyethylene pipes, particularly for fast-transients events. The elastic wave-speed in polyethylene pipes is significantly underestimated if it is calculated using the static modulus of elasticity (in this case, $E = 0.7$ to 1.0 GPa) rather than the dynamic modulus of elasticity (in this case, $E = 1.35$ GPa). This is because, instead of a wave speed of 380 m/s, the value of 300 m/s is used, and, consequently, the estimated maximum pressure increase (given by Joukovsky formula, $\Delta H = a_0 V/g$) is one-fourth of that observed.

Several tests have been carried out by manufacturers in polyethylene pipes, and it has been observed that this material resists up to three-times the nominal design pressure for fast-loading-rates (Marshall et al. 1998). This raises the question of whether a viscoelastic model or an elastic model should be used. The answer, certainly, depends upon the level of accuracy required for the analysis. For design, the classical elastic models provide a conservative estimate of the maximum transient pressure. This means that polyethylene pipes can sustain pressures higher than design values. In cases where hydraulic transients are used for leak detection and calibration, an accurate numerical model is essential. Thus it is crucial in these cases for the viscoelastic behaviour of the pipe wall to be included into the hydraulic simulation.

4.2.3 Transient circumferential strains. While the circumferential strain is measured at the outside wall of the pipe, collected pressure refers to the pressure in the fluid (i.e. at the inner pipe wall). The pipe has a thick wall with a standard diameter ratio, SDR = 11. For thick wall pipes, stress and strain distributions cannot be considered uniform throughout the wall. Solutions for the displacement and stress fields across the wall are described, in the theory of elasticity, by the Lamé solutions (Eringen 1967). In the current case, the inner pipe diameter is 0.0506 m, the outer diameter is 0.063 m, and Poisson’s ratio $\nu$ is 0.46 (given by the manufacturer) and a linear assumption for the circumferential strain distribution is acceptable (Covas et al. 2004b). The ratio outside/inside strain ratio in the pipe wall is 66% (Covas et al. 2004b). Gally et al. (1979) neglected this ratio and thought that observed differences between measured and calculated strains were due to physical pipe constraints.

![Figure 18. Circumferential strain at location 5: data vs. numerical results ($Q_0 = 1.0$ l/s).](image-url)
In figures 18, 19 and 20, the circumferential strain time variation obtained by numerical analysis is compared with measurements at the same three locations (i.e. SG5, SG8 and SG1). Both the retarded and the total strains calculated by the numerical model are presented in these figures. The retarded strain is calculated and presented separately. The retarded strain is one-fourth of the total strain, contributing significantly to the mechanical behaviour of the pipe wall.

Similar to the piezometric head variations, calculated strains agree well with the measured strains. However, at locations 5 and 1, the total numerical strain is slightly higher than that measured and, at location 8, the inverse phenomenon is observed. This can be due to: (i) a non-uniform pipe wall thickness and inner diameter, or non-perfectly circular cross section; (ii) different axial constraints at the three locations, although the pipe is similarly fixed and completely constrained; (iii) slight bending in the pipe wall; (iv) non-uniform stress distribution in pipe wall. In general, there is a good agreement both in the amplitude and phase of strain variation at the three locations along the pipe.

This good agreement between numerical results and observed data was obtained by fitting the creep function \( J(t) \) to the collected-pressure data. If this function had been determined experimentally by a mechanical test, a higher dispersion would be expected not only due to the error associated with the test, but also due to unsteady friction effects not accounted for in the material test. The unsteady friction phenomenon has a similar effect on pressure variations as the non-elastic behaviour of pipe walls. Thus, it is remarkably difficult to distinguish the viscoelastic effect
of the pipe wall from the unsteady-friction effect. The experimental measurement of the creep function in a creep or dynamic test would allow these two effects to be separated.

Another issue, that could be raised when implementing this viscoelastic model, is the time-uncertainty associated with the creep-function. Although this function can be determined experimentally for each material and temperature, the mechanical characteristics of polymers depend on long-term properties. Indeed, the creep compliance depends on the stress time-history of the pipe, namely on the loading frequency and amplitude (material fatigue), on the temperature and on the average loading amplitude and rate. Furthermore, the creep compliance of a pipe wall is dependent on other external factors, such the axial and circumferential constraints of the pipe. Certainly, a buried pipe does not respond the same way as an unburied pipe. For these reasons, the creep compliance is a function with several sources of uncertainty that has to be calibrated for each situation using collected data (valid only for a certain period of time) and not just by relying on a creep or a dynamic test of a pipe sample.

4.3 Complete model considering viscoelasticity and unsteady friction

The calibration followed herein is a two-step procedure. The first step consists of the creep function calibration for laminar conditions using Trikha’s (1975) formula to describe unsteady friction. The second is the model verification for turbulent conditions by using Trikha’s (1975) and Vardy et al.’s (1993) formulae and by calibrating Vitkovsky et al.’s (2000) formula.

4.3.1 Calibration for laminar flow. The creep function was calibrated for laminar conditions \(Q_0 = 0.056 \text{ l/s}; Re = 1400\) considering unsteady friction effects (see creep coefficients in table 2). Unsteady friction was simulated by Trikha’s formula. Numerical results obtained for \(\Delta T = 10 \text{ s}\) calibration are compared with collected data. Three numerical solutions and the respective LSE are presented in figure 21: (i) the classic water hammer solution; (ii) results of the implementation of Trikha’s formula only; and (iii) results of the combination of pipe-viscoelasticity and unsteady friction. The latter results show excellent agreement with experimental data (i.e. LSE = 0.0004 m), unlike the results of (i) and (ii) (i.e. LSE = 1.03 and 0.46 m, respectively).

4.3.2 Verification for smooth-wall turbulent flow. Assuming a good approximation of the creep-function was obtained with the calibration for laminar flow and \(\Delta T = 10 \text{ s}\), this function was verified for smooth-wall turbulent conditions \(Q_0 = 1.008 \text{ l/s}; Re = 25 000; f.Re = 648\) using Trikha’s (1975) and Vardy et al.’s (1993) formulae and for the calibration of Vitkovsky et al.’s (2000) formula.

The inverse solver was used for the calibration of Vitkovsky et al.’s decay coefficient \(k’\). Calibrated decay

![Figure 21. Piezometric head at location 1 for laminar conditions, \(Q_0 = 0.054 \text{ l/s}\). Collected data vs. numerical results.](image-url)
coefficients $k'$ (i.e. $k' = 0.028, 0.030$ and $0.033$, respectively) are within the expected range of values for single-phase flows (Brunone et al. 1995, Bughazem and Anderson 1996, 2000).

Numerical results for seven cases are presented in figure 22: (i) the classic water hammer solution; (ii) the results obtained using Trikha’s formula; (iii) the results obtained using Vardy et al.’s formula; (iv) the results obtained using Vitkovsky et al.’s formula with $k' = 0.03$; (v) the results obtained combining Trikha’s formula with viscoelasticity; (vi) the results obtained combining Vardy et al.’s formula with viscoelasticity; and (vii) Vitkovsky et al.’s formula ($k' = 0.03$) with viscoelasticity.

Firstly, unsteady friction cannot \textit{per se} generate the total damping, phase-shift and curve-shape observed in transient pressure fluctuations, as can be seen in the solutions without viscoelasticity obtained using Trikha’s, Vardy et al.’s and Vitkovsky et al.’s (i.e. LSE = 80.42, 128.25 and 93.54 m, respectively). Secondly, the results of Trikha’s and Vitkovsky’s (with $k' = 0.03$) unsteady friction formulations agree fairly well with each other both in the elastic and viscoelastic solutions. Finally, results obtained combining unsteady friction models and viscoelasticity fit well with observed data (with LSE $\leq 0.57$ m), though with a slightly higher damping than the actual observed data. This means that the unsteady friction contribution is reasonably well quantified by any of these formulations in both laminar and smooth-wall turbulent flows as long as the decay coefficient of Vitkovsky’s formula is adequately calibrated.

Table 2. Best fitted creep coefficients $j_k$ for a four-element Kelvin–Voigt model. Calibration for laminar conditions considering Trikha’s unsteady friction formulation.

<table>
<thead>
<tr>
<th>$\Delta T$ (s)</th>
<th>$\tau_0$ (10 Pa)</th>
<th>Creep coefficients $j_k$ (10 Pa) for the retardation times indicated below:</th>
<th>Least square error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.70</td>
<td>$\tau_1 = 0.05$ s</td>
<td>0.0804</td>
</tr>
<tr>
<td>10</td>
<td>0.70</td>
<td>$\tau_2 = 0.5$ s</td>
<td>0.0805</td>
</tr>
<tr>
<td>20</td>
<td>0.70</td>
<td>$\tau_5 = 10$ s</td>
<td>0.0801</td>
</tr>
</tbody>
</table>

Fixed parameters before calibration. Corresponding wave speed $a_0$ of 395 m/s.

Figure 22. Piezometric head at location 1 for \textit{smooth-wall turbulent conditions}, $Q_0 = 1.008$ l/s. Collected data vs. numerical results.
5. Thames Water case study

The developed hydraulic linear-viscoelastic solver is calibrated and validated using transient data collected at the Thames Water (TW) pipeline. For this case, the linear-viscoelastic model is used including unsteady friction. Vardy et al.’s (1993) formulation is used to describe fluid frictional and inertial effects. The upstream boundary condition is described by the pressure data collected at transducer T8 (i.e. the one located 468.0 m from the upstream end). The creep function is calibrated for a transient test with $Q_0 = 3.1$ l/s, and the model is verified for another transient test for a different flow rate, $Q_0 = 2.0$ l/s.

5.1 Model calibration for $Q_0 = 3.1$ l/s

The creep function was calibrated using the transient solver and data collected at transducer T1 during $\Delta T = 50$ s. Considering $u_0 = 350$ m/s and $\tau_c(s) = (0.05; 0.5; 5)$, best fitted creep coefficients were $J_k (10 \text{ Pa}) = (0.090; 0.076; 0.335)$. Numerical results obtained at transducers T1, T5 and T8 for $Q_0 = 3.1$ l/s are presented in figure 23a. The transient event is well described with $\text{LSE} = 0.097$ m. Nevertheless, numerical results at T1 and T5 have several ‘discrepancies’ (see dashed circles in figure 23a). These discrepancies are mainly due to a slight time lag between the arrival of the ‘travelling wave front’ generated at the
downstream end and measurements at transducer T8 used as boundary conditions in the simulation. This induces artificial reflections of the pressure wave that propagates downstream (Covas 2003).

5.2 Model verification for \( Q_0 = 2.0 \) l/s

A transient event was simulated in TW pipeline, for \( Q_0 = 2.0 \) l/s, using the calibrated creep function from the \( Q_0 = 3.1 \) l/s test. The transient was generated by the downstream valve BV1 closure (at \( t = 1.0 \) s) and opening (at \( t = 40 \) s). Obtained numerical results and collected data at transducers T1, T5 and T8 are presented in figure 23b. Numerical results agree well with collected data. Figure 23b also presents numerical results obtained after calibrating the creep function for this flow (\( Q_0 = 2.0 \) l/s). Differences between calculated pressures using either of the two calibrated creep functions are negligible and both agree fairly well with collected data.

6. Conclusions

Classic elastic models are usually used to describe pipe material and protection devices in pipeline systems design. These models provide a conservative estimate of a maximum transient pressure in viscoelastic pipes such as polyethylene pipes. Thus, viscoelastic pipes such as polyethylene can sustain pressures higher than design values. However, in cases where hydraulic transients are used for calibration or leak detection, a more accurate numerical model is essential and the viscoelastic behaviour of the pipe wall should be accounted for in the simulations.

This paper presents a mathematical model for the calculation of water hammer in polyethylene pipes. The model was validated with experimental data collected from the IC pipeline laboratory rig and under quasi-field conditions at TW utilities. The incorporation of the viscoelastic behaviour of pipe walls in the transient-flow equations is achieved by adding a term associated with a retarded strain in the pipe wall to the classical water hammer equations. The strain–stress relationship was determined by a creep function. A generalized Kelvin–Voigt model with six elements was calibrated using a genetic algorithm approach.

The numerical results obtained by the elastic and the viscoelastic models were compared with collected data. The pressure fluctuation obtained with the viscoelastic model showed excellent agreement with the experimental data both in terms of phase and amplitude, whereas the classic elastic solution showed large discrepancies. The circumferential strain was also monitored for the same tests and compared with the results of the viscoelastic model. The results fitted well with the data, again demonstrating the accuracy of the model.

Although there is good agreement between the data and the numerical results, it should be noted that this was obtained by fitting the creep function \( J(t) \) to the experimental data considering frequency-dependent head losses (unsteady friction). The latter has a dissipative and dispersive effect on the pressure wave, similar to the viscoelastic behaviour of pipe walls. The next step in this work is to distinguish and, if possible, quantify the influence of each of these two phenomena—unsteady friction and pipe wall viscoelasticity—during pressure transients in polyethylene pipes, and eventually, extend this conclusion to other polymer pipe materials.

Acknowledgements

The results presented here were achieved through a joint research project between the University of Exeter and Imperial College London supported by the UK Engineering and Physical Sciences Research Council (Inverse Transient Analysis for Pipe Roughness Calibration and Leak Detection). Additionally, Didia Covas gratefully acknowledges the financial support of Fundação da Ciência e Tecnologia (FCT, Portugal) and Instituto Superior Técnico (IST, Portugal).

References

Covas, D., Inverse transient analysis for leak detection and calibration of water pipe systems—modeling special dynamic effects, PhD thesis, Imperial College of Science, Technology and Medicine, University of London, 2003.
### Appendix

#### Notation

The following symbols are used in this paper: 

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>effective wave speed ((\text{m/s}))</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>elastic wave speed ((\text{m/s}))</td>
</tr>
<tr>
<td>( D )</td>
<td>pipe inner diameter ((\text{m}))</td>
</tr>
<tr>
<td>( e )</td>
<td>pipe wall thickness ((\text{m}))</td>
</tr>
<tr>
<td>( E_d )</td>
<td>dynamic modulus of elasticity ((\text{Pa}))</td>
</tr>
<tr>
<td>( E_k )</td>
<td>dynamic modulus of the springs ((\text{Pa}))</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Darcy-Weisbach steady state friction factor ((-))</td>
</tr>
<tr>
<td>( g )</td>
<td>gravity acceleration ((\text{m/s}^2))</td>
</tr>
<tr>
<td>( H )</td>
<td>piezometric head, ( H = p/\gamma + z ) ((\text{m}))</td>
</tr>
<tr>
<td>( h' )</td>
<td>head loss per unit length ((-))</td>
</tr>
<tr>
<td>( H_o )</td>
<td>steady state piezometric head ((\text{m}))</td>
</tr>
<tr>
<td>( J )</td>
<td>creep-compliance ((\text{Pa}^{-1}))</td>
</tr>
<tr>
<td>( J_0 )</td>
<td>instantaneous or elastic creep-compliance ((\text{Pa}^{-1}))</td>
</tr>
<tr>
<td>( J_k )</td>
<td>creep of the springs of the Kelvin–Voigt elements, ( J_k = 1/E_k ) ((\text{Pa}^{-1}))</td>
</tr>
<tr>
<td>( K )</td>
<td>bulk modulus of elasticity of the fluid ((\text{Pa}))</td>
</tr>
<tr>
<td>( L )</td>
<td>length of the pipeline ((\text{m}))</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure of the fluid ((\text{Pa}))</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>initial steady-state pressure ((\text{Pa}))</td>
</tr>
<tr>
<td>( Q )</td>
<td>flow rate ((\text{m}^3/\text{s}))</td>
</tr>
</tbody>
</table>

---


Q₀ = initial steady-state flow rate \( (m^3/s) \)
Re = Reynolds number, \( Re = \frac{VD}{\nu'} \) \((-)\)
S = pipe cross section \( (m^2) \)
T = temperature of the fluid \( (^\circ C) \)
t, t', t'' = time \( (s) \)
u = average velocity of the fluid \( (m/s) \)
x = coordinate along the pipe axis \( (m) \)
ρ = fluid density \( (kg/m^3) \)
α = dimensionless parameter (function of pipe cross-section dimensions and constraints) \((-)\)
ν = Poisson’s ratio (ratio between axial and circumferential strain) \((-)\)
ν' = kinematic fluid viscosity \( (m^2/s) \)

ε = strain; circumferential total strain \( (m/m) \)
ε₀ = initial strain \( (m/m) \)
i₀ = initial strain \( (m/m) \)
i = instantaneous elastic strain \( (m/m) \)
eᵣ = retarded strain \( (m/m) \)
e = strain; circumferential total strain \( (m/m) \)

σ = stress; circumferential stress \( (Pa) \)
σ₀ = initial stress \( (Pa) \)

τₖ = retardation time of the dashpots, \( τₖ = \frac{\etaₖ}{Eₖ} \) \( (s) \)

ηₖ = the viscosity of the dashpots \( (kg/sm) \)

Δt = time-step increment \( (s) \)

ΔT = sample length used for calibration \( (s) \)

Δx = space-step increment \( (m) \)