Measurement of hydraulic transients in a metal pipe rig: effects of free-air, cavitation and pipe axial-deformation

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ABSTRACT

The current paper aims at the analysis and discussion of the uncertainties associated with the numerical model and the experimental hydraulic transients tests carried out in a metal pipe rig, installed in the Hydraulic Laboratory of the Civil Engineering Department of Instituto Superior Técnico (Lisbon, Portugal), for the analysis of transient events both for academic (i.e., lecturing under/postgraduate courses) and research purposes. The pipe system is made of coiled copper with 100 m length and 20 mm of inner diameter. An experimental programme was carried out for the collection of transient pressures at three locations (at the upstream end, at a middle section and at the downstream end) and of steady state flow rates during a set of transients. Collected transient data were primarily analysed to determine basic parameters (i.e., steady state head losses, valve closure time and wave speed) and to check the accuracy of measurements. Afterwards, other mathematical modelling parameters (i.e., valve opening, decay coefficients of unsteady friction models) were calibrated. Finally, data were compared with numerical results carried out by an in-house developed hydraulic transient solver. Discrepancies between data and numerical results were observed, associated with the presence of free-air in the fluid, pipe axial deformation and transient cavitation. Conclusions are drawn concerning the calibration of transient solvers based on physical measurements.

Keywords: water hammer, mathematical model, calibration, experimental tests, uncertainties.

NOTATION

The following symbols are used in this paper:

- $c$ = pipe wave speed (ms$^{-1}$);
- $D$ = pipe inner diameter (m);
- $e$ = pipe wall thickness (m);
- $E$ = Young modulus of the pipe (Pa);
- $f_s$ = Darcy–Weisbach steady-state friction factor (–);
- $g$ = gravity due to acceleration (ms$^{-2}$);
- $h_f$ = head loss per unit length (–);
- $h_{fs}$ = steady-state component of the head loss per unit length (–);
- $h_{fu}$ = unsteady-state component of the head loss per unit length (–);
\( H \) = piezometric-head, \( H = p/\gamma + z \) (m);
\( K \) = bulk modulus of elasticity of the liquid (Pa);
\( k' \) = decay coefficient (-);
\( p, p_0 \) = pressure of the mixture gas-liquid for time \( t \) and for initial steady state (Pa);
\( Q \) = flow-rate (m\(^3\)s\(^{-1}\));
\( R \) = coefficient \( R = f_D / 2DS \) (m\(^{-1}\));
\( S \) = pipe cross-sectional area (m\(^2\));
\( t \) = time (s);
\( x \) = coordinate along the pipe axis (m);
\( \alpha, \alpha_0 \) = ratio gas volume/liquid volume for time \( t \) and for initial steady state (-);
\( \nu \) = velocity (ms\(^{-1}\));
\( \rho \) = liquid density (kg.m\(^{-3}\));
\( \rho_g \) = gas density in steady state conditions (kg.m\(^{-3}\));
\( \Delta t, \Delta x \) = time step (s) and space step (m);
\( \nu' \) = kinematic viscosity (ms\(^{-2}\));
\( \theta \) = relaxation coefficient variable between 0 and 1(-).

1 INTRODUCTION

The control of hydraulic transients in pressurized water supply and wastewater systems is a major concern for engineers and pipe system managers, for reasons related to risk, safety and efficient operation. Examples of problems caused by hydraulic transients are the occurrence of instabilities in the operation of systems due to the transient sub-atmospheric pressures and, consequently, the occurrence of cavitation, or the pipe burst due to the occurrence of overpressures.

Hydraulic transient analysis can be analysed by means of simplified formulas (e.g., Joukowsky and Michaud formulas), classic water hammer simulators (e.g., commercial models) and by using more complete simulators (non-conventional models). Typically, results obtained by the commercial models, although based on classic water hammer, are satisfactory in terms of design of pressurised pipe systems, as maximum and minimum pressures occurred in the systems can be reasonably well estimated and surge protection devices can be designed whenever necessary to control transient pressures. However, these models are not accurate enough for the diagnosis and analysis of existing systems, mainly because these models cannot describe unconventional phenomena, such as cavitation, two-phase flow, unsteady friction and nonlinear rheological behaviour of the pipe; in these cases, it is necessary to use more complex models incorporating the full description of these phenomena, which are not commercially available and which are much more model calibration-dependent.

The current paper aims at the experimental and numerical analysis of the pipe flow behaviour during transient events carried out in a metal pipe rig under controlled laboratory conditions, at the Hydraulic Laboratory of the Civil Engineering Department of Instituto Superior Técnico (IST) (Lisbon, Portugal). The paper includes a brief description of the developed hydraulic transient solver, the description of the experimental facility used and the set of experimental tests carried out, and the comparison of experimental tests and the mathematical model results. Finally, the main uncertainties associated with these hydraulic transients modelling are discussed.
2 HYDRAULIC TRANSIENT SOLVER

A mathematical model for the calculation of hydraulic transients in pressurised pipes has been developed based on the classical theory of water hammer. This model allows the calculation of hydraulic transients in systems of the type "reservoir-pipe-valve" induced by downstream valve closure. Equations that describe the behaviour of fluid in the pipe during transient events are based on the mass balance and the momentum conservation principles - independently of initial and boundary conditions. These equations are (Chaudhry, 1987; Covas, 2003; Wylie and Streeter, 1993):

Continuity Equation: \[
\frac{Q}{S} \frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} + \frac{c^2}{gS} \frac{\partial Q}{\partial x} = 0
\] (1)

Momentum Equation: \[
\frac{1}{gS} \frac{\partial Q}{\partial t} + \frac{Q}{gS^2} \frac{\partial Q}{\partial x} + \frac{\partial H}{\partial x} + h_f = 0
\] (2)

In most of the engineering applications, convective terms can be neglected, thus these equations can be further simplified to a hyperbolic system of equations. Assumptions considered are (Chaudhry, 1987; Wylie and Streeter, 1978):

(i) the flow is one-dimensional with a pseudo-uniform velocity profile;
(ii) fluid is one-phase, homogenous and compressible and the changes in fluid density and temperature during transients are negligible
(iii) head losses during transient flow are calculated similarly to steady-state by friction formulas (e.g. Colebrook-White’s or Hazen Williams’s);
(iv) the pipe material has a linear rheological elastic behaviour;
(v) the dynamic fluid-pipe interaction is neglected assuming a constrained pipe without any axial movement;
(vi) the pipe is straight, uniform, with constant cross-section, without lateral in/out flow.

The friction losses term is decomposed into steady-state and the unsteady-state friction components \( h_f = h_{fs} + h_{fu} \). There are several formulas for the calculation of both components. Two numerical schemes were implemented for the calculation of the steady-state component (first-order and second order-schemes, see Table 1) and three formulations (Trikha, 1975; Vardy and Brown, 1995; Vardy et al., 1993; Vitkovsky et al., 2000) to estimate the unsteady-state component (see Table 2).

<table>
<thead>
<tr>
<th>Implementation type</th>
<th>Term ( h_{fs} )</th>
<th>Term ( h_{fu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-order approximation</td>
<td>( h_{fs} = R\Delta t Q_A</td>
<td>Q_A</td>
</tr>
<tr>
<td>Second-order approximation</td>
<td>( h_{fs} = R\Delta t Q_P</td>
<td>Q_A</td>
</tr>
</tbody>
</table>
Table 2 – Numerical schemes to estimate the unsteady-state component: $h_{fu}$

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Term $h^+_{fu}$</th>
<th>Term $h^-_{fu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitkovsky et al. (2000)</td>
<td>$h_{fu} = \frac{k}{g} \left( \frac{\partial v}{\partial t} + c \left</td>
<td>\frac{\partial v}{\partial x} \right</td>
</tr>
<tr>
<td></td>
<td>$\frac{\partial v}{\partial x} = \frac{v_P - v_A}{\Delta x}$</td>
<td>$\frac{\partial v}{\partial x} = \frac{v_{P'} - v_{B'}}{\Delta x}$</td>
</tr>
<tr>
<td>Trikha et al. (1975)</td>
<td>$h_{fu} = \frac{16}{gD^2} (Y_1 + Y_2 + Y_3)$ with $Y_{i(P)} = Y_{i(P)} \frac{4v_i}{D} \left( \frac{\Delta t}{\Delta x} \right) + \frac{m_i}{S} (Q_P - Q_{P'})$</td>
<td>in which $n_i = m_i = 0$ for steady state; $n_1 = 800; m_1 = 40; n_2 = 200; m_2 = 8.1; n_3 = 26.4; m_3 = 1$ during transients.</td>
</tr>
</tbody>
</table>

The Method of Characteristics (MOC) has been used for solving these equations. MOC transforms the hyperbolic system of differential equations into a system of ordinary differential equations valid along the characteristic lines, $C^+$ and $C^-$, known as the compatibility equations. The stability of this method restricts the time and the space step to the Courant-Friedrich-Lewy (CFL) condition. As the fluid velocity is much lower than the wave speed, this can be neglected, leading to approximate straight characteristics lines (i.e., $dx/dt = \pm c$) forming a rectangular grid. Accordingly, the pipe is divided into sections with length $\Delta x = c \Delta t$.

Compatibility equations are valid for the interior sections of the pipe, being necessary to specify additional equations for the boundaries of the system. In the current case, a system of the type “reservoir-pipe-valve” has been implemented.

3 DATA COLLECTION AND ANALYSIS

3.1 Experimental facility description

An experimental pipe rig was assembled in the Laboratory of Hydraulics and Water Resources, Department of Civil Engineering and Architecture (DECivil), at IST, for analysis of transient events both for academic (i.e., lecturing under/postgraduate courses) and research purposes (Figure 1a).

The pipe rig has a simple configuration of the type “reservoir-pipe-valve”. The pipe system is made of coiled cooper with approximately 100 m of length, 20 mm of inner diameter and 1 mm of pipe-wall thickness. The system is supplied from a storage tank with 125 l of capacity by pump with nominal flow rate of 4.5 m$^3$/h and nominal elevation of 43 m. The rig was assembled with a portable metal frame (with four wheels), 1 m depth, 2 m length and 1.6 m height, for lecturing purposes in undergraduate and post-graduate courses.

Immediately downstream of the pump, there is a hydro-pneumatic vessel (HPV), made of stainless steel, with 60 l of capacity and designed for the nominal pressure of 6 bar.
At the upstream end there is a set of valves for changing the system configuration, which allows three different configurations: Type I: Tank – pump – pipe – HPV (bifurcation) – pipe (spiral) – valve; Type II: Tank – pump – HPV – pipe (spiral) – valve; Type III: Tank – pump – pipe (spiral) – valve.

At the downstream end of the pipe, there is a system of valves – a ball valve with DN 3/4” and globe valve with DN15 ½ - that allow the generation of water hammer and the control of the flow rate, respectively; there is also a valve to purge the air (Figure 1d).

The facility is equipped with instrumentation for collecting steady flow data and transient pressure data. Steady state flow rates are measured by a rotameter (Figure 1b) and transient pressures are measured by three strain-gauge type pressure transducers located in different sections of the pipe (T1: at the upstream end, T2: at a middle section and T3: at the downstream end) and using a data acquisition system (Picoscope) with four channels (Figure 1c).

![General view](a) general view  ![Set of valves at the downstream end](b) set of valves at the downstream end  ![Data acquisition system](c) data acquisition system  ![Tank, pump and HPV at the upstream end](d) tank, pump and HPV at the upstream end

Figure 1 – View of the experimental copper pipe rig.
3.2 Data analysis: cavitation and free air

A set of experimental tests was carried out for the configuration Type II and for flow rates between 200 and 1000 l/h. Transient pressures collected at transducer T3 are presented in Figures 2 and 3. For this configuration of the system, there is no transient cavitation for flow rates lower than 500 l/h (Figures 2a, 3b), however, cavitation occurs for higher flow values, being the minimum measured pressure at the downstream transducer (T3) equal to -9 m (Figures 2b, 3b). Additionally, small vibrations were observed in transient pressures that were generated by the pipe coil not being rigidly fixed to the supports.

These tests have been analysed in terms of maximum observed pressure, $\Delta H_{max}$, and wave speeds – calculated based on maximum pressures, $c^*$, and observed based on traveling times between transducers, $c^{**}$. Results are presented in Table 3 and Figure 4.

Maximum observed piezometric heads increase with the flow rate (Table 3 and Figure 4a), being the ratio $\Delta H/Q = 359321$. This value corresponds to an average wave speed $c^* = 1106$ m/s, being $c^* = \Delta HgS/Q$. This wave speed is significantly lower than the theoretical one for monophasic fluids in copper pipes ($E = 1.07-1.31 \times 10^{11}$ Pa) with 20 mm diameter, that is approximately 1250 m/s.
Table 3 – Estimated wave speeds and corresponding overpressures for configuration Type II

<table>
<thead>
<tr>
<th>Test</th>
<th>$Q_0$ (l/h)</th>
<th>$\Delta H_{\text{max}}$ observed $^{(0)}$ (m)</th>
<th>$c^\circ$ $^{(1)}$ (m/s)</th>
<th>$\Delta H_j^{(1)}$ (m)</th>
<th>Error $^{(4)}$ (%)</th>
<th>$c**$ $^{(2)}$ (m/s)</th>
<th>$\Delta H_j^{(3)}$ (m)</th>
<th>Error $^{(4)}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>200</td>
<td>19.80</td>
<td>1098</td>
<td>1130</td>
<td>20.36</td>
<td>-2.88%</td>
<td>1115</td>
<td>20.10</td>
</tr>
<tr>
<td>A2</td>
<td>300</td>
<td>28.50</td>
<td>1054</td>
<td>1059</td>
<td>28.64</td>
<td>-0.51%</td>
<td>1048</td>
<td>28.34</td>
</tr>
<tr>
<td>A3</td>
<td>400</td>
<td>38.50</td>
<td>1067</td>
<td>1038</td>
<td>37.41</td>
<td>2.82%</td>
<td>1165</td>
<td>41.98</td>
</tr>
<tr>
<td>A4</td>
<td>600</td>
<td>62.49</td>
<td>1155</td>
<td>1211</td>
<td>65.47</td>
<td>-4.78%</td>
<td>1310</td>
<td>70.85</td>
</tr>
<tr>
<td>A5</td>
<td>800</td>
<td>78.10</td>
<td>1083</td>
<td>1130</td>
<td>81.48</td>
<td>-4.33%</td>
<td>1092</td>
<td>78.72</td>
</tr>
<tr>
<td>A6</td>
<td>900</td>
<td>90.80</td>
<td>1119</td>
<td>1240</td>
<td>100.61</td>
<td>-10.80%</td>
<td>1165</td>
<td>94.47</td>
</tr>
</tbody>
</table>

$^{(0)}$ Maximum observed piezometric head based on the first transient pressure wave (before cavitation)

$^{(1)}$ Wave speed estimated based on the maximum observed piezometric head Frizel Joukowsky formula.

$^{(2)}$ Wave speed estimated based on the travelling time of pressure wave between transducers

$^{(3)}$ Frizel Joukowsky overpressure calculated using $c**$;

$^{(4)}$ Error $= (\Delta H_{\text{max}} - \Delta H_j)/\Delta H_{\text{max}}$

Discrepancies were observed between maximum collected overpressures for the non-cavitating flows, $\Delta H_{\text{max}}$, and maximum pressures calculated according Joukowsky’s formula using estimated wave speeds, $\Delta H_j$; these discrepancies varied between 3% and 11% (measurements were lower). The reason for this is that the pipe is arranged in a coil configuration, not being constrained from any movement along its axis and, as the transient propagates, the pipe deforms axially along its development and increases its length, reducing maximum observed pressures (see hypothesis v, in section 2).

Wave speeds were estimated as well based on the travelling time of pressure wave between transducers T3-T2 and T2-T1, $c**$; these varied between 1038 and 1310 m/s (Table 4 and Figure 4a). Since, nothing has changed between tests, the only cause for this is the presence of free air in the fluid introduced by the cavitation at the downstream end flow control valve. For two-phase flows, with very small percentages of free air, the wave speed can be estimated based on (Chaudhry, 1987; Chaudhry et al., 1990):

![Figure 4](image)

(a) maximum head vs flow rate  
(b) Estimated waves speed for different tests
\[ a_m = \sqrt{\frac{l}{\rho (1 - \frac{\rho g \rho_p}{p}) + \rho g \rho_p \left( \frac{\alpha_0 q g}{p} \right) \left( \frac{L}{K_j} \alpha \frac{E_p}{\rho_p} \right)}} \]  

(3)

A small amount of air as 0.01% (in volume) can justify observed wave speed differences (see hypothesis ii, in section 2). Figure 5 presents the variation of wave speed with the percentage of dissolved gas: the wave speed difference between the monophasic fluid and the two-phase mixture with 0.03% of gas is from 1250 and 850 m/s, that is 32% less.

![Figure 5](image)

**Figure 5 – Theoretical wave speed as a function of the volumetric ratio of free-air, \( \alpha_0 \)**

### 4 NUMERICAL VS EXPERIMENTAL RESULTS

The developed mathematical model was used to analyse and numerically simulate the behaviour of the system and to compare results with collected data. Based on the results of the experimental tests, the behaviour of the system in the Type I configuration was simulated. The test with \( Q = 400 \text{ lh}^{-1} \) was used and corresponding valve closure time was 0.041 s (note that the reflection time \( 2L/c \) is 0.18 s, thus it is a fast manoeuvre).

Collected data at transducers T2 and T3 were compared with numerical results obtained using second-order approximation for the steady-state friction losses and three unsteady friction modelling formulations:

(i) neglecting the losses;

(ii) calculating unsteady-friction according Vitkovsky *et al.* (2000) formulation;

(iii) Trikha *et al.* (1975) formulation.

For this transient test, calibrated wave speed was 1150 ms\(^{-1}\). The results are presented in Figure 6.

The analysis of Figure 6 shows that numerical results obtained for \( Q_0 = 400 \text{ lh}^{-1} \) cannot reproduce the maximum and minimum pressures observed for any of the unsteady friction formulations. Only if the flow arte is reduced by 10% is it possible to obtain a better fitting. This is a numerical way to reproduce the axial deformation of the pipe using the classical transient solver developed.
Thus, the flow rate was reduced to 360 lh\(^{-1}\) (instead of 400 lh\(^{-1}\)) which hardly changes the steady state pressures as total headlosses along the pipe are minimum (~0.5 m). Results are presented in Figure 7.

None of the results calculated by any of the formulations can reproduce the maximum pressure observed in the measurements. Regarding the experimental facility, the pipe is not rigidly fixed (to ensure that it remains motionless during transient events), violating one of the basic assumptions of the classical theory of water hammer - the hypothesis (vi) (see section 2).

The best simulation results were obtained using second-order approximation for the calculation of steady friction losses and Vitkovsky’s (2001) formulation for unsteady

Figure 6. Experimental versus numerical results for conf. Type I and Q\(_{0} = 400\) lh\(^{-1}\)
friction losses with $k' = 0.05$. However, for slower transient events, the axial movement of the pipe is less evident and there are no secondary waves visible as in the fastest tests, and the formulation of Trikha et al. (1975) reproduces more accurately slow transient events.

![Graph](image)

(a) at transducer T2

![Graph](image)

(b) at transducer T3.

Figure 7. Experimental versus numerical results for conf. Type I and $Q_0 = 360 \text{ lh}^{-1}$

5 CONCLUSIONS AND FINAL REMARKS

The description of the 1-D hydraulic transient solver used was presented. The experimental facility and set of experimental tests carried out were described. Results were analysed and discussed by comparing data with the transient solver results considering the second-order approximation and neglecting the unsteady friction and calculated according to the formulations of Vitkovsky et al. (2000) and Trikha et
al. (1975). The best simulation results were obtained using Vitkovsky’s formulation with $k’=0.05$ after reducing the initial flow rate by 10% in order to reproduce the axial deformation of the pipe coil. However, for the slower transient events, the axial movement of the pipe is less evident and there are no secondary waves as visible in the fastest transient events, and Trikha et al. (1975) formulation describes more accurately the slower transient events.

This paper shows that the simulation results of mathematical models, even after model calibration, do not always fit with the experimental tests. This is due to, firstly, the nature of the measurements and uncertainty of the equipment used. For instances, the transducer error that in the current experiments was 0.25% of the maximum scale (25 bar), that is an uncertainty of 0.60 m. On the other hand, it can be due to the uncertainties of the model itself (e.g., value of wave speed, formulations for calculating unsteady friction losses) and of the dynamic effects not incorporated in the transient solvers (e.g., presence of inevitable free air, reflections from the movement of pipe or transient cavitation).

6 ACKNOWLEDGMENTS

The authors wish to acknowledge the financial support of the Portuguese Foundation for Science and Technology (FCT) through the project PTDC/ECM/112868/2009.

7 REFERENCES


