Fluid-structure interaction in pipe coils during hydraulic transients

David Ferràs$^{1,2}$, Dídia I.C. Covas$^1$, and Anton J. Schleiss$^2$

$^1$Instituto Superior Técnico, Lisboa (Portugal)
$^2$Laboratory of Hydraulic Constructions (LCH), École Polytechnique Fédérale de Lausanne (Switzerland)

Abstract

A study of fluid-structure interaction in a coiled pipe system is carried out. The aim is the understanding and description of the effect of the coil movement on the transient wave. The method is based on the implementation of a four-equation model, neglecting radial inertia, flexure and torsion of the piping system. Two conceptual models have been developed: the first simplifying the coil to a straight pipe with a moving valve, and a second model assuming independent axial deformation in each coil ring. Although the results of the second approach fit better with measurements, the first model relies on a more solid physical basis.

Keywords: friction coupling, fluid-structure interaction; hydraulic transient; junction coupling; Poisson coupling; stress-strain analysis; four-equation model.

1 Introduction

The present research focuses on the analysis of hydraulic transient flow in coiled pipe systems. The aim is the characterization of the Fluid-Structure Interaction (FSI) phenomena occurring between the pipe structure and the inner fluid. Three interaction mechanisms are analysed: the shear-stresses generated between the fluid and pipe-wall, the axial movement of the pipe induced by its radial deformation during pressure surges and the pipe movement generated by an imbalance of forces at the junctions and boundaries.

Fluid-structure interaction in piping systems consists of the transfer of momentum and forces between the structure and the contained liquid during unsteady flow (Wiggert & Tijseling, 2001). In Skalak (1955) an extended theory of FSI beyond Joukowski’s essential theory was introduced, establishing the theoretical basis for FSI in straight pipes. Skalak (1955) combined motion equations for a thin cylindrical tube with classic water-hammer equations, taking into account both fluid and pipe inertia.
Although FSI occurs in any structure, it is important to have the right criteria in order to decide when FSI effects are actually important. For this purpose the main FSI dimensionless parameters are (Tijsseling, 1996): the Poisson ratio ($\nu$), the ratio between the pipe radius and the pipe-wall thickness ($\frac{r_e}{t}$), ratio between the solid and the fluid densities ($\frac{\rho_s}{\rho_f}$) and the ratio between the solid Young’s modulus and the fluid bulk modulus ($\frac{E}{K}$). Also Tijsseling (1996) stated that FSI may be of importance when fluid and solid wave celerities are of the same order of magnitude, provided that the transient excitation is sufficiently rapid. Therefore, the ratio between fluid pressure wave and solid stress wave celerities ($\frac{a_f}{a_s}$) must be considered as well. In the specific case of coil systems the ratio between coil radius and pipe radius ($\frac{R}{r}$) and the ratio between flow velocity and fluid wave celerity ($\frac{V}{a_f}$) have to be also taken into account (Ferras et al., 2014).

Pipe coils have many industrial engineering applications, being typically used in most heat exchange systems, like cooling systems in power plants, industrial and commercial refrigerators, solar water heaters or radiators for automotive industry. The incorporation of the pipe coil behaviour in hydraulic transient analysis through FSI has never been carried out, being the novel contribution of the current paper the numerical and experimental investigation of pipe coil behaviour during hydraulic transients.

2 Background theory

2.1 Waves propagation modes

Skalak (1955), in his development of FSI by integral transforms, found an infinite number of wave propagation modes. These modes represent the degree of freedom of pipe movement. A classification of one-dimensional FSI models can be made following the basic pipe movement equations (Tijsseling, 1996):

- **Two-equation model**: this model describes the first wave propagation mode, corresponding to the pressure waves calculated only in the fluid.
- **Four-equation model**: this model is the second mode in which axial stress waves in the solid are also taken into account together with fluid conservation principles.
- **Six-equation model**: includes also radial inertia forces.
- **Eight-equation model**: allows the description of movement at pipe changes of direction (e.g. in elbows or curves).
- **Fourteen-equation model**: describes axial motion, flexure and torsional motion of three-dimensional systems.

In the present study, radial inertia, flexure and torsion are neglected and the fluid-structure interaction in the coil system is described by a four-equation model. Such assumption is based on a previous study (Ferras et al., 2014) where stress-strain laws were derived for coils subjected to static inner pressure loads. The study concluded that torsional and bending movements in a pipe coil are negligible in comparison to axial or circumferential displacements. Moreover, the axial stress generated for the inner pressure load is equivalent to the axial stress produced in a free moving straight pipe with closed ends.

2.2 FSI coupling

A transient flow is defined by the intermediate-stage flow, when the flow conditions are changed from one steady-state condition to another steady-state (Chaudhry, 1987). In the four-equation
model, two transient events are coupled, i.e. the propagation of the pressure wave in the fluid and the propagation of the axial stress wave in the pipe-wall. The interaction between the two transient events is described by three main coupling mechanisms (Tijsseling, 1996): Poisson coupling, friction coupling and junction coupling. The first mechanism is based on the axial deformation of the pipe caused by the radial load produced by the inner pressure. Friction coupling, arises from the shear stress between the pipe-wall and the fluid. Junction coupling results from unbalanced local forces and by changes in the fluid momentum that occur in pipe curves, Tee junctions or cross section changes. During the development of the model, the three coupling mechanisms were analysed and implemented.

2.3 Four-equation model

Skalak (1955) introduced a solution of a 1-D four-equation non-dispersive model as an extension of Joukowsky’s method. This model was developed by Lavooij & Tijsseling (1991) in all the basic coupling mechanisms (Poisson, junction and friction coupling) solving the following set of equations:

\[
\frac{\partial V}{\partial t} + g \frac{\partial H}{\partial x} = -\frac{f}{4r} (V-U)|V-U|
\]

\[
\frac{\partial V}{\partial x} + \frac{g}{a_f^2} \frac{\partial H}{\partial t} = \frac{2\nu}{E} \frac{\partial S}{\partial t}
\]

\[
\frac{\partial U}{\partial t} - \frac{1}{\rho_s} \frac{\partial S}{\partial x} = \frac{\rho_f A_f f}{\rho_s A_s 4r} (V-U)|V-U|
\]

\[
\frac{\partial U}{\partial x} - \frac{1}{\rho_f a_f^2} \frac{\partial S}{\partial t} = -\frac{\rho_f g r \nu}{c E} \frac{\partial H}{\partial t}
\]

Equations 1 and 2 are the fluid momentum and mass conservation equations, whereas Equations 3 and 4 are the homologous pipe conservation equations. \(H\) is the fluid pressure head, \(V\) is the fluid axial velocity, \(S\) is the solid axial stress, \(U\) is the solid axial velocity, \(r\) the pipe radius, \(g\) is the gravity acceleration, \(f\) is the Darcy friction coefficient, \(E\) the Young’s modulus of elasticity of the pipe, \(\nu\) the Poisson ratio, \(\rho_f\) and \(\rho_s\) are respectively fluid and solid densities, and \(a_f\) and \(a_s\) are the wave propagation celerities in the fluid and in the solid, respectively. The wave celerity in the solid corresponds to the axial stress wave being described by

\[
a_s = \sqrt{\frac{E}{\rho_s}}
\]

whereas \(a_f\) is the fluid wave celerity, which for a non-anchored coil is given by (Ferras et al., 2014):

\[
a_f = \sqrt{\left[\left(\frac{1}{K} + (2-\nu) \frac{r}{Ee} + 1.3252 \cdot 10^{-11}\right)\rho_f\right]^{-1}}
\]

3 Data collection

The experimental set up is composed of a copper pipe of nominal diameter \(D = 0.02 \text{ m}\), pipe-wall thickness \(e = 0.001 \text{ m}\) and pipe length \(L = 105 \text{ m}\). The torus radius is \(R = 0.45 \text{ m}\) and 36 rings compose the entire coil. Strain gauges were installed in the middle section of the pipe in order to carry out strain measurements in the axial and in the circumferential directions for different positions of the cross-section. Young’s modulus of elasticity and Poisson ratio were experimentally determined by measuring stress-strain states over a straight pipe sample for the experimental range.

of pressures. The obtained values were Young’s modulus of elasticity $E = 105 \text{ GPa}$ and Poisson ratio $\nu = 0.33$.

Fig. 1: Scheme of the copper pipe coil facility used for the experimental data collection

As referred in Section 1, several dimensionless numbers indicate when FSI effect might be important. The following table shows the value of these parameters for the copper coil facility:

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$a_f/a_s$</th>
<th>$r/c$</th>
<th>$\rho_s/\rho_f$</th>
<th>$E/K$</th>
<th>$R/r$</th>
<th>$V/a_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>0.368</td>
<td>10</td>
<td>8.96</td>
<td>47.95</td>
<td>50</td>
<td>0.00028</td>
</tr>
</tbody>
</table>

Fig. 2 depicts measured hydraulic head immediately upstream the valve in comparison with the numerical results obtained by the classic water-hammer model for an initial flow rate $Q_0 = 400 \text{ l/h}$. There is a notable discrepancy on wave amplitude. The present study approaches the solution of such discrepancy by assuming its source is the structural behaviour of the coiled pipe system as well as the movement of the downstream end valve.
4 Model development

4.1 Characteristic grid in a four-equation model

The set of partial differential Equations 1 to 4 are transformed to ordinary differential equations (compatibility equations) by applying the method of characteristics (MOC). Resulting equations can be easily integrated over a characteristic grid.

However, in a four-equation model two different waves propagate with different celerities over the same characteristic grid: fluid pressure wave and solid stress wave. Two main approaches can be used: wave celerity adjustment in order to keep Courant numbers equal to one by achieving a ratio between celerities of integer numbers, as suggested by Schwarz (1978), Wiggert (1986) or Bergant et al. (2008); or by applying either temporal or spatial interpolation over the grid, as followed, for example, by Fan (1989), Elansary & Contractor (1990), Bouabdallah & Massouh (1997) or Ghodhlan & Hadj-Taieb (2013). Tijsseling (2002) proposed a third approach, the resolution of an exact solution by means of a mathematical recursion. Methods based on interpolations introduce numerical dispersion and diffusion, while the exact solution is oriented only to verification and validation, as for large simulation periods it is computationally expensive. The method applied herein is based on the celerities adjustment, though, special attention was given on validation, as this method introduces phase shift in the transient pressure wave.

For instance, when applying Equations 5 and 6 to the copper facility (see characteristics in Table 1), the celerities obtained are 1261 m/s for the fluid pressure wave and 3423 m/s for the solid stress wave. Two different integer numbers ratio were tested \( \frac{a^*_{f}}{a^*_{s}} = \frac{1}{3} \) (being \( a^*_{f} = 1141 \text{ m/s} \) for \( a^*_{s} = 3423 \text{ m/s} \)) and a more accurate \( \frac{a^*_{f}}{a^*_{s}} = \frac{4}{11} \) (being \( a^*_{f} = 1245 \text{ m/s} \) for \( a^*_{s} = 3423 \text{ m/s} \)).
Inevitably such adjustment will lead to a small phase error arisen from the adjusted fluid wave celerity.

Fluid and solid densities were corrected according to the modified celerities applying the following equations (Lavooij & Tijseling (1991)):

\[
\rho_f^* = k_1 \frac{(a_f^2 + a_s^2) + \sqrt{(a_f^2 + a_s^2)^2 - 4(1 + k_1 k_2 a_f^2 a_s^2)}}{2(1 + k_1 k_2 a_f^2 a_s^2)} \\
\rho_s^* = k_2 \frac{(a_f^2 + a_s^2) - \sqrt{(a_f^2 + a_s^2)^2 - 4(1 + k_1 k_2 a_f^2 a_s^2)}}{2a_f^2 a_s^2}
\]

where \( \rho_f^* \) is the adjusted fluid density and \( \rho_s^* \) the adjusted solid density, \( a_f^* \) and \( a_s^* \) the modified wave celerities, and \( k_i \) are parameters given by: \( k_1 = \left( \frac{1}{K_f} + 2 \frac{H}{E} \left( \frac{1 - \nu^2}{\nu^2} \right) \right)^{-1} \), \( k_2 = E \) and \( k_3 = 2\nu^2 \frac{H}{E} \).

The adjustment of wave speeds allows calculations to lie in the grid points. However, in the boundaries and in their vicinities, temporal or spatial interpolations are unavoidable. Temporal interpolations were carried out herein as depicted in Fig 3.

![Scheme of the characteristic lines for the wave celerities adjustment](image)

**Fig. 3:** Scheme of the characteristic lines for the wave celerities adjustment \( \frac{a_f^*}{a_s^*} = \frac{1}{3} \)

### 4.2 Boundary conditions and junction coupling

Several boundary conditions can be taken into account in a reservoir-pipe-system. The boundary conditions for a fixed infinite reservoir at the upstream boundary are

\[
\begin{align*}
H[0,j] &= H_{res} \\
U[0,j] &= 0
\end{align*}
\]
Model development

while for a fixed valve at the downstream boundary are

$$
\begin{align*}
U[l,j] &= 0 \\
V[l,j] &= \tau(t)\sqrt{\Delta H}
\end{align*}
$$

(10)

where $\tau(t)$ is a function describing the valve closure, the coefficients in the squared brackets correspond to space and time coordinate, 0 stands for the upstream boundary and $l$ for the downstream boundary, and $H_{res}$ is the pressure head in the reservoir.

For a boundary condition of a moving valve, the balance of forces must be carried out taking into account the movement of the valve. Hence, the second law of Newton is applied, describing the rate of change of momentum at the valve as the unbalance of forces over the valve between the fluid pressure and the pipe-wall stress:

$$
\rho_f g \Delta H A_f - S[l,j] A_s = m_v \ddot{U}[l,j]
$$

(11)

in which $m_v$ is the mass of the valve, $A_f$ and $A_s$ are, respectively, the fluid and solid cross-sectional areas, and $\ddot{U}$ the axial acceleration of the solid.

The consequent movement of the pipe induces an axial stress wave that propagates throughout the pipe (junction coupling). Assuming static conditions, though, Ferras et al. (2014) showed that the axial stress in a toroidal pipe due to inner pressure is equivalent to the axial stress of a non anchored straight pipe with closed ends, being $S[l,j] = \frac{2C}{R}$, which is indeed the expression presented at Equation 11 considering a massless valve. However, in dynamic conditions, the inertia of the moving element must be taken into account. By rearranging Equation 11 and considering the change of flow rate due to valve closure, the following boundary conditions are achieved:

$$
\begin{align*}
S[l,j] &= \rho_f g \frac{A_f \Delta H}{A_s} - \frac{m_v}{A_s} \ddot{U}[l,j] \\
V[l,j] - U[l,j] &= \tau(t)\sqrt{\Delta H}
\end{align*}
$$

(12)

4.3 Model verification for Delft Hydraulics FSI benchmark problem

A verification of the basic implementation was carried out by means of the simulation of the well-known Delft Hydraulics FSI benchmark problem from Tijsseling & Lavooij (1990) and Lavooij & Tusseling (1991). From the set of FSI problems presented in these papers, Problem A is the most suitable for the verification of the developed four-equation model. It consists of a reservoir-pipe-valve system with length $L = 20$ m, inner radius $r = 398.5$ mm, pipe-wall thickness $e = 8$ mm, Young’s modulus $E = 210$ GPa, solid density $\rho_s = 7900$ kg/m$^3$, Poisson ratio $\nu = 0.30$, bulk modulus $K_f = 2.1$ GPa, fluid density $\rho_f = 1000$ kg/m$^3$ and initial flow velocity $V_0 = 1$ m/s. The valve is closed in one time-step, and both boundary conditions fixed and free moving valve are analysed.

Corresponding wave speeds are $a_f = 1024.7$ m/s and $a_s = 5280.5$ m/s, giving a ratio of $a_f/a_s = 0.194$. Like in Tijsseling (2002), two different simulations have been carried out: the first simulation approximating the ratio between celerities to $1/5 = 0.2$, and a second simulation considering a more accurate ratio of $13/67 = 0.194$. Figures 4 and 5 show the output of both simulations, either for a fixed and a moving downstream boundary, as well as the Joukowsky solution.
Fig. 4: Delft Hydraulics benchmark Problem A with fixed boundaries

Fig. 5: Delft Hydraulics benchmark Problem A with a free moving valve

In the first case, the ratio $\frac{a_f}{a_s} = \frac{1}{5}$ allows no interpolations at the boundaries for the characteristic lines corresponding to the fluid pressure wave propagation, however the higher the adjustment
of the wave speed is, the greater the phase shift error becomes. For a ratio $\frac{a_1}{a_2} = \frac{13}{67}$, the wave speed remains almost with the same value, though, the interpolations at the boundaries are more intensive and, consequently, numerical diffusion and dispersion increase.

When simulating a frictionless system, results show two different phenomena: the Poisson coupling beat, described in Tijseling (1997), for a system with $\nu = 0.3$ and fixed boundaries (Fig. 6a); and also a different kind of beat produced in junction coupling, when a massless valve is allowed to move and null Poisson ratio is considered $\nu = 0$ (Fig. 6b).

![Fig. 6: Delft Hydraulics benchmark Problem A: (a) pressure output for $\nu = 0.3$ and fixed valve (Poisson coupling beat); (b) pressure output for $\nu = 0$ and a free moving valve.](image)

Both beats represented in Fig. 6 show a quite different structural behaviour and its effect in the fluid wave. Comparing the two graphs, the pressure variation amplitude in a fixed valve with Poisson coupling is higher in regard to pressure values but lower in regard to axial stress values. In addition, there seems to be a 90° phase shift between the axial stress wave and the fluid pressure wave, while in the case of free moving valve there is no apparent shift. The reason lies in the boundary condition imposed on the free moving valve, which balances the forces of the pressure over the valve with the axial stress of the pipe-wall, thus establishing a direct relation between both variables.

5 Model application

5.1 Modelling assumptions

The coil pipe system, when pressurized, increases the fluid volume due to both the axial and the circumferential deformation of the pipe-wall. Conversely, for negative pressures, the pipe reduces its fluid volume. In coil-pipes, due to the axial deformation, this effect is stronger than in straight pipes with anchored boundaries. The consequent response of such “breathing” effect of the coil over the hydraulic transient wave is a smoothing of the pressure peaks, resulting on a reduction of the wave amplitude.

Axial strains of a pipe coil for inner pressure loads in static conditions can be assumed to be equivalent to the ones of a straight pipe with closed ends, though, with a modified wave celerity (Ferras et al., 2014). Two modelling approaches applying the developed four-equation model are analysed. The first model is built considering that the coil can be simplified to a single straight
pipe with a moving valve at the downstream end. The second considers that the downstream end valve is fixed and adds internal conditions along the pipe to describe the behaviour if each ring.

5.2 Model-1: straight pipe with moving valve

The implemented four-equation model of Section 4 was used to describe the transient pressures in the coiled copper facility. A reservoir-pipe-valve system with a free moving valve was considered in order to represent the “breathing” effect of the coil due to the axial deformation of the straight pipe. Fig. 7 depicts the schematic of the Model-1.

![Fig. 7: Model-1: single straight pipe with a free moving valve at the downstream end](image)

The model has the following characteristics: pipe length of $L = 105$ m, pipe inner diameter $D = 2$ cm, pipe-wall thickness $e = 1$ mm, modulus of elasticity $E = 105$ GPa, fluid bulk modulus $K = 2.19$ GPa, fluid density $\rho_f = 1000$ kg/m$^3$, solid density $\rho_s = 8960$ kg/m$^3$, Poisson ratio $\nu = 0.33$, initial flow velocity $V_0 = 0.354$ m/s and initial Darcy friction factor $f = 0.035$ (smooth wall pipe).

Fig. 8 depicts the model results for a moving massless valve in comparison with the classical water-hammer solution and measured data.

![Fig. 8: Results of Model-1: a) simulated piezometric head at the pipe downstream end for a free moving massless valve versus results of classic water-hammer solver and collected pressure data; b) detail of the first peak](image)
Piezometric head obtained by the four-equation model, although being smoothed due to the non-instantaneous closure of the valve, is subdivided into three stages. A first Stage-1 in which the pressure is lower than the classic two-equation model. Stage-2 with higher pressure and, finally, Stage-3 with a pressure decrease. The first pressure drop is caused by the movement of the valve in the downstream direction after the first pressure surge. Afterwards, as the solid axial stress wave travels approximately 3 times faster than the fluid pressure wave, at around one third of the pressure peak there is an increase of pressure resulting from the negative axial stress which is pulling the pipe upwards, producing this “pumping” effect. Finally, the axial stress wave bounces back pushing again the valve and producing the last pressure drop over the pressure surge.

In order to assess the effect of the moving valve, a sensitivity analysis has been carried out for the valve mass ($m_v$). A set of 100 simulations from $m_v = 0$ kg until $m_v = 1000$ kg was carried out. Results are shown in Fig. 9.

![Feasible solution region of Model-1 output for a free moving valve with variable mass](image)

Fig. 9: Feasible solution region of Model-1 output for a free moving valve with variable mass

Fig. 9 shows the band range of possible solutions of the four-equation model pressure output next to the downstream end for a free moving valve of variable mass. The bold dashed line indicates the minimum valve mass threshold modelled, which is equal to the already presented massless valve solution. The dotted line depicts the output for the maximum valve mass threshold modelled, which is $m_v = 1000$ kg. As the mass valve increases, results tend to the solution of Poisson coupling with a fixed valve. It is interesting to point out that, due to the dispersion effect of the mass valve, the maximum pressure peak does not occur for an infinite mass valve (fixed valve) nor for a massless valve, but somewhere in-between.

In order to determine the best simulation, the Mean Squared Error (MSE) was computed for the first pressure peak (within the time slot 0.2 to 0.5 s, see Fig. 9). Fig. 11 depicts the MSE in function of the mass valve variation. The lowest MSE corresponds to a valve mass $m_v = 121$ kg, which does not correspond to the valve actual weight (as the valve is quite small, weighting 300 gr), however
represents the valve constraints as the valve and the pipe at the downstream end are partially fixed (not rigidly fixed) as it can be seen in Fig. 10.

Fig. 10: Pipe downstream end with the valve anchors and pipe brackets

The four-equation model also allows the assessment of axial stress output. However such variable cannot be directly compared with measured data, as the experimental tests only provide strain measurements from the strain-gauges. Consequently either computed stress is converted to strain or the other way around, measured strain to stress. This second option was adopted using the following conversion based on the stress-strain equations:
\[
\begin{align*}
\epsilon_c &= \frac{\sigma_c}{E} - \frac{\nu}{E} \sigma_a \\
\epsilon_a &= \frac{\sigma_a}{E} - \frac{\nu}{E} \sigma_c \\
\sigma_a &= \frac{E \epsilon_a}{1 - \nu^2} + \epsilon_c
\end{align*}
\]

(13) (14)

In which \(\epsilon_c\) is the circumferential strain, \(\epsilon_a\) the axial strain, \(\sigma_c\) the circumferential stress and \(\sigma_a\) the circumferential strain.

Results of Equation 14 are depicted in Fig. 12.

Fig. 12: Comparison of axial stress model output for a free moving valve with a mass of 121 kg with axial stress computed from strain measurements, at the middle section of the pipe.

The axial stresses depicted in Fig. 12 show quite a good matching between measurements and numerical results. However, discrepancies arise from the wave propagation. Computed axial stress presents a wave shift with a delay induced since the valve closure. Also the wave attenuation is presented different in both sources, while measured axial stresses present a smooth constant attenuation from one peak to another, computed stresses peaks oscillates as result of the interaction with the fluid pressure, having the maximum stress in the second peak.

5.3 Model-2: analogue mechanical model

In order to better describe the observed structural behaviour of the pipe coil during the transient pressure wave propagation, an analogue mechanical model was build using the four-equation FSI model. The approach consisted of applying the concept that the rings of the coil vibrate independently from each other according to their inner pressure load in each time-step.
To describe these independent vibrating rings two sorts of models were combined: 1) a main two equation model representing a straight pipe with a total length of $L = 105 \, m$, discretized by 38 nodes with each inner node representing a coil ring; and 2) 36 four-equation sub-models describing the rings behaviour. These nested FSI sub-models were build as straight pipes of length equal to the ring perimeter (i.e., $\pi \, m$) and with closed but free moving ends. The valve in the main pipe is fixed. A schematic of the model is depicted in Fig. 13.

Fig. 13: Model-2: independent vibrating rings by nested FSI sub-models which assume straight pipes with closed free moving ends

The coupling is carried out by considering the centre node of the ring models as the inner node of the main system. In each time-step, the pressure of the rings is equal to the pressure of the entire nested models, consequently the free moving ends stretch or shrink the pipes describing the ring behaviour. This effect produces an increase or decrease of pressure which is transferred to the main pipe. The coupling is carried out during the first positive pressure surge. The effect of the moving closed ends can be calibrated by adding inertia to the pipe boundaries (considering the mass of the coil rings). Finally, velocity must be recomputed in each time-step and each node of the main system.

With the aim of assessing the sensitivity of model output to the rings inertia, a set of simulations was carried out by varying the mass of the rings, from 0 to 40 $kg$. Fig. 14 depicts this set of simulations showing the feasible solution band for the varying mass in the rings. As it can be observed, the mechanical model proposed allows the adjustment of the pressure peaks by considering the independent expansion and contraction of the rings according to a certain mass load.
Fig. 14: Model-2 output considering a varying mass of the rings in comparison with measured pressure data at the downstream end

The simulation with better fitting to measurements (i.e. a lower MSE) was selected from the sequence presented above and calibrated. The calibration process consisted of distributing a varying mass throughout the coil rings in order to get the best fitting for the first pressure peak. For this purpose a sensitivity analysis of the mass distribution in the rings was carried out. The analysis consisted of a set of simulations by enabling the free movement of massless rings throughout the coil except one fixed ring. Each ring was assessed. A total of 36 simulations was carried out. The output allowed the analysis of the effect of the fixed ring over the pressure output. Fig. 16 depicts hydraulic head during the first peak for the mentioned set of simulations.

Fig. 15 shows the results of the sensitivity analysis for a varying average mass of the rings.
5 Model application

Fig. 15: Mean square error values in function of ring mass variation. MSE was computed for the first pressure peak.

Fig. 16: Model-2 output for a set of simulations by allowing the free movement of the rings except one. The position of the fixed ring is changed in each simulation.
The sensitivity of the model to the mass ring distribution is shown in the following Fig. 17 representing the MSE computed by taking as reference the solution of massless rings throughout the coil. Thereby, the lower is MSE the closer the model output is to the massless rings solution, and consequently, the lower is the sensitivity of the model to the assessed ring.

![MSE from previous simulations](image)

**Fig. 17:** MSE from the previous simulations of Fig. 16. MSE was computed taking as reference the massless rings solution.

As it can be seen in the previous Fig. 17 the closer are the rings to the upstream boundary, the less sensitive is the model to the ring movement. Taking into account the sensitivity analysis, a manual calibration was carried out by distributing the mass over the rings. The goal of this calibration was to get the best fit for the first pressure peak. The following Fig. 18 shows the distribution of mass over the rings after calibration.
Fig. 18: Mass distribution of the rings throughout the coil after calibration

Fig. 19 depicts both the best simulation from Fig. 14 and the calibrated simulation.

Fig. 19: Model-2 output after calibration, either with homogeneous rings mass or distributed, and in comparison with measured pressure data at the downstream end
As it can be seen in Fig. 19, results from the simulation with homogeneous mass load are quite accurate in regard to the wave amplitude and phase, however do not represent the wave shape, being quite similar to classic theory. On the other side, the model calibration by distributing the mass load allows a very good fitting in the first pressure peak. Nonetheless, the achieved wave shape remains quite stable over the propagated pressure peaks.

6 Results discussion

As it has been explained in Section 5, two models with different conceptual assumptions were developed with the aim to describe the FSI in a copper coil facility. Model-1 simplifies the coiled pipe system to a straight pipe with a moving valve in its downstream end, while Model-2 describes the independent vibrating rings by assuming an analogue mechanical model. Figs. 20 and 21 show the results of the best simulations from both models (with lower MSE at the first pressure peak) at two pipe sections: at the downstream and at the intermediate section. The best simulation of model-1 is with a valve mass of 121 kg, while for Model-2 the average distributed mass over the rings is 2.8 kg (i.e., a total mass of 100.8 kg over the coil).

Fig. 20: Pressure outputs at the downstream boundary from the best simulations of Model-1 and Model-2 in comparison with measurements
Fig. 21: Pressure outputs at the middle section of the pipe from the best simulations of Model-1 and Model-2 in comparison with measurements

Figs. 20 and 21 show the uncertainties of both models in comparison with measured data at the pipe downstream end and at the middle section. Although Model-1 enables the representation of the pressure variation according to pipe-wall axial deformation, the model does not describe with accuracy the shape of the pressure wave, consequently the mean squared error is quite high in comparison to Model-2 for both measuring points. On the other side, Model-2 shows a very good fitting with measured data for the first pressure peak, as the distributed mass of the rings allows an accurate calibration of the pressure wave shape. However, the calibrated wave shape in the first pressure peak does not evolve according to the observed propagation of the transient event. The reason may be that other physical phenomena, relevant during the propagation of the water-hammer wave, have not been taken into account, such as unsteady friction, anelastic behaviour of the pipe-wall, viscoelasticity of the anchorages of the coil rings or friction between the coil structure and its supporting media.

The main difference between the two models analysed is how junction coupling is considered. In Model-1 the junction coupling is focused on the balance of forces over the valve (boundary condition). In Model-2, junction coupling is done based on the balance of forces in each coil ring (internal conditions). In the experimental coil system the moving elements are the rings and not the valve, hence Model-2 is more faithful to the real phenomenon. However, the mechanical model proposed does not solve entirely the FSI problem, as solid variables are only partially solved. Therefore, stress and strains, and so forth pipe-wall movement, cannot be analysed by means of Model-2. On the other side, Model-1, although less accurate, can be used to analyse both fluid and solid variables. There is a logical argument followed from some simplifying assumptions applied in both basic theories, beam theory and water-hammer theory, allowing the solution of a simple four-equation model composed of a straight pipe with a moving end. The axial stress waves are sent out from this moving end allowing an easy understanding of their propagation and their effect over the water-hammer wave.
7 Conclusions

The present paper aims to develop a mathematical model that describes the fluid structure interaction occurring in a coil pipe system during hydraulic transient events. The approach followed is based on the implementation of a four-equation model, which takes into account the effect of axial stress waves throughout the pipe-wall. The assumption of neglecting the flexure and torsional motion of the coil is based on a previous study of stress-strain analysis in coiled pipe systems (Ferras et al., 2014).

In the four-equation model two transient events are coupled, i.e. the solid and fluid transients in the pipe axial direction. Fluid interacts with its containing structure through three different coupling mechanisms: Poisson, junction and friction coupling. After the implementation of a basic FSI four-equation model and its verification by means of the Delft Hydraulics benchmark Problem A (Tijsseling & Lavooij, 1990; Lavooij & Tusseling, 1991), two models were developed with the goal to adapt to the coil singularities. The main difference between the two models is how junction coupling is considered. A first attempt was carried out by assuming a single straight pipe with a moving valve at its downstream end (Model-1). A second model (Model-2) was developed by fixing the valve and considering that the moving elements are the coil rings, which vibrate independently according to their inner pressure. Both models were calibrated by varying the mass of their moving elements.

Model-2 enables a distributed mass on the coil rings, allowing a greater capability for calibration. Consequently, the fitting of Model-2 with measurements after calibration is more accurate than Model-1. Nonetheless, the good adjustment achieved in the first pressure peak is not transmitted to the subsequent pressure peaks. The shape of the propagated pressure wave remains approximately constant during its propagation, just varying its amplitude according to friction losses. On the other side, Model-1 does describe the effect of FSI during the entire wave propagation, modifying the wave shape due to dispersion and dissipation induced by the axial stress variation throughout the pipe. However, its lower parametrization does not allow a better fitting with measurements, showing lower performance and a higher error (i.e., mean squared error). Even though, this model has the advantage to entirely solve the solid and fluid variables, hence, both computed pressure and stresses can be compared versus experimental data.

In a real system, the phenomenon of FSI analysed herein is combined with other phenomena such as unsteady friction, rheological behaviour of the pipe-wall, viscoelasticity of the anchorages of the coil rings or friction between the coil structure and its supporting media. The analysis of these different phenomena combined with FSI should improve the understanding on the distinction of the best approach.

Acknowledgements

This research is supported by the Portuguese Foundation for Science and Technology (Fundação para a Ciência e a Tecnologia) through the project ref. PTDC/ECM/112868/2009 “Friction and mechanical energy dissipation in pressurized transient flows: conceptual and experimental analysis” and the PhD grant ref. SFRH/BD/51932/2012 also issued by FCT under IST-EPFL joint PhD initiative.
References


