Damping Analysis of Hydraulic Transients in Pump-Rising Main Systems

Alexandre Kepler Soares¹; Dídia I. C. Covas²; and Helena M. Ramos³

Abstract: This paper focuses on the analysis of unsteady pipe flows caused by a pump trip in a water pipeline system. Field experiments have been carried out to collect transient pressure and steady-state flow rate data in the Prado–Instituto Politécnico da Guarda (IPG) pumping system located in Guarda, Portugal. Observed transient pressures were compared with numerical results. Results obtained were excellent both in terms of damping and phase shift of the transient pressure signal, provided that unsteady friction effects were taken into account and an appropriate downstream end boundary condition was considered. The downstream end boundary condition was described by small tanks with variable level and with free discharge into the storage tank; the variation of water level in these small tanks results in the relief of extreme transient pressures (conversely to what is observed in the line-packing effect) and changes the shape of transient pressure waves. This analysis has shown that classical water hammer theory (neglecting unsteady friction) and the consideration of a constant-level reservoir at the downstream end do not accurately describe the observed transient behavior of pressurized pipe systems. DOI: 10.1061/(ASCE)HY.1943-7900.0000663. © 2013 American Society of Civil Engineers.

CE Database subject headings: Transient flow; Water hammer; Water pipelines; Skin friction.

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Introduction

Hydraulic transient analysis is important in the design of pressurized pipe systems to guarantee their security, reliability, and good performance for various normal operating conditions. Transients can be caused by valve maneuvers, pump trips or start-up, or the occurrence of sudden pipe ruptures. The prediction of maximum transient pressures is used to verify whether pipe materials, pressure classes, and wall thicknesses are sufficient to withstand predicted pressure loads to avoid pipe rupture or system damage. Verification of minimum allowable pressures is important to prevent air release, cavitation, and water column separation, and, consequently, to avoid pipe collapse or pathogenic intrusion into the system. When severe transients cannot be avoided, either the pipe system layout or pipe characteristics are changed, or surge-protection devices are specified (e.g., air vessels), so as to reduce the extreme pressures to within acceptable limits. Usually, the decision is the most economical and reliable solution that yields an acceptable transient pressure response. Typically, classical water hammer analysis is carried out.

Hydraulic transient analysis is equally important in the operation stage of an existing system for the diagnosis of malfunction problems or the causes of pipe bursts. For this case, it is extremely important to use accurate hydraulic transient solvers that incorporate additional effects that are not typically available on commercial software (e.g., unsteady skin friction, pipe nonelastic rheological behavior). An example of this is inverse transient analysis carried out for system calibration or for leak detection, the success of which very much depends on the use of calibrated and accurate transient solvers (Stephens et al. 2004, 2005, 2011; Vítkovský et al. 2007; Savic et al. 2009; Covas and Ramos 2010).

Different approaches can be used for carrying out hydraulic transient analysis: simplified formulations for estimating extreme pressures and pressure envelopes (e.g., the Joukowsky formula for rapid maneuvers and the Michaud formulation for slow maneuvers), classical transient solvers that are based on a set of simplifications, and complete transient solvers that take into account different dynamic effects (e.g., nonelastic pipe-wall behavior, fluid-structure interaction, or cavitation). Results obtained by most commercial software are based on the classic water hammer theory; these are satisfactory to estimate extreme pressures and allow the design of surge-protection devices. However, such models are frequently very imprecise for the analysis and diagnosis of existing systems (Ramos et al. 2004; Covas et al. 2005).

In this study, transient pressurized pipe flows caused by the pump trip are analyzed. Field tests were carried out collecting transient pressures and steady-state flow rate in the Prado–Instituto Politécnico da Guarda (IPG) water-pumping system located in Guarda, Portugal. Numerical results obtained were compared with transient pressures measured. Results obtained were excellent provided that unsteady friction effects were taken into account and the downstream end boundary condition was defined by three small tanks with variable levels and free discharge to the atmosphere (i.e., to the downstream end storage tank).

The objective of this paper is twofold. First, it aims at showing the complexity of the hydraulic transient solver calibration when
dealing with real-life systems with different boundary conditions and uncertainties associated with the system physical characteristics. Second, the paper aims to demonstrate that some simplifications considered in transient analysis, such as considering steady-state friction only and constant-level reservoir, are not reasonable and cannot describe the pipe system behavior, which is very important for the diagnosis of existing problems and for inverse transient analysis.

Guarda Water Pipeline System

The case study consists of a pipe-rising main between two storage tanks: the Prado tank and the IPG tank. Prado pumping station is composed of three submersible pumps and two centrifugal pumps, installed in parallel. Immediately downstream of each pump, there is an automatic control valve and a gate valve. The surge-protection device installed is a pressure-relief valve with 200-mm diameter. Figs. 1 and 2 present the simplified schematic of the Guarda pipe system and the pipe profile.

The main pipeline is made of cast iron with a nominal diameter of 500 mm and a length of 2,225 m. The pipeline has an air valve installed at an intermediate section (point C in Fig. 2).

At the downstream end of the pipeline, an electromagnetic flow-meter is installed, followed by a reduction to a 400-mm PVC pipe that connects to three 200-mm PVC pipe branches. These three PVC pipes are vertical with free discharge to the IPG storage tank. The total length of the pipeline, from the Prado pumping station to the IPG reservoir, is approximately 2,241 m.

The data-acquisition system was composed of two pressure transducers (pressure range of 0–25 bar, 0.2% accuracy of total range), two data-acquisition boards with four channels each, an ultrasonic portable flowmeter, and two notebooks. Fig. 1 shows the location of pressure and flow rate measurement sections, in which both P1 and P2 refer to the pressure transducer locations (acquisition frequency of 50 Hz), and Q1 and Q2 are the location of the ultrasonic flowmeter and the electromagnetic flowmeter, respectively.

Some features of the Guarda water pipeline system, such as details of pumps, the relief valve, and the pumping station, as well as of the measurement sections at the IPG tank, are presented in Fig. 3.

Hydraulic Transient Solver

Elastic Model

Equations that describe the one-dimensional transient-state flows in closed conduits are the momentum and continuity equations [Eqs. (1) and (2), respectively]. Because the flow velocity and pressure (dependent variables) in transient flows are functions of time and space (independent variables), these equations are a set of two hyperbolic, partial differential equations (Chaudhry 1987; Almeida and Koelle 1992; Wylie and Streeter 1993):

\[
\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + h_f = 0
\]

(1)

\[
\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0
\]

(2)
Fig. 3. Guarda water pipeline system: (a) Prado pumping station; (b) submersible pumps and relief valve; (c) centrifugal pumps; (d) pressure and flow rate measurement points in Prado pumping station; (e) ultrasonic flowmeter; (f) pressure transducer location in Prado pumping station; (g) electromagnetic flowmeter and inlet PVC pipe in IPG reservoir; (h) crosspiece and PVC pipe branches in IPG reservoir; (i) pressure transducer in IPG reservoir; (j) free discharge outlet
where \( x \) = coordinate along the pipe axis; \( t \) = time; \( H \) = piezometric head; \( Q \) = flow rate; \( A \) = pipe cross-sectional area; \( a \) = celerity (or elastic wave speed); \( g \) = gravity acceleration; and \( h_f \) = head loss per unit length.

Considering an elastic behavior of the pipe wall, the wave speed for monophasic fluids (i.e., liquids) can be estimated by (Wylie and Streeter 1993)

\[
a = \sqrt{\rho [1 + \psi(D/e)] (K_2/E_0)}
\]

where \( K_2 \) = bulk modulus of elasticity of the fluid; \( \rho \) = mass density of the fluid; \( E_0 \) = Young’s modulus of elasticity of the pipe; \( D \) = pipe inner diameter; \( e \) = pipe-wall thickness; and \( \psi \) = dimensionless parameter that depends on the elastic properties of the conduit (i.e., cross-section dimensions, pipe axial constraints, and Poisson’s coefficient). For gas-liquid mixtures, wave speed also depends on the gas initial concentration, pressure, and temperature (Chaudhry 1987).

The set of differential Eqs. (1) and (2) can be solved by the method of characteristics, which allows the transformation of these equations into a set of total differential equations valid along the characteristic lines, \(\frac{dx}{dt} = \pm a\)

\[
C^\pm: \frac{dH}{dt} \pm \frac{a}{gA} \frac{dQ}{dt} \pm a \cdot h_f = 0
\]

Using a rectangular computational grid (Fig. 4), these simplified equations can then be numerically solved by the following scheme:

\[
C^\pm: (H_{i,t} - H_{i+1,t-\Delta t}) \pm \frac{a}{gA} (Q_{i,t} - Q_{i+1,t-\Delta t}) \pm a \Delta h_f = 0
\]

valid along \( \Delta x/\Delta t = \pm a \), respectively.

To take into account unsteady friction effects, the friction losses, \( h_f \), have been separated into two components

\[
h_f = h_{fs} + h_{fa} = \frac{fQ^2}{2gDA^2} + h_{fa}
\]

where \( h_{fs} \) = head loss for steady-state conditions (expressed in terms of square flow rate for turbulent flows); \( h_{fa} \) = head loss for unsteady-state conditions; and \( f \) = Darcy-Weisbach friction factor calculated for turbulent and laminar flow by (Swamee 1993):

\[
f = \left\{ \left( \frac{64}{R} \right)^8 + 9.5 \ln \left( \frac{\varepsilon}{3.7 D} + \frac{5.74}{R^{0.5}} \right) - \left( \frac{2500}{R} \right)^{0.125} \right\}^{-16}
\]

where \( \varepsilon \) = pipe roughness; and \( R \) = Reynolds number.

With regard to the head loss for unsteady-state conditions, two one-dimensional models, developed by Vítkovský et al. (2000) and Vardy and Brown (2007), are considered. The unsteady friction model proposed by Brunone et al. (1991) and improved by Vítkovský et al. (2000) is adequate for turbulent flows, is parameter-dependent, and is a function of both local and convective accelerations. Two numerical schemes can be implemented. Brunone et al. (1991) proposed grid interpolations, whereas a general scheme was used by Vítkovský et al. (2000) without grid interpolations. Brunone et al. (1991):

\[
h_{fa} = \frac{K_3}{gA} \left( \frac{\partial Q}{\partial t} - a \frac{\partial Q}{\partial x} \right)
\]

Vítkovský et al. (2000):

\[
h_{fa} = \frac{k'}{gA} \left( \frac{\partial Q}{\partial t} + a \cdot SGN(Q) \left( \frac{\partial Q}{\partial x} \right) \right)
\]

where \( K_3 \) and \( k' \) = decay coefficient; and \( SGN = \) operator for the sign of the average flow rate.

In Brunone et al.’s model, the characteristic lines \( C^+ \) and \( C^- \) represent two straight lines having different slopes (Fig. 5). The equation \( C^+ \) is valid if \( dx/dt = a/(1 + K_3) \), and the equation \( C^- \) is valid if \( dx/dt = -a \). Thereby, to lead a second compatibility equation (equation \( C^+ \)) in terms of the same two unknown variables at point \( P \), an interpolation procedure has to be applied to determine piezometric head and discharge at both points \( A \) and \( A' \).

The major disadvantage of the interpolations is that they introduce an artificial numerical damping to the solution. Higher-order interpolations are also possible, but they can introduce fluctuations, a more undesirable situation than the mathematical damping provided by the linear interpolation (Wylie and Streeter 1993).

To eliminate grid interpolations, Vítkovský et al.’s formula, which is an improvement of Brunone et al.’s formulation and more appropriate for turbulent flows in rough pipes, has been used to calculate the unsteady-state component of the head losses. This formulation requires the calculation of derivatives [Eq. (9)], and the characteristic grid, without interpolations and straight characteristic lines with slopes \( dx/dt = \pm a \), is shown in Fig. 4.

A numerical scheme proposed by Covas (2003) is used to calculate both the convective term [Eq. (10)] and the local term [Eq. (11)]:

\[
C^\pm, \frac{\partial Q}{\partial x} = \frac{Q_{i,t-\Delta t} - Q_{i+1,t-\Delta t}}{\Delta x}
\]

Fig. 4. Characteristic grid with specified time intervals

Fig. 5. Characteristic lines in the \( x, t \)-plane using Brunone’s \( K_3 \) unsteady friction model
where \( \theta = \) relaxation coefficient. If \( \theta = 0 \), the flow time derivative becomes explicit and unstable for certain conditions; if \( \theta > 0 \), the numerical scheme is implicit and unconditionally stable. To minimize computer storage and increase computational speed, the author have considered \( \theta = 1 \), and the same assumption has been adopted in this study.

Zielke (1968) presented a formulation of the wall shear stress, which relates the unsteady friction component to the local acceleration and a weighting function as follows:

\[
h_{fu}(t) = \frac{16\nu}{gD^2} \left( \frac{\partial V}{\partial t} + W \right)(t)
\]

where \( \nu = \) kinematic viscosity; \( V = \) average velocity; \( W = \) weighting function; and \( * \) denotes convection, which indicates that the unsteady friction term is a convolution of past fluid accelerations with a weighting function. Recently, Vardy and Brown (2007) formulated the weighting function as follows:

\[
W(\lambda) = \sum_{i=1}^{17} m_i e^{-n_i \lambda}
\]

where \( m_i = A^* m_i^*; \ n_i = B^* + n_i^*; \) and \( \lambda = 4\nu t/D^2 \). The coefficients \( m_i^*; n_i^*; A^*; \) and \( B^* \) are defined as in Vardy and Brown (2007).

### Transients in Multipipe Systems: Generalized Elastic Model

The set of compatibility equations [Eq. (5)] can be solved numerically by a general simplified linear form for the linear-elastic conduit useful for complex multipipe systems

\[
C^+ = C + C_{a, h, \theta}
\]

\[
C^- = C + C_{a, h, \theta}
\]

where \( i = \) pipe section; and \( C, C_a, C_h, \) and \( C_{\theta} = \) coefficients that depend on the numerical scheme used to describe steady-state friction and the unsteady friction model adopted. These constants, in a generic form, can be defined as follows (Covas 2003; Soares et al. 2008):

<table>
<thead>
<tr>
<th>Friction model</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state friction</td>
<td>( C_p = Q_{i-1, r-\Delta t} + C_{a, h, r-\Delta t} + C_{\theta} )</td>
</tr>
<tr>
<td>Frictionless</td>
<td>( C_{p} = 0 )</td>
</tr>
<tr>
<td>First-order accuracy</td>
<td>( C_{p} = -R\Delta t(Q_{i-1, r-\Delta t}) )</td>
</tr>
<tr>
<td>Unsteady friction</td>
<td>No unsteady friction</td>
</tr>
<tr>
<td>Vinkovský et al. model</td>
<td>( C_{p} = 0 )</td>
</tr>
<tr>
<td>Vardy-Brown model</td>
<td>( C_{p} = gA\Delta t(16\nu/gD^2)\sum_{i=1}^{17}(e^{-n_i\Delta t})\sum_{j=1}^{17}(m_{kj}/A)Q_{i,j-1} )</td>
</tr>
</tbody>
</table>

### Table 1. Coefficients \( C_{p1}, C_{p2}, C_{N1}, \) and \( C_{N2} \)

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These parameters may be plotted with $\omega$ for different values of specific speeds $N_s$ (Fig. 6):

$$F_h = \frac{h}{n^2 + q^2}; \quad F_b = \frac{b}{n^2 + q^2}; \quad N_s = \frac{N_b Q_b}{H_R^{1/4}}$$  \hspace{1cm} (22)

In this study, the suction line of the pump is short compared with the discharge line, and thus the water hammer waves in this line may be neglected. Referring to Fig. 7, the following equations can be written

$$H_{pi} = H_{suc} + H_p - \Delta H_{pv} \quad (23)$$

$$\Delta H_{pv} = C_v|Q_{pi}|Q_{pi} | \quad (24)$$

$$Q_p = Q_{pi} = C_n + C_{ai}H_{pi} \quad (25)$$

$$C_n = Q_B - C_{ai}H_B - R\Delta t|Q_B| \quad (26)$$

where $H_{suc} =$ height of the liquid surface in the suction reservoir above datum; $H_p =$ pumping head at the end of the time step; $\Delta H_{pv} =$ head loss in the discharge valve; and $C_v =$ head-loss coefficient in the valve.

**Internal Condition: Check Valve**

A check valve was installed downstream of the pump. Further experimental analysis in the actual system is necessary to determine valve coefficients related to the inertial effects of accelerating and decelerating flow through the valve closing; however, this was outside of the scope of the current study. The check valve was described by an in-line valve with a steady-state orifice equation, and the closure time was calibrated using observed data. A schematic representation of the in-line valve is shown in Fig. 8. According to Wylie and Streeter (1993), the following equations are defined for a single check-valve modeling:

For positive flow $\rightarrow (C_A - C_B) \geq 0$

$$Q_{pi} = -C_{iv}(B_A + B_B) + \sqrt{C_{iv}^2(B_A + B_B)^2 + 2C_{iv}(C_A - C_B)} \quad (27)$$

For negative flow $\rightarrow (C_A - C_B) < 0$

$$Q_{pi} = C_{iv}(B_A + B_B) - \sqrt{C_{iv}^2(B_A + B_B)^2 - 2C_{iv}(C_A - C_B)} \quad (28)$$

in which

$$C_A = H_A + BQ_A \quad (29)$$

$$C_B = H_B - BQ_B \quad (30)$$

$$B_A = B + R'|Q_A| \quad (31)$$

$$B_B = B + R'|Q_B| \quad (32)$$

Fig. 6. Pump characteristic curves for different specific speeds: (a) pumping pressure head; (b) net torque

Fig. 7. Pump with short suction line

Fig. 8. Dynamic check valve modeled as an in-line valve
\[ R' = \frac{f \Delta x}{2gDA^2} \]  \hspace{1cm} (33) \\
\[ B = \frac{a}{gA} \]  \hspace{1cm} (34) \\
\[ C_{ie} = \left( \frac{Q_0 \tau}{2} \right)^2 / (2H_0) \]  \hspace{1cm} (35)

where \( \tau \) = dimensionless valve-opening coefficient specified as a function of time (\( \tau = 1 \) for fully opening; \( \tau = 0 \) for valve completely closed); and \( H_0 \) = steady-state drop in hydraulic grade line across the valve with a flow of \( Q_0 \) when \( \tau = 1 \).

**Downstream Boundary Condition: Variable-Level Tank**

The last reach of the pipeline discharges to the atmosphere before the IPG storage tank. The final sections of the pipeline can be described as variable-level tanks with free discharge to the atmosphere and constant cross-sectional area. Similar consideration has been presented in the literature by Di Santo et al. (2002) for rising mains with an air chamber. However, the authors have not taken into account the unsteady friction losses. The following equations can be written for the downstream boundary condition (Fig. 9):

\[
\begin{align*}
Q_p &= C_p - C_a H_{pi} - R \Delta t Q_0 |Q_0| \\
C_p &= Q_0 + C_a H_{pi} - R \Delta t Q_0 |Q_0| \\
Q_d &= H_{pi} \sqrt{2g(H_{pi} - H_T)} \\
Q_d &= H_{pi} + 0.5(\Delta t / A_s)(Q_{pi} + Q_s) \\
Q_d &= 0
\end{align*}
\]  

(36a) \hspace{1cm} (36b)

where \( Q_d \) = discharge into the downstream storage tank; \( A_s \) = cross-sectional area of the vertical tank; \( H_T \) = elevation of the tank top; \( H_s \) = tank water level at the beginning of time step; and \( Q_s \) = flow rate at the beginning of time step.

**Model Calibration**

**Pump-Motor Unit Inertia**

The Prado pumping station is composed of five pumps installed in parallel. The field tests have been carried out by using only pump 1, which is a submersible pump with the following nominal parameters: pump discharge \( Q_R \) = 300 m³/h, pumping head \( H_R \) = 105 m, power \( P_R \) = 110 kW, rotational speed \( N_R \) = 3,000 rpm, and efficiency \( \eta_R \) = 0.78. Considering these nominal values, the pump-motor inertia was calculated by the Thorley and Faithfull (1992) formulation, in which \( I = I_1 + I_2 \), where \( I_1 \) = estimated inertia of the pump impeller and fluid; and \( I_2 \) = inertia of the motor, given by

\[
I_1 = 0.038 \left( \frac{P_R}{(N_R/1,000)^{0.96}} \right) = 0.038 \left( \frac{110}{300} \right)^{0.96} = 0.146 \text{ kg} \cdot \text{m}^2
\]  \hspace{1cm} (37)

\[
I_2 = 0.0043 \left( \frac{P_R}{(N_R/1,000)^{1.48}} \right) = 0.0043 \left( \frac{110}{300} \right)^{1.48} = 0.888 \text{ kg} \cdot \text{m}^2
\]  \hspace{1cm} (38)

The total inertia for the pump-motor unit was accordingly estimated as \( I = 1.034 \text{ kg} \cdot \text{m}^2 \).

**Check-Valve Closure**

Because of the pump trip, the flow in the discharge line reduces rapidly to zero. To prevent reverse flow through the pump, the check valve installed downstream of the pump closes completely. This causes a stoppage of the flow with the corresponding pressure rise. In this study, the time of closure and the corresponding \( \tau \) versus \( t \) curve were calibrated. Fig. 10(a) depicts the calibrated check-valve closure, and Fig. 10(b) the respective flow rate at location Q1. After the pump trip, the check valve closes completely in 1.77 s.

**Wave-Speed Estimation**

The second parameter to be determined and calibrated is the elastic wave speed, which can be estimated by theoretical formulas with the modulus of elasticity provided by the manufacturer of the pipes

**Fig. 9. Variable-level tank**

**Fig. 10. (a) Calibrated check-valve closure; (b) the respective flow rate at location Q1**
(Chaudhry 1987; Wylie and Streeter 1993). The celerity was estimated as 1,130 m/s by using Eq. (3) and by considering the following parameters for cast-iron pipes: pipe class K9 with elastic joint, nominal diameter of 500 mm, external pipe diameter of 532 mm; iron wall thickness $e = 9$ mm, cement lining thickness of 4.5 mm (i.e., total wall thickness of 13.5 mm), internal diameter of 505 mm; bulk modulus of elasticity of water $K_2 = 2.19$ GPa; mass density of water $\rho = 999$ kg/m$^3$; modulus of elasticity of the cast-iron pipe $E_0 = 170$ GPa; and Poisson’s coefficient of the cast-iron pipe equal to 0.25. Whereas the PVC section length is very short when compared with the total length of the pipeline (16/2,241 = 0.7%), its effect was taken into account. The wave speed was estimated as 428 m/s considering the parameters for the PVC pipes: nominal diameter of 200 mm, external pipe diameter of 222 mm; pipe-wall thickness $e = 8.9$ mm, internal diameter of 204.2 mm; modulus of elasticity of the PVC pipe $E_0 = 3.6$ GPa; and Poisson’s coefficient equal to 0.46.

The wave speed of 1,130 m/s resulted in a computational scheme with $\Delta t = 0.009823$ s, $\Delta x = 11.1$ m (202 pipe reaches between the pump and the downstream storage tank), and number of Courant-Friedrichs-Lewy stability condition equal to 1, which indicates that wave-speed adjustments/grid interpolations were not used.

To analyze the pressure transients in the system, the following two scenarios have been considered:

1. Constant-level reservoir as the downstream boundary condition and calculating the head losses with and without the unsteady friction component—in this case, the IPG tank is directly linked to the pipeline.
2. Three variable-level tanks as the downstream boundary condition and considering unsteady friction effects—in this case, the three cells of the IPG storage tank are not linked to the pipeline, which discharges to the atmosphere (actual system).

All the field tests were carried out with the relief valve out of service (i.e., by closing the gate valve immediately at the downstream end). In this way, such a protection device did not influence the system behavior, which reduced the uncertainties related to the different effects in pressure variations, such as the attenuation and dispersion attributable to unsteady friction and the pressure relief attributable to the variable-level tanks.

**Scenario 1: Constant-Level Reservoir as Downstream Boundary Condition**

In the first attempt to calibrate the transient solver, a constant-level reservoir was assumed as the downstream end boundary condition. The transient event was simulated by using both the classic elastic model (with only a steady-state friction term) and the elastic model taking into account unsteady friction (UF) losses (Vítkovský et al. 2000; Vardy and Brown 2007). The decay coefficient of the Vítkovský model, $k'$, was calibrated as 0.020. With regard to steady-state friction losses, the actual pipe roughness, $\varepsilon$, was determined on the basis of steady-state measurements of pressure head on both points P1 and P2. The head loss for steady-state conditions between points P1 and P2 was determined as 0.85 m, which is the difference between the piezometric heads in P1 (874.00 + 104.35 = 978.35 m) and P2 (977.50 m). The Darcy-Weisbach formulation has been used for head-loss calculations ($L = 2.225$ m; $Q_0 = 72$ L/s; $D = 0.505$ m), in which the friction factor was determined by the Swamee (1993) formulation [Eq. (7)]. The pipeline roughness, $\varepsilon$, was thus determined as 2 mm. Comparisons between numerical results obtained by using the elastic transient solver with collected pressure data immediately downstream of the check valve (location P1) are presented in Fig. 11. Transient pressures observed in the location P2 are presented in Fig. 12, as well as the numerical results obtained by the elastic model with a constant-level reservoir as the downstream end boundary condition.

Initial minimum pressures are described accurately by the elastic models, but the attenuation of transient pressures cannot be described by any of these models, even when unsteady friction is incorporated in the transient solver. Compared with the classic elastic model (with quasi-steady friction), the elastic model with unsteady friction results in higher attenuation of pressure peaks.

![Observed pressure heads at location P1 versus numerical results of both classic elastic model and elastic model taking into account unsteady friction (constant-level reservoir as downstream boundary condition) ($Q_0 = 72$ L/s; $R \approx 182,000$)](image_url)
and delays the transient pressure wave; however, the shape of the transient pressure signal cannot be described (see the first pressure peak, “Detail” in Fig. 11) and tends to get worse as the transient propagates along the pipeline.

Possible phenomena, such as viscoelasticity and fluid-structure interaction (pipe displacements), could occur in the PVC pipes because PVC is characterized by viscoelastic rheological behavior. Further discussion on the behavior of the transient pressure signal is needed. First, the viscoelasticity effect smooths down maximum/minimum transient pressures. This behavior is not verified in the transient pressures observed, thus excluding the application of a viscoelastic transient solver. There is a delay in the pressure wave, which can be ascribed to the unsteady friction losses effect because the majority of the pipeline is composed of cast-iron pipes (2,225 m of cast-iron pipes and 16 m of PVC pipes). However, the unsteady friction models used in this paper are not able to reproduce the pressure relief observed in the first pressure peak (“Detail” in Fig. 11), which propagates along time.

The key question is, thus, the physical justification for the pressure relief in observed maximum transient pressures (which is exactly the opposite behavior observed in the line-packing effect). When scrutinizing the transient pressures measured at location P2 (Fig. 12), it is shown that a pressure relief occurs after the pressure wave has arrived at that location. In addition, the pressure head established in approximately 0.5 m after 150–160 s, which indicates that the water column in the PVC vertical pipe branches is approximately 0.5 m—this is less than the vertical length of the final sections of the pipeline (three PVC pipe branches with a nominal diameter of 200 mm and a vertical length of approximately 3.0 m). It has been considered that, after the pump was switched off and while the pressure wave did not arrive at the downstream end, water was still delivered to the downstream end storage tank. After the pressure wave has arrived at the downstream end of the pipeline, the discharge into the downstream storage tank approaches zero, and the water column in the PVC pipe branches oscillates. The pressure wave travels along the pipeline from the downstream to upstream direction and passes by location P1 with a relief in the maximum pressure and behavior slightly distorted, as is shown in the observed pressure data (“Detail” in Fig. 11).

This is because there is a loss of water volume while the first pressure wave does not reach the final section of the pipeline. After the pressure wave reaches the downstream end, the water level inside the three PVC pipes oscillates and then relieves the maximum/minimum pressures. The solution proposed in this study is thus to model the water-level variation in the final section by variable-level tanks with free discharge to the atmosphere. This solution does not require any characteristic grid modification during simulations because the water column in the pipeline shrank and changing grids can produce discontinuities in the numerical results.

Scenario 2: Variable-Level Tanks with Free Discharge at the Downstream End

A second attempt to calibrate the hydraulic transient solver was carried out by using three vertical tanks with variable levels and free discharge into the IPG tank as the downstream end boundary condition. The inner diameter of the three small vertical tanks was equal to the inner diameter of the three PVC pipe branches in the final section of the pipeline: 204.2 mm. The elevation of the tank top was assumed as the difference between the elevations of both the IPG tank (977.50 m) and Prado pumping station (874.00 m)—that is, \( H_T = 103.5 \) m.

The head losses were calculated by using both the steady and unsteady components, in which Vítkovský et al.’s (2000) formulation has been used for the unsteady friction losses component.

Fig. 13 shows the observed transient pressures for location P1 and the numerical results obtained by using the elastic model with the decay coefficient adjusted as \( k' = 0.033 \). The numerical results fit extremely well with collected pressure data, and the model can describe transient pressure wave attenuation, dispersion, and shape. The pressure relief in the first maximum pressure peak can be described by the model when the downstream end boundary condition is defined as three variable-level tanks. However, some discrepancy still remains, possibly attributable to the description of the boundary condition (rigid) neglecting the elasticity of both water and pipe wall. Transient pressures observed in location P2 are presented in Fig. 14, as well as the numerical results obtained by the elastic model considering UF and three variable-level tanks as the downstream end boundary condition. The adoption of variable-level tanks gives the pressure head variation observed at the downstream end of the pipeline. Fig. 15 shows the dimensionless observed traces \( \left( \frac{H - H_0}{\Delta H_f} \right) \) where \( H_0 = \text{steady-state head} \); and \( \Delta H_f = \text{theoretical Joukowsky overpressure:} \) \( \Delta H_f = aQ_0/gA \) at locations P1 and P2, as well as the computed pressure heads and discharge to the downstream end storage tank for the first 10 s. Although the pressure head falls because of the pump trip, the discharge into the IPG tank remains at 72 L/s in the first 2 s.

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**Fig. 12.** Observed pressure heads at location P2 versus numerical results of both classic elastic model and elastic model taking into account unsteady friction (constant-level reservoir as downstream end boundary condition) \( (Q_0 = 72 \text{ L/s}; R \approx 182,000) \)

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**Fig. 13.** Observed pressure heads at location P1 versus numerical results of the elastic model taking into account unsteady friction and three variable-level tanks as downstream end boundary condition \( (Q_0 = 72 \text{ L/s}; R \approx 182,000) \)
Numerical results of the elastic model were fitted to the observed pressures, it is not accurate for water-pumping system diagnosis, on the safe side for design purposes because it predicts higher overpressures, it is not satisfactory. Although such an assumption is approximate 0.6 s.

Conclusions

The current paper presented field tests and numerical analysis of water hammer in a water-pumping system. A hydraulic transient solver that takes into account unsteady friction and three variable-level tanks as downstream end boundary condition \( (Q_0 = 72 \text{ L/s}; R \approx 182,000) \) data in terms of damping and phase shift of pressure waves when unsteady friction effects were taken into account. However, the key factor was the modeling of the pressure relief observed on maximum pressure heads. After the pump was switched off and while the pressure wave did not arrive at the downstream end, water was still delivered to the downstream end tank. After the pressure wave has arrived at the downstream end of the pipeline, the flow reverses and the water column in the PVC pipe branches oscillates. This was overcome by modeling the final sections of the pipeline, composed with three PVC pipe branches, as three small vertical tanks with a variable level and a free discharge into the downstream end reservoir. The numerical results have shown that the pressure head at the downstream end of the pipeline oscillated and caused a pressure relief.

Analyses carried out in this work have shown that unsteady friction effects in water pipeline systems have to be better studied. The use of new numerical methods instead of the well-known method of characteristics can be also the solution for the problems related to the discrepancies between numerical results and observed data when considering the variable-level tanks.

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Notation

The following symbols are used in this paper:

- \( A \) = pipe cross-sectional area;
- \( A_s \) = cross-sectional area of the vertical tank;
- \( A^*, B^* \) = Vardy-Brown weighting function coefficients;
- \( a \) = elastic wave speed;
- \( C_v \) = coefficient of head losses in the valve;
- \( D \) = pipe inner diameter;
- \( E_0 \) = Young’s modulus of elasticity of pipe;
- \( e \) = pipe-wall thickness;
- \( F_h, F_b \) = head and torque Suter parameters;
- \( f \) = Darcy-Weisbach steady-state friction factor;
- \( g \) = gravitational acceleration;
- \( H \) = piezometric head;
- \( H_R \) = nominal pumping head;
- \( H_s \) = tank water level at the beginning of time step;
- \( H_{\text{vac}} \) = height of the liquid surface in the suction reservoir above datum;
- \( H_T \) = elevation of the tank top;
- \( H_0 \) = steady-state drop in hydraulic grade line across the valve;
- \( h_j \) = head loss per unit length;
- \( h_{fs} \) = head loss for steady-state conditions;
- \( h_{fu} \) = head loss for unsteady-state conditions;
- \( I \) = pump-motor inertia;
- \( K_s \) = bulk modulus of elasticity of the fluid;
- \( K_3 \) = decay coefficient of Brunone et al.’s (1991) unsteady friction formulation;
- \( k' \) = decay coefficient of Vítkovsky et al.’s (2000) unsteady friction formulation;
- \( L \) = pipe length;
$m_i$, $n_i$ = exponential sum coefficients;  \\
$N$ = rotational speed;  \\
$N_R$ = nominal pump rotational speed;  \\
$N_s$ = specific rotational speed;  \\
$P_R$ = nominal pump power;  \\
$Q$ = flow rate;  \\
$Q_d$ = discharge into the downstream reservoir;  \\
$Q_R$ = nominal pump discharge;  \\
$Q_s$ = flow rate at the beginning of time step;  \\
$Q_0$ = initial steady-state flow rate;  \\
$R$ = Reynolds number;  \\
$T$ = net torque;  \\
$t$ = time;  \\
$V$ = average velocity;  \\
$W$ = weighting function;  \\
$x$ = coordinate along the pipe axis;  \\
$z$ = elevation;  \\
$\Delta H_J$ = theoretical Joukowsky overpressure;  \\
$\Delta H_{Ps}$ = head loss in the discharge valve;  \\
$\Delta t$ = time-step increment;  \\
$\Delta x$ = space-step increment;  \\
$\varepsilon$ = pipe-wall roughness;  \\
$\eta_R$ = nominal pump efficiency;  \\
$\theta$ = relaxation coefficient for the flow-time derivative calculation (Vítkovský’s unsteady friction numerical scheme);  \\
$\lambda$ = dimensionless time (= $4\nu t/D^2$);  \\
$\nu$ = kinematic viscosity;  \\
$\rho$ = mass density of the fluid;  \\
$\tau$ = dimensionless valve-opening coefficient;  \\
$\psi$ = dimensionless parameter (function of pipe cross-section dimensions and constraints); and  \\
$\omega$ = angle of pump operation.

References


