Nonlinear Theories of Beams and Shells

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Acknowledgements

CNPq, CAPES and EPUSP, Brasil
IST, ICIST and GRICES, Portugal
DFG, Germany
CISM, Italy

Prof. Peter Wriggers (Germany)
Dr. Carlos Tiago (Portugal)
Dr. Eduardo Campello (Brasil)
M. Sc. Evandro Dasambiagio (Brasil)
Lecture Topics

1st Lecture (Monday):
1. Introduction
2. Elements of Solid Mechanics
3. Finite Rotations

2nd Lecture (Monday):
4. Basic Beam Theory

3rd Lecture (Monday):
5. Basic Shell Theory
6. Initially curved shells
Lecture Topics

4th Lecture (Tuesday):
7. Advanced Beam Models
8. Advanced Shell Models

5th Lecture (Tuesday):
10. Beam and Shell Dynamics

6th Lecture (Friday):
11. EFG Method
12. Final Remarks
Introduction

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Some Pioneers on Beams and Shells

Motivation

Galileo Beam (1564-1642)

Firth of Forth Bridge (Scotland – 1890)

Akashi Kaikyo Bridge (Japan – 1998 AD)

Tacoma Narrows Bridge (USA -1940 AD)
Motivation

- Pantheon (Rome – 27 BC)
  World’s largest concrete dome

- Sancta Sophia (Instanbul – 537 AD)

- San Pietro (Rome – 1564 AD)
  World’s largest church

- Santa Maria del Fiore (Florence -1434 AD)
  World’s largest masonry dome
Motivation (Europe)

Frauenkirche (Dresden - 1743 and 2000 AD)
St-Paul’s Cathedral (London – 1708 AD)

St-Isaac’s Cathedral (St-Petersburg – 1808 AD)
Les Invalides (Paris – 1670 AD)
Motivation

National Shrine of Brazil (ca. São Paulo – 1955)
World’s second largest church

Notre Dame de la Paix de Yamoussoukro (Ivory Coast – 1990 AD) world’s tallest church

São Paulo’s Cathedral (1954 AD)
Motivation

Taj Mahal (Agra – 1641AD)

Capitol (Washington DC – 1800 AD)
World’s largest cast iron dome

Dome of the Rock (Jerusalem – 691AD)

La Rotonda de Palladio (Vicenza – 1591 AD)
Motivation

Grötzingen (Germany)

Xochimilco (Mexico)

Munich

Sydney Opera House
Motivation

Kingdome (Seattle – 1976)
World’s largest RC dome

Louisiana Superdome (New Orleans – 1975)
World’s largest steel frame dome

Superior Dome (Michigan – 1991)
World’s largest wood dome

O₂ Dome (London – 2000)
World’s largest cable supported dome
Motivation (Brazil)

Brazilian Parliament (Brasília)

NovoMuseu (Curitiba)

Contemporary Art Museum (Niterói)

Japanese Immigration Gate (Santos)
Motivation

ship's propeller

sheet metal forming

aircraft engine
Motivation

automobile industry
Motivation

automobile industry
Motivation

food industry
Motivation

aerospace industry
Motivation

aerospace industry
Features

- Fully nonlinear beam and shell models with large strains and rotations.
- Consistent derivation from Solid Mechanics with internal constrains.
- Beam and shell model assumptions are the only kinematical approximations (geometrically exact models).
- Vector-like rotation parameterization (easy connection of beam and shell models).
- Basic quantities: 1st Piola-Kirchhoff stress tensor and displacement gradient tensor.
- Cross-sectional resultant formulation.
Features

Variationally consistent formulation of equilibrium (or motion).

Two equilibrium weak forms: virtual work vs. virtual power.

Two rotation approximations: total and incremental.

Variationally consistent derivation of the tangent bilinear weak form in Statics. Symmetric (VWT) or non-symmetric (VPT) tangent operators are obtained.

Small strain beam model: rigid cross section model (6 DOF’s) and model with cross section out-of-plane warping (7 DOF’s).

Consistent large strain beam model. Additional parameters are introduced at model level. General in-plane distortion and out-of-plane warping of the cross section are allowed for.
Features

- Consistent small strain shell model with constant thickness (6 DOF’s).
- Constant thickness shell model (6 DOF’s). Formulation of a plane stress condition consistent with large strains.
- Variable thickness shell model suitable for large strain elasticity and plasticity (8 DOF’s). Additional parameters are introduced at model level.
- Higher-order shell models suitable for sandwich and composite shells. Additional parameters must be introduced at model level.
- Unified framework for beam and shell models suitable for conjunct programming within a FE code.
Features

- Initial beam and shell curvatures described by means of an initial stress-free configuration (no need of curvilinear coordinates, Christoffel symbols and differential geometry, no issue on physical components)

- Simple momentum and energy conserving schemes for the time integration of the equations of motion.

- Variationally consistent derivation of the tangent bilinear weak form in Dynamics.

- Kirchhoff-Love shell model using the same framework and MEF.

- Smooth beam and shell approximation with EFG method.