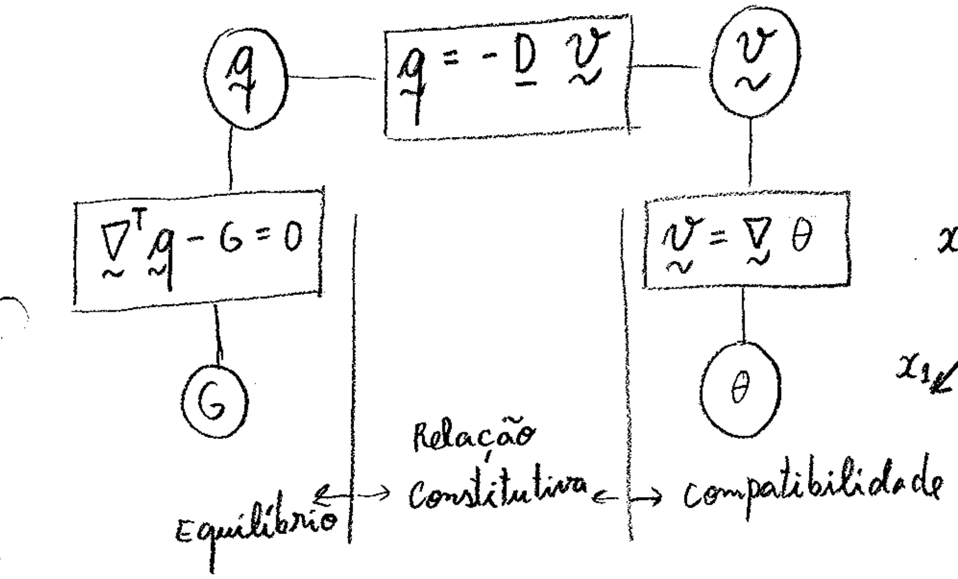
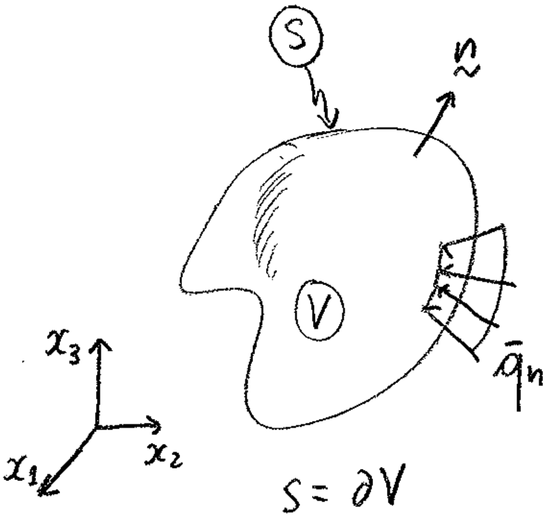
O Problema da condução de calor

A forma **forte** é dada por

Em V





$$S = \partial V$$

$$S = S_q \cup S_\theta$$

Em S

$$q_n + \bar{q}_n = 0 \qquad\qquad \theta = \bar{\theta}$$

onde

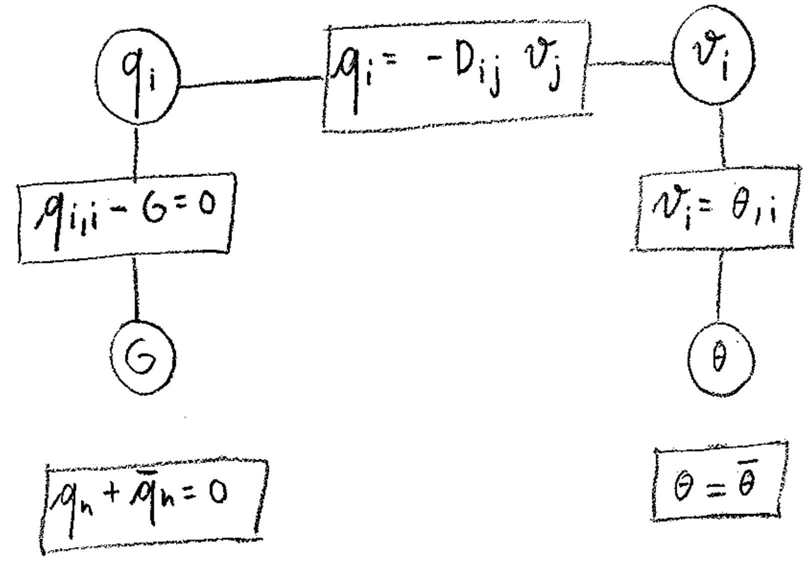
$$\underset{\sim}{\nabla} = \begin{Bmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{Bmatrix} ; \quad \underset{\sim}{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} ; \quad \underline{D} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} ; \quad \underset{\sim}{v} = \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix}$$

$$q_n = \underset{\sim}{q}^T \underset{\sim}{n}$$

$\bar{q}_n$ é o fluxo prescrito que <u>entra</u> no corpo

G é o calor <u>fornecido</u> por unidade de volume.

Em notação indicial escreve-se



e $q_n = q_i n_i$

A forma fraca das equações de equilíbrio é dada por

$$\int_V \delta\theta \left(\underset{\sim}{\nabla}^T \underset{\sim}{q} - G \right) dV - \int_{S_q} \delta\theta \left(q_n + \bar{q}_n \right) dS_q = 0 \qquad (1)$$

Considerando que

$$\int_V f\, \underset{\sim}{\nabla}^T \underset{\sim}{g}\, dV = \int_V f\, g_{i,i}\, dV = \int_V (f\, g_i)_{,i}\, dV - \int_V f_{,i}\, g_i\, dV =$$

$$= \int_S f\, g_i\, n_i\, dS - \int_V f_{,i}\, g_i\, dV = \int_S f\, g_n\, dS - \int_V f_{,i}\, g_i\, dV =$$

$$= \int_S f\, g_n\, dS - \int_V \underset{\sim}{\nabla}^T f\, \underset{\sim}{g}\, dV \;, \quad \forall\, f, g \quad \text{e} \quad g_n = \underset{\sim}{g}^T \underset{\sim}{n}$$

Vem, fazendo $f = \delta\theta$ e $\underset{\sim}{g} = \underset{\sim}{q}$

$$\int_S \delta\theta \, q_n \, dS - \int_V \underset{\sim}{\nabla}^T \delta\theta \, \underset{\sim}{q} \, dV - \int_V \delta\theta \, G \, dV - \int_{S_q} \delta\theta \, q_n \, dS_q - \int_{S_q} \delta\theta \, \bar{q}_n \, dS_q = 0 \Leftrightarrow$$

$$\Leftrightarrow \cancel{\int_{S_q} \delta\theta \, q_n \, dS_q} + \int_{S_T} \delta\theta \, q_n \, dS_T - \int_V \underset{\sim}{\nabla}^T \delta\theta \, \underset{\sim}{q} \, dV - \int_V \delta\theta \, G \, dV - \cancel{\int_{S_q} \delta\theta \, q_n \, dS_q} - \int_{S_q} \delta\theta \, \bar{q}_n \, dS_q = 0$$

Impondo $\delta\theta \big|_{S_\theta} = 0$,

$$-\int_V \underset{\sim}{\nabla}^T \delta\theta \, \underset{\sim}{q} \, dV - \int_V \delta\theta \, G \, dV - \int_{S_q} \delta\theta \, \bar{q}_n \, dS_q = 0 \qquad (2)$$

A discretização de T é efectuada através de

$$\theta = \underline{\Psi} \, \underset{\sim}{\theta} \qquad (3)$$

Se $\underset{\sim}{q} = \underset{\sim}{q}(\theta)$ é <u>não-linear</u>, então a discretização de (2) conduz a

$$-\int_V \underset{\sim}{\nabla}^T(\underline{\Psi}\,\delta\underset{\sim}{\theta})\,\underset{\sim}{q}\,dV - \int_V \underline{\Psi}\,\delta\underset{\sim}{\theta}\,G\,dV - \int_{S_q} \underline{\Psi}\,\delta\underset{\sim}{\theta}\,\bar{q}_n\,dS_q = 0 \Leftrightarrow$$

$\underline{B} = \underset{\sim}{\nabla}\underline{\Psi}$

$$\Leftrightarrow -\delta\underset{\sim}{\theta}^T\int_V (\underset{\sim}{\nabla}\underline{\Psi})^T\,\underset{\sim}{q}\,dV - \delta\underset{\sim}{\theta}^T\int_V \underline{\Psi}^T G\,dV - \delta\underset{\sim}{\theta}^T\int_{S_q} \underline{\Psi}^T\,\bar{\underset{\sim}{q}}_n\,dS_q = 0 \Leftrightarrow$$

$$\Leftrightarrow \delta\underset{\sim}{\theta}^T\underbrace{\left\{-\int_V \underline{B}^T\,\underset{\sim}{q}\,dV - \int_V \underline{\Psi}^T G\,dV - \int_{S_q} \underline{\Psi}^T\,\bar{q}_n\,dS_q\right\}}_{\underset{\sim}{r}(\underset{\sim}{\theta})} = 0 \Leftrightarrow$$

$$\Leftrightarrow \delta\underset{\sim}{\theta}^T\,\underset{\sim}{r}(\underset{\sim}{\theta}) = 0 \Leftrightarrow \delta\underset{\sim}{\theta}^T = \underset{\sim}{0} \quad \vee \quad \underset{\sim}{r}(\underset{\sim}{\theta}) = \underset{\sim}{0}$$

Se $\underset{\sim}{q} = -\underline{D}\,\underset{\sim}{\nabla}$, i.e., $\underset{\sim}{q} = \underset{\sim}{q}(\theta)$ é linear, então a discretização de (2) conduz a

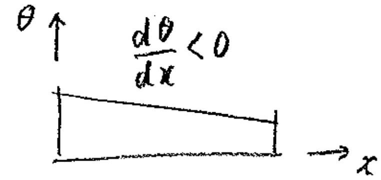
$$-\int_V \underset{\sim}{\nabla}^T(\underline{\Psi}\,\delta\underset{\sim}{\theta})\left(-\underline{D}\,\underset{\sim}{\nabla}(\underline{\Psi}\,\underset{\sim}{\theta})\right)dV - \int_V \underline{\Psi}\,\delta\underset{\sim}{\theta}\,G\,dV - \int_{S_q} \underline{\Psi}\,\delta\underset{\sim}{\theta}\,\bar{q}_n\,dS_q = 0 \Leftrightarrow$$

$$\Leftrightarrow \delta\underset{\sim}{\theta}^T\left\{\underbrace{\int_V \underline{B}^T\underline{D}\,\underline{B}\,dV}_{\underline{K}}\,\underset{\sim}{\theta} - \underbrace{\int_V \underline{\Psi}^T G\,dV}_{\underset{\sim}{f}^V} - \underbrace{\int_{S_q} \underline{\Psi}^T\,\bar{q}_n\,dS_q}_{\underset{\sim}{f}^{S_q}}\right\} = 0 \Leftrightarrow$$

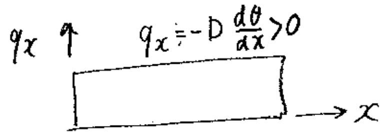
$$\Leftrightarrow \delta\underset{\sim}{\theta}^T\left\{\underline{K}\,\underset{\sim}{\theta} - \left(\underset{\sim}{f}^V + \underset{\sim}{f}^{S_q}\right)\right\} = 0 \Leftrightarrow \delta\underset{\sim}{\theta}^T = \underset{\sim}{0} \quad \vee \quad \underline{K}\,\underset{\sim}{\theta} = \underset{\sim}{f}^V + \underset{\sim}{f}^{S_q}$$

Num problema 1D com a distribuição de temperaturas

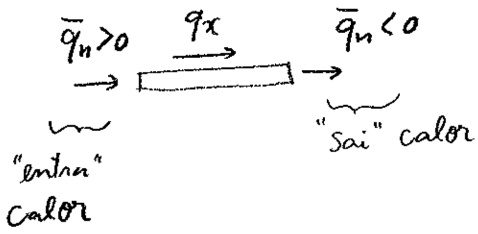
θ



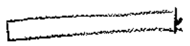
q_x



O fluxo é



Nas extremidades tem-se



$q_n = \underbrace{n_x}_{=-1} \underbrace{q_x}_{>0} < 0$

A C.F. é

$\underbrace{q_n}_{<0} + \underbrace{\bar{q}_n}_{>0} = 0$

$q_n = \underbrace{n_x}_{=+1} \underbrace{q_x}_{>0}$; A C.F. é $\underbrace{q_n}_{>0} + \underbrace{\bar{q}_n}_{<0} = 0$

Bibliografia:

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Bathe, "The FE Procedures", pag. 642

Cook, "Concepts and applications of the FEM...", pag. 454

Reddy, "An introduction to the FEM", 3ª edition, pag. 458