

GBTUL 1.0 β

Buckling and Vibration Analysis of Thin-Walled Members

GBT THEORETICAL BACKGROUND

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1. Introduction

The code GBTUL 1.0 β , which implements recently developed formulations of the Generalized Beam Theory (GBT), performs linear buckling (bifurcation) and vibration analyses of elastic thin-walled members.

The objective of the *GBT Theoretical Background* is to provide the user an insight into some theoretical aspects of GBT that are helpful for using the program. References are made to publications where the GBT formulations are expounded in detail.

GBTUL 1.0 β (acronym for “GBT at the Technical University of Lisbon”) is a freeware code, meant to provide the users with a graphical and easy-to-use structural analysis tool. Being based on GBT, it allows the users to benefit from the technique’s unique modal features. For more information, including access to the electronic forms of the manuals referenced above, visit the program website <http://www.civil.ist.utl.pt/gbt>.

2. GBT: Overview

Generalised Beam Theory (GBT) may be viewed as either (i) a *bar theory* incorporating cross-section in and out-of-plane deformations or (ii) a *folded-plate theory* that includes plate rigid-body motions (*e.g.*, [1]-[3]). By decomposing the member deformed configurations or buckling/vibration mode shapes into linear combinations of longitudinally varying cross-section *deformation modes*, GBT provides a general and elegant approach to obtain accurate solutions for several structural problems involving prismatic thin-walled members – moreover, one also obtains the contributions of each deformation mode, a feature enabling a much clearer interpretation of the structural response under consideration.

Consider the prismatic thin-walled member depicted in Figure 1(a), which has a supposedly arbitrary *open*¹ cross-section – also shown is the member global coordinate system X - Y - Z (longitudinal, major and minor axis). Moreover, local coordinate systems x - s - z are adopted in each wall, as shown in Figure 1(b), where (i) x (parallel to X) and s define the wall mid-plane and (ii) z is measured along the thickness h – when expressed in this coordinate system, the mid-line displacement field components are designated as u , v and w . Moreover, the member Young’s moduli, Poisson’s ratios, distortion modulus and volumetric mass density are, for

¹ *I.e.*, a cross-section that does not contain closed cells.

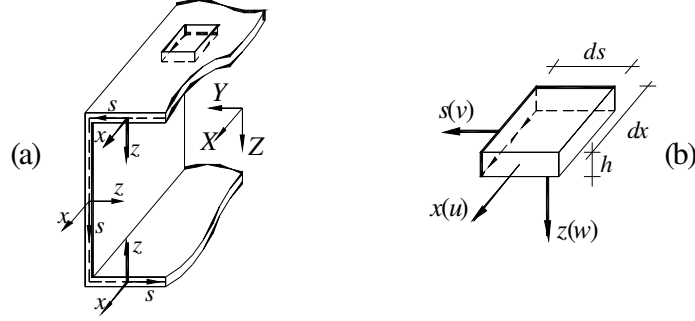


Figure 1: (a) Arbitrary prismatic open-section thin-walled member and global coordinate system, and (b) wall element with its local coordinate system and displacement components.

materials with special orthotropy², $E_{xx}(s)$, $E_{ss}(s)$, $\nu_{xs}(s)$, $\nu_{sx}(s)$, $G_{xs}(s)$ and $\rho(s)$, respectively. In compliance with classical thin-walled beam theory [4], the displacement components are expressed as

$$u(x, s, t) = u_k(s)\zeta_{k,x}(x, t) \quad v(x, s, t) = v_k(s)\zeta_k(x, t) \quad w(x, s, t) = w_k(s)\zeta_k(x, t) \quad (1)$$

where (i) $(\cdot)_{,x} = \partial(\cdot)/\partial x$, (ii) $u_k(s)$, $v_k(s)$ and $w_k(s)$ are the cross-section deformation mode components (*i.e.*, the functions defining the GBT deformation modes) and (iii) $\zeta_k(x, t)$ are the amplitude functions describing their variation both along the longitudinal direction and with time t . In addition to the classical Kirchhoff-Love hypotheses, in linear buckling and vibration analyses of thin-walled members one commonly assumes null mid-surface membrane shear strains ($\gamma_{xs}^M = 0$ – “Vlasov’s hypothesis”) and transverse extensions ($\epsilon_{ss}^M = 0$).

In the next sub-sections, the systems of differential equations representing the member static (for buckling analysis) or dynamic (for vibration analysis) equilibrium are presented and briefly discussed.

2.1 Buckling analysis

GBTUL 1.0 β incorporates a formulation of GBT [5] that performs linear buckling analyses and which handles members subjected to uniform or non-uniform (longitudinally) axial force and bending moments (for the latter, the effect of the ensuing shear stresses is also accounted) and uniform bimoments.

Considering the member total potential energy as the sum of (i) the internal energy and the (ii) potential of the applied (pre-buckling) loading, by the application of the Principle of Virtual Work one is led to

² For isotropic members, like steel profiles, $E_{xx}(s) = E_{ss}(s) \equiv E$, $\nu_{xs}(s) = \nu_{sx}(s) \equiv \nu$, $G_{xs}(s) \equiv G$, and $\rho(s) \equiv \rho$.

$$C_{ik}\phi_{k,xxxx} - D_{ik}\phi_{k,xx} + B_{ik}\phi_k - \lambda [X_{jik}(W_j^0\phi_{k,x})_{,x} - X_{jki}^\tau(W_{j,x}^0\phi_k)_{,x} + W_{j,x}^0 X_{jik}^\tau\phi_{k,x}] = 0 \quad (2)$$

where C_{ik} , D_{ik} , B_{ik} and X_{jik} , X_{jik}^τ are the cross-section linear and geometrical stiffness matrices, which mathematical definitions and physical meaning are presented in Table 2.1, and $\phi_k(x)$ are the longitudinal amplitude functions of the modal components (for static equilibrium, $\zeta_k(x,t) \equiv \phi_k(x)$). Vector W_j^0 ($W_j^0(x)$, with $j=1\dots 4$), contains the resultants of the applied pre-buckling stresses, namely (i) axial force ($W_1^0(x) \equiv N(x)$), (ii) major and minor axis bending moments ($W_2^0(x) \equiv M_Y(x)$, $W_3^0(x) \equiv M_Z(x)$), and (iii) bimoment ($W_4^0 \equiv B$).

The solution of (2) yields the buckling load parameters (λ values) and mode shapes ($\phi(x)$ functions).

2.2 Vibration analysis

In addition to buckling, GBTUL 1.0 β performs free vibration analyses of unloaded³ members [3]. By the application of the Hamilton's Principle, one gets the dynamic equilibrium eigensystem, which reads

$$C_{ik}\zeta_{k,xxxx} - D_{ik}\zeta_{k,xx} + B_{ik}\zeta_k - Q_{ik}\zeta_{k,xxtt} + R_{ik}\zeta_{k,tt} = 0 \quad (3)$$

where it can be noted that (3) and (2) share the first three terms (*i.e.*, the linear stiffness matrices), while Q_{ik} and R_{ik} are inertia matrices (see Table 2.1). Assuming vibration modes with synchronous configurations exhibiting sinusoidal variation in time, the modal amplitude functions can be expressed, for each vibration mode, as

$$\zeta_k(x,t) = \phi_k(x)Y(t) = \phi_k(x)Y_0 \sin(\omega t) \quad (4)$$

where ω is the corresponding angular frequency. Finally, substituting (4) into (3) one is led to

$$C_{ik}\phi_{k,xxxx} - D_{ik}\phi_{k,xx} + B_{ik}\phi_k - \omega^2(R_{ik}\phi_k - Q_{ik}\phi_{k,xx}) = 0 \quad (5)$$

which solution yields the vibration modes frequencies (ω values) and shapes ($\phi(x)$ functions).

³ For the application of GBT to the vibration analysis of *loaded* members see [3] and [6]-[7].

2.3 GBT cross-section matrices

The tensors presented in equations (2), (3) and (5) are mathematically defined and their physical meaning is explained in Table 2.1. For every analysis, GBTUL 1.0 β provides these matrices in both (i) Screen 2 (“Modal Selection”), the components being displayed in dialogue boxes, and (iii) the output text file *Matrices.txt*.

3. Cross-Section Analysis

After assumption (1), the member displacements have been represented as sums of contributions of cross-section-defined *modal components* (*i.e.*, functions $u_k(s)$, $v_k(s)$ and $w_k(s)$) to which are associated the due longitudinal amplitude functions (*i.e.*, $\zeta_k(x,t)$) – however those modal components, or *GBT deformation modes*, have not yet been defined. In the scope of any GBT application, the Cross-Section Analysis is the procedure in which the deformation modes of the cross-section under analysis are determined.

GBTUL 1.0 β incorporates a Cross-Section Analysis procedure that handles arbitrary *open* cross-sections, as those depicted in figure 3.1(a) (*e.g.*, see [3],[8]). *Closed* cross-sections, *i.e.*, containing closed cells (see figure 3.1(b)), are thus outside the scope of applicability of version 1.0 of GBTUL, although several GBT formulations dealing with this type of sections have already been developed [9]-[10]. Furthermore, because the present formulation is based on the hypotheses mentioned in the previous section (*i.e.*, $\gamma_{xs}^M = \varepsilon_{ss}^M = 0$) it yields only the so-called *conventional modes* – thus, not providing *shear modes* or *transverse extension modes* (*e.g.*, [11]), which influence in buckling or vibration analysis is generally negligible.

Table 2.1: Definition and physical meaning of the GBT cross-section matrices

| Matrix | Definition | Physical Meaning | |
|--|--|--------------------------|--|
| C_{ik} | $C_{ik} = \int_S \left[E_{xx} h u_i u_k + \frac{E_{xx} h^3}{12(1-\nu_{xs}\nu_{sx})} w_i w_k \right] ds$ | Linear Stiffness | Generalized warping (primary and secondary) |
| B_{ik} | $B_{ik} = \int_S \frac{E_{ss} h^3}{12(1-\nu_{xs}\nu_{sx})} w_{i,ss} w_{k,ss} ds$ | | Transverse bending of the walls |
| D_{ik} | $D_{ik} = \int_S \left[\frac{G h^3}{3} w_{i,s} w_{k,s} - \frac{E_{xx} \nu_{xs} h^3}{12(1-\nu_{xs}\nu_{sx})} (w_i w_{k,ss} + w_k w_{i,ss}) \right] ds$ | | Generalized torsion of the walls |
| X_{jik} | $X_{jik} = \int_S \frac{E_{xx} h}{C_{jj}} u_j (v_i v_k + w_i w_k) ds \quad j = 1 \dots 4$ | Geometrical Stiffness | Stiffness degradation due to acting membrane longitudinal stresses |
| X_{jik}^τ | $X_{jik}^\tau = \int_S \frac{E_{xx}}{C_{jj}} M_j w_{i,s} w_k ds \quad j = 2 \dots 3$ | | Stiffness degradation due to acting membrane shear stresses |
| Q_{ik} | $Q_{ik} = \int_S \left[\rho h u_i u_k + \frac{\rho h^3}{12} w_i w_k \right] ds$ | Inertia | Inertia associated with out-of-plane movement (translational and rotational) |
| R_{ik} | $R_{ik} = \int_S \left[\rho h (v_i v_k + w_i w_k) + \frac{\rho h^3}{12} w_{i,s} w_{k,s} \right] ds$ | | Inertia associated with in-plane movement (translational and rotational) |
| Notes: | | | |
| <ul style="list-style-type: none"> The integrations are carried over the cross-section mid-line coordinate s, which domain is here defined as S. The term M_j (appearing in the definition of X_{jik}^τ) is explained in [5]. | | | |

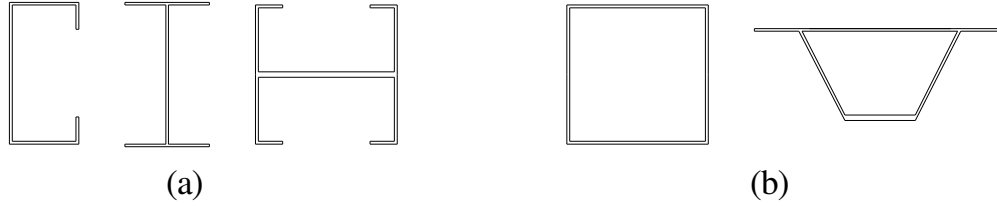


Figure 3.1: Examples of member cross-sections: (a) open (handled by GBTUL 1.0) and (b) closed (not handled by GBTUL 1.0)

3.1 Cross-Section Analysis: Brief overview

The performance of the GBT cross-section analysis involves a sequential procedure comprising the following major steps, which are expounded at length in references [3] and [8]. At this point, it is relevant to classify the open cross-sections as *branched* or *unbranched*, depending on whether they include or not branching nodes, *i.e.* nodes shared by more than two walls (*e.g.*, C, Z or U-sections are *unbranched*, while I or T-sections are *branched*). The major steps involved in the cross-section analysis are briefly described next:

- (i) Cross-section discretisation into (i₁) $n+1$ *natural* nodes (ends of the n walls forming the cross section) and (i₂) m *intermediate* nodes (located within the walls). In *branched* sections, these must be still subdivided into *independent* and *dependent* – as for *unbranched* sections, all the natural nodes are *independent*.
- (ii) Determination of the *initial shape functions* $u_i(s)$, $v_i(s)$ and $w_i(s)$, by imposing (ii₁) unit warping displacements ($u=1$) at each independent natural node and (ii₂) unit flexural displacements ($w=1$) at each intermediate node – note that no functions are associated with the dependent natural nodes (their displacements depend solely on those of the independent nodes, hence the designation). The cross-section *extremity* nodes are always treated as both natural (independent or dependent) and intermediate. The total number of initial shapes functions is

$$N_d = n + 1 - d + m, \quad (6)$$

where d is the number of dependent natural nodes ($d=0$ for unbranched sections).

- (iii) Calculation of the cross-section ($N_d \times N_d$) stiffness and mass matrices, on the basis of the initial shape functions and applied loading. One

obtains fully populated matrices with components that exhibit no obvious structural meaning.

- (iv) In order to uncouple the member equilibrium equation system as much as possible and, at the same time, have matrix components with clear structural meanings, one simultaneously diagonalises the linear stiffness matrices C_{ik} and B_{ik} . This leads to the determination of the N_d cross-section *deformation modes* (the *final shape functions* $u_k(s)$, $v_k(s)$ and $w_k(s)$) and to the evaluation of the associated cross-section modal mechanical, mass and geometrical properties – several of the new matrix components have a very clear/illuminating structural meaning. This process is the GBT “trademark” and makes it possible to express the equilibrium equations in *modal* form, thus leading to a fair amount of interpretation and numerical implementation advantages.

3.2 Nodal discretization in GBTUL 1.0 β

From all the steps described in 3.1, only the first involves inputs to be provided by the user, namely the *nodal discretization*: (i) the number and location of the intermediate nodes, and (ii) the classification of natural nodes into independent and dependent (as natural nodes, their positions are automatically assigned to the walls ends) – the first has direct implications on the number and quality of the deformation modes obtained, while the second, though not influencing the resulting mode shapes, must obey certain criteria. It is important to mention that the formulations presented in [5] and [8] suggest an intuitive procedure to guarantee (implicitly) an efficient classification of the natural nodes (into *dependent* or *independent*), involving the division of the section into sub-sections of several *orders* – these are inputs of GBTUL 1.0 β , in the `Section Walls` table, at Screen 1.

Next, some explanations are given on how to discretize the cross-sections, having in mind the use of GBTUL 1.0 β – for the sake of convenience, unbranched and branched sections are treated separately.

- Unbranched sections

In the unbranched sections, all the natural nodes are independent, and thus, all the walls orders are 0. The intermediate nodes should be placed mainly in the walls that are expected to be more susceptible to undergo local deformation. For most cases, 3 or 4 nodes are enough for accurate results. Figure 3.2 shows the modes obtained for a C-section for two different nodal discretizations: (A) no intermediate

nodes (apart from the two associated with the two extremities, figure 3.2(a)), and (B) one intermediate node at each flange and three at the web (figure 3.2(b)). As the figure shows, (i) although the first 6 modes are fairly similar for both discretizations, for the finer one (*i.e.*, (A)) (ii) modes **7-8** display a better “quality” and (iii) there were obtained 4 additional modes, namely **9-13**.

- Branched Sections

The criterion for the placement of intermediate nodes is the same as for the unbranched sections.

Concerning the walls *orders* (and implicitly, the classification of the natural nodes into dependent and independent), the user must first choose an *unbranched sub-section* that (i) should contain as many branching nodes as possible and (ii) must not include aligned walls sharing the same branching node – to the walls making up this sub-section it is attributed *order=0*. This ensures that the branching nodes can be treated as natural nodes of the unbranched sub-section. Then, the same criteria is used to select, among the remaining walls, further unbranched sub-sections – to every new sub-section starting from a branching node belonging to an earlier defined subsection with order n , it is assigned the order $n+1$.

Figure 3.3 depicts two examples of the application of this method to branched cross-sections. Tables containing some of the cross-section inputs are also presented – note that the walls are numbered according to their orders.

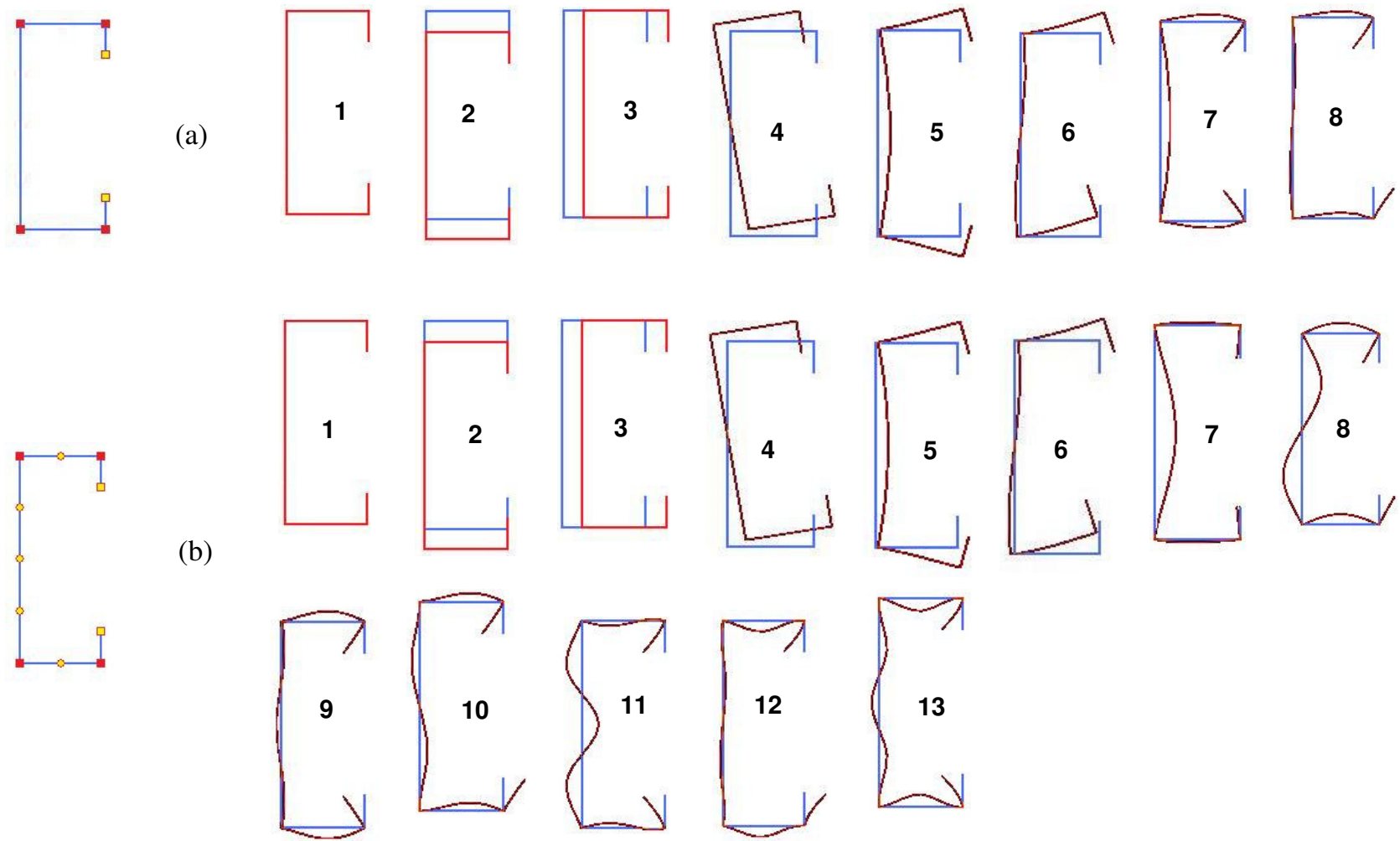
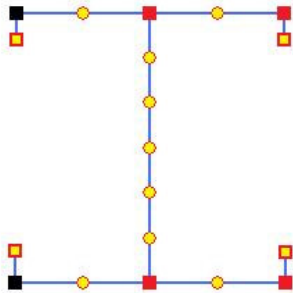
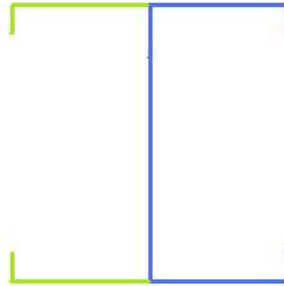


Figure 3.2: In-plane configurations of C-section deformation modes: (a) discretization (1), and (b) discretization (2)



Nodal discretizations



Walls orders

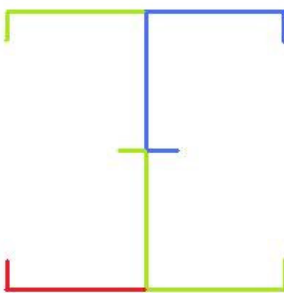
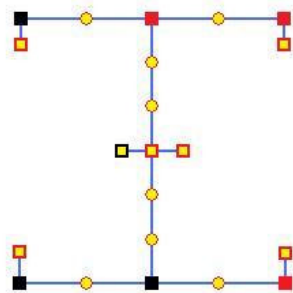
| #Wall | #Node i | #Node j | Order | Inodes |
|-------|---------|---------|-------|--------|
| 1 | 1 | 2 | 0 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 4 | 0 | 5 |
| 4 | 4 | 5 | 0 | 1 |
| 5 | 5 | 6 | 0 | 0 |
| 6 | 3 | 7 | 1 | 1 |
| 7 | 7 | 8 | 1 | 0 |
| 8 | 4 | 9 | 1 | 1 |
| 9 | 9 | 10 | 1 | 0 |

Walls properties

Natural: ■ indepe. ■ dep.
 End: ■ indepe. ■ dep.

0: blue
 1: green
 2: red

Intermediate: ●



| #Wall | #Node i | #Node j | Order | Inodes |
|-------|---------|---------|-------|--------|
| 1 | 1 | 2 | 0 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 4 | 0 | 2 |
| 4 | 4 | 5 | 0 | 0 |
| 5 | 3 | 6 | 1 | 1 |
| 6 | 6 | 7 | 1 | 0 |
| 7 | 4 | 8 | 1 | 0 |
| 8 | 4 | 9 | 1 | 2 |
| 9 | 9 | 10 | 1 | 1 |
| 10 | 10 | 11 | 1 | 0 |
| 11 | 9 | 12 | 2 | 1 |
| 12 | 12 | 13 | 2 | 0 |

Figure 3.3: Branched cross-section discretization: two examples

4. Mode Selection

The cross-section analysis yields a set of N_d (see eq. (6)) deformation modes, which represent the possible cross-section deformation patterns to be accounted for. As figure 3.2 shows, there are three different natures of deformation modes obtained: (i) the first 4 are the classical rigid-body *global* modes – axial extension (mode **1**), major and minor axis bending (modes **2** and **3**) and torsion (mode **4**) – (ii) modes **5-6** are *distortional*, invariably associated with fold-line motions, and (iii) the remaining are *local-plate* modes, which involve exclusively wall bending (their number is equal to the number of intermediate modes considered, m). Table 4.1 summarizes the correspondence between numbers and natures of modes – note that only sections with at least $n=4$ walls have distortional modes.

Table 4.1: Classification of GBT deformation modes ($n>3$)

| Mode Number | Classification |
|-------------------|--------------------|
| 1 | Axial extension |
| 2 | Major axis bending |
| 3 | Minor axis bending |
| 4 | Torsion |
| 5 to n+1-d | Distortional modes |
| (remaining m) | Local-plate modes |

From the N_d deformation modes obtained, one can select any sub-set of n_d ($1 \leq n_d \leq N_d$) of such modes to use in the problem solution – namely, the ones expected to be relevant in the particular context. Some comments about this modal selection follow:

- (i) By considering only the rigid-body modes **1-4**, systems (2) or (5) become exactly the same of the classical thin-walled beam theory, due to Vlasov [4].
- (ii) As it is well known, mode **1** (axial extension) is never relevant for the solution of buckling analysis, which means it shall always be withdrawn from the modal selections. However, it should be included for vibration analysis, if axial vibration modes are to be determined.

(iii) For buckling analyses of columns with cross-section displaying at least one axis of symmetry (e.g., the C-section), only the odd-number modes (1, 3, 5, etc.) may be considered in the analysis. In fact, for these type of sections, the odd modes are always symmetrical (see figure 3.2), and the ensuing buckling modes are also symmetrical.

5. Member Analysis

After performing the cross-section analysis and the ensuing mode selection, systems (2) and (5) are expressed in the GBT modal coordinates and have dimension $n_d \times n_d$. The solution of the buckling, or vibration, problem involves, thus, solving a coupled system of eigenvalues (λ or ω) and eigenfunctions ($\phi_k(x)$) – this task, named *Member Analysis*, is usually performed by one of two methods, both available in GBTUL 1.0 β (Screen 3): (i) the *Analytical Solution* or (ii) the *Numerical Solution*. While the first involves solving (2) or (5) analytically, which is only possible for a few particular situations – namely (i) buckling analysis of simply supported members subjected to longitudinally uniform stresses and (ii) vibration analysis of simply supported members –, the latter involves a longitudinal discretization of the member into GBT-based finite elements, and it is always applicable. In the next two sub-sections these two procedures are briefly explained.

5.1 Analytical solution

As mentioned above, the analytical solution is only applicable for two types of analysis: (i) buckling analysis of simply supported members subjected to longitudinally uniform stresses and (ii) vibration analysis of simply supported members (*i.e.*, with locally and globally pinned and free-to-warp sections).

If a given member is acted by loads causing longitudinally uniform stresses (*i.e.*, a combination of constant axial force, bending moments and bimoment), which means $W_j^0(x) \equiv W_j^0$, system (2) becomes

$$C_{ik} \phi_{k,xxxx} - D_{ik} \phi_{k,xx} + B_{ik} \phi_k - \lambda X_{jik} W_j^0 \phi_{k,xx} = 0 \quad (7)$$

Like system (5), concerning dynamic equilibrium, system (7) admits an exact solution of the form ($k=1 \dots n_d$)

$$\phi_k(x) = A_k \sin\left(\frac{n_h \pi x}{L}\right) \quad (8)$$

which is valid for simply supported members. The substitution of (8) into (7) or (5) yields a system of eigenvalues (λ or ω) and eigenvectors (A_k), defining the buckling or vibration modes sought. Since this system has a relatively low dimension, namely $n_d \times n_d$, the computational time required for its resolution is very low – this is why Analytical Solution is always to be used, whenever possible, rather than Numerical Solution, a much slower process.

5.2 Numerical solution – GBT-based finite element

The derivation of the GBT-based beam finite element, which is explained in detail in references [5] or [7], involves the variational (or weak) counterpart of (2), or (5). Within the domain of a finite element of length L_e , the amplitude function $\phi_k(x)$ is approximated by means of linear combinations of Hermite cubic polynomials, *i.e.*,

$$\phi_k(x) = d_{k,1}^e \Psi_1(\tilde{x}) + d_{k,2}^e \Psi_2(\tilde{x}) + d_{k,3}^e \Psi_3(\tilde{x}) + d_{k,4}^e \Psi_4(\tilde{x}) \quad (9)$$

where $d_{k,1}^e = \phi_{k,x}(0)$, $d_{k,2}^e = \phi_k(0)$, $d_{k,3}^e = \phi_{k,x}(1)$, $d_{k,4}^e = \phi_k(1)$, $\tilde{x} = x/L_e$ and

$$\Psi_1 = L_e(\tilde{x}^3 - 2\tilde{x}^2 + \tilde{x}), \quad \Psi_2 = 2\tilde{x}^3 - 3\tilde{x}^2 + 1, \quad \Psi_3 = L_e(\tilde{x}^3 - \tilde{x}^2), \quad \Psi_4 = -2\tilde{x}^3 + 3\tilde{x}^2 \quad (10)$$

which means that each finite element has 4 degrees of freedom per node, hence a total of $4 \times n_d$. If the member is discretized into n_e such finite elements, the total number of global generalized degrees of freedom is approximately equal to $2 \times n_d \times (n_e + 1)$, a number that is usually between 100 and 1000.

When Numerical Solution is used, the modal nature of GBT allows the consideration of different boundary conditions to different modes. Standard boundary conditions⁴, *i.e.* those involving total restriction of displacements ($\phi_k=0$) and/or derivatives ($\phi_{k,x}=0$), are easily taken into account in the process of assembling the global equilibrium eigensystem. GBTUL 1.0 β offers the possibility to assign 4 different types of support (boundary) conditions to 4 different types of modes (any combination is allowed). The 4 support conditions are: (i) simply supported (“S-S”), (ii) clamped-free (or cantilever, “C-F”), (iii) clamped-clamped (“C-C”) and (iv) clamped-supported (“C-S”) – see Table 5.1. The four sets of modes to which the mentioned support conditions can be applied independently are: (i) mode 2 (major axis bending), (ii) mode 3 (minor axis bending), (iii) modes 4+distortional (torsion and distortional modes), and (iv) local-plate modes.

⁴ For GBT-based buckling analyses of members with non-standard support conditions (*e.g.*, elastic point springs), see [12].

Table 5.1: The 4 support conditions available in GBTUL1.0 β

| Support Conditions | |
|--------------------|---|
| S-S | $\phi_k(0)=\phi_k(L)=0$ |
| C-F | $\phi_k(0)=\phi_{k,x}(0)=0$ |
| C-C | $\phi_k(0)=\phi_{k,x}(0)=\phi_k(L)=\phi_{k,x}(L)=0$ |
| C-S | $\phi_k(0)=\phi_{k,x}(0)=\phi_k(L)=0$ |

6. Deformed Configuration Representation

Once problem solution is found, GBTUL 1.0 β provides, on Screen 4, the deformed configurations, corresponding to the member buckling or vibration modes, by both (i) graphical 2D and 3D representations, and (ii) modal participation factors and diagrams. The next subsections concern these two types of output.

6.1 Graphical representations (2D and 3D)

The graphical representations available concern the (i) cross-section (2D) and (ii) whole member (3D) deformed configurations associated to the resulting buckling or vibration mode. Because these configurations are given as the sum of the individual contributions of the n_d deformation modes selected for the analysis, at this stage it is possible to further select a sub-set of m_d modes ($1 \leq m_d \leq n_d$) to be included in the graphical representations – this allows the perception of the role played by any mode (or sub-set of modes) to the overall deformation.

For this purpose, Screen 4 includes the mode selection tool GBT Modes, which applies to both 2D and 3D deformed configurations.

6.2 Modal participation diagrams

As implied by eq. (1), each deformation mode provides, in association with the corresponding amplitude function, an individual contribution to the member overall deformed configuration (*i.e.*, buckling or vibration mode). One easy and intuitive method to assess the contribution of each deformation mode, is provided by the *modal participation factor* (P_k) concept. For a given GBT deformation mode k , one has

$$P_k = \frac{\int_L |\phi_k(x)| dx}{\sum_{i=1}^{n_d} \int_L |\phi_i(x)| dx} \quad (11)$$

where the numerator and denominator are the sum of the contributions of (i) that deformation mode (k) and (ii) all the n_d deformation modes included in the analysis to the member cross-section deformed configurations associated with the vibration mode under consideration.

Although, in general, the P_i values give no information on the modal amplitude function *shapes*, the cumulative P_k vs L modal participation diagrams provide a quite good assessment of the variation of the buckling/vibration mode nature and characteristics with the member length. For instance, a member length range with $P_2+P_4 \approx 1.0$ corresponds to (major axis) flexural-torsional vibration modes – moreover, the P_2 and P_4 values indicate how relevant are the mode **2** (flexure) and mode **4** (torsion) participations in those vibration modes.

As an illustrative example, figure 5.1 represents the P_k vs L diagrams corresponding to the first (i) buckling (figure 5.1(a)) and (ii) vibration (figure 5.1(b)) modes, within the range $1 \leq L \leq 1000 \text{ cm}$, obtained for a lipped channel (C-section) steel beam (see figure 3.2(b), for the section deformation modes). These results are part of a study presented in reference [7].

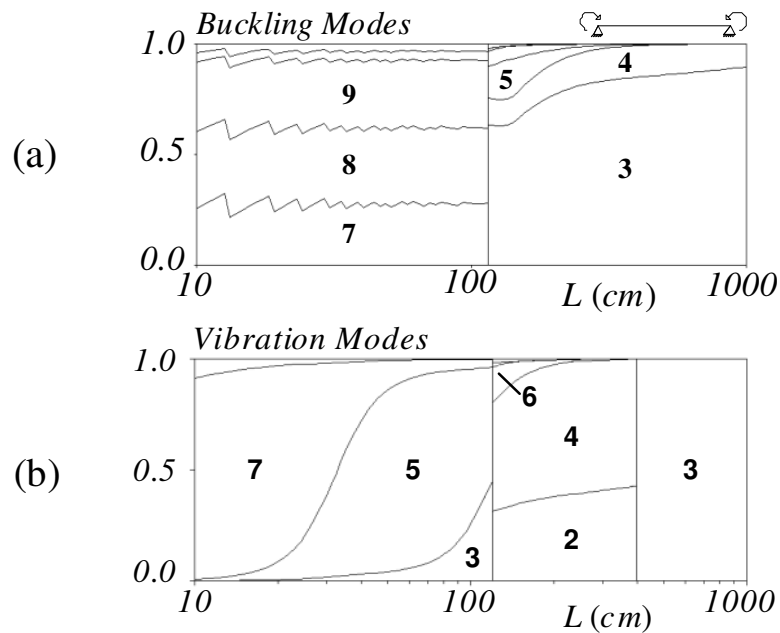


Figure 5.1: C-section beam modal participation diagrams: (a) critical buckling modes (uniform major axis bending) and (b) fundamental vibration modes

The interpretation of the diagrams in figure 5.1 is as follows:

- (i) Figure 5.1(a) shows that, for $L < 120\text{cm}$, the beam critical buckling mode is governed by local-plate modes **7-9** (fairly equal contributions). For $L \geq 120\text{cm}$, modes **3+4** (i.e., flexural-torsional buckling) prevail – for the lower lengths, there are also small contributions from local-plate and distortional modes (e.g., mode **5**).
- (ii) The fundamental vibration mode shape (figure 5.1(b)) is (ii₁) *local-plate* (mode **7** predominant), for $L < 25\text{cm}$ (very short beams), (ii₂) *distortional* (mode **5** governs, with relevant contributions from modes **3** and **7**), for $25 < L < 120\text{cm}$, (ii₃) *flexural-torsional-distortional* (combines modes **2**, **4** and **6**), for $120 < L < 400\text{cm}$, and (ii₄) *purely flexural* (mode **3**), for $L > 400\text{cm}$.

GBTUL 1.0 β displays, at Screen 4, (i) the higher 3 values of P_k , for the length L selected, and (ii) the modal participation diagrams corresponding to the whole length range analysed – the latter are similar to those displayed on figures 5.1(a)-(b), with the exception that the mode numbers are not presented (a small drawback of the program which is expected to be overcome in future versions). Moreover, all the P_k - L values are printed in the file *Results.txt*, enabling their use to build modal participation diagrams with other applications (e.g., Microsoft Excel).

References

- [1] R. Schardt (1989) *Verallgemeinerte Technische Biegetheorie*, Springer-Verlag, Berlin. (German)
- [2] D. Camotim, N. Silvestre, R. Gonçalves and P.B. Dinis (2004). “GBT analysis of thin-walled members: new formulations and applications”, *Thin-Walled Structures: Recent Advances and Future Trends in Thin-Walled Structures Technology*, J. Loughlan (ed.), Canopus Publishing, Bath, 137-168.
- [3] D. Camotim, N. Silvestre and R. Bebiano (2007). “GBT local and global vibration analysis of thin-walled members”, *Dynamics of Plated Structures: Analysis and Design*, N.E. Shanmugam and C.M. Wang (eds.), Woodhead Publishing Ltd., Cambridge, 36-76.
- [4] B.Z. Vlasov (1959), *Thin-Walled Elastic Bars*, Fizmatgiz, Moscow. (English translation: Israel Program for Scientific Translation, Jerusalem, 1961).
- [5] R. Bebiano, N. Silvestre and D. Camotim (2007). “GBT formulation to analyze the buckling behaviour of thin-walled members subjected to non-uniform bending”, *International Journal of Structural Stability and Dynamics*, **7**(1).
- [6] N. Silvestre, D. Camotim (2006), “Vibration behaviour of axially compressed cold-formed steel members”, *Steel and Composite Structures* **6**, 221-236.

- [7] R. Bebiano, N. Silvestre and D. Camotim (2008), “Local and global vibration of thin-walled members subjected to compression and non-uniform bending”, *Journal of Sound and Vibration*. In press.
- [8] P.B. Dinis, D. Camotim and N. Silvestre (2006), “GBT formulation to analyse the buckling behaviour of thin-walled members with arbitrarily ‘branched’ open cross-sections”, *Thin-Walled Structures*, **44**, 20-38.
- [9] N.M.F. Silva, D. Camotim and N. Silvestre (2008). “GBT cross-section analysis of thin-walled members with arbitrary cross-sections: a novel approach”, Proceedings of Fifth International Conference on Thin-Walled Structures (ICTWS 2008 - Brisbane, 18-20/6), submitted for publication.
- [10] R. Gonçalves, P.B. Dinis and D. Camotim (2005), “GBT formulation to analyze the stability of thin-walled members with fully arbitrary cross-section shapes”, CD-ROM Proceedings of the 2005 Joint ASME/ASCE/SES Conference on Mechanics and Materials (MCMAT 2005 - Baton Rouge, 1-3/6).
- [11] N. Silvestre, D. Camotim (2006), “Local-plate and distortional postbuckling behavior of cold-formed steel lipped channel columns with intermediate stiffeners”, *Journal of Structural Engineering*, 132(4), 529-540.
- [12] D. Camotim, N. Silvestre, C. Basaglia and R. Bebiano (2008), “GBT-based buckling analysis of thin-walled members with non-standard support conditions”, *Thin-Walled Structures*. In press.