SITE-EFFECT ASSESSMENT USING ACCELERATION TIME SERIES
APPLICATION TO SÃO SEBASTIÃO VOLCANIC CRATER

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To my family, old and new
ABSTRACT

The present thesis concerns the analysis of acceleration time series and their use in site-effect assessment, namely at São Sebastião volcanic crater. There is historical evidence that São Sebastião volcanic crater, at Terceira Island, Azores, exhibits stronger ground motion during seismic events than its surrounding areas.

A review of the most important concepts regarding the characterization of seismic motion, soil behavior under cyclic loading and site effect is made.

A constitutive relation based on the Ramberg-Osgood model was implemented in the finite-element code PLAXIS, allowing the accountance of hysteretic damping in two-dimensional dynamic analyses.

During two seismic crises in 1997 and 1998 it was possible to obtain several records at three different places inside the volcanic crater: Escola, Junta and Misericórdia. Comparison between seismic records allowed a primary identification and analysis of site effects inside the crater. One dimensional parametric analyses were made in order to explain differences in the acceleration time series recorded at the mentioned places. Two of the most usual site-effect assessment techniques were used, namely the reference-site spectral ratio and horizontal-to-vertical spectral ratio. Finally, two-dimensional finite-element meshes were defined in PLAXIS having as goal the assessment of possible ground motion amplification due to subsurface topographic features.

Keywords:

Site effects

Acceleration time series

Soil behavior under cyclic loading

Ramberg-Osgood model

São Sebastião
RESUMO

A presente dissertação diz respeito à análise de séries temporais de aceleração e ao seu uso na avaliação de efeitos de sítio, nomeadamente na cratera vulcânica de São Sebastião. Existem dados históricos indicando que a cratera vulcânica de São Sebastião, na ilha Terceira, Açores, exibe movimento sísmico superior às áreas vizinhas.

É feita uma revisão dos conceitos mais importantes relativos à caracterização do movimento sísmico, ao comportamento dos solos sob carregamento cíclico e aos efeitos de sítio.

Uma relação constitutiva baseada no modelo de Ramberg-Osgood foi implementada no programa de elementos finitos PLAXIS, permitindo a consideração do amorteceimento histerético em análises dinâmicas bi-dimensionais.

Durante duas crises sísmicas em 1997 e 1998, foi possível obter vários registos acelerométricos em três sítios distintos da cratera: Escola, Junta e Misericórida. A comparação entre os registos acelerométricos permitiu uma identificação e análise preliminares dos efeitos de sítio na cratera. Uma análise paramétrica uni-dimensional foi feita com vista à explicação das diferenças nos registos obtidos nas diferentes estações acelerométricas. Duas das técnicas experimentais para a avaliação de efeitos de sítio foram usadas: a razão espectral usando um sítio de referência e a razão espectral entre as componentes vertical e horizontal do movimento sísmico obtidos na mesma estação. Finalmente, foram geradas malhas bi-dimensionais de elementos finitos, tendo como fim a avaliação da influência da topografia subsuperficial no movimento sísmico.

Palavras-Chave:
Efeitos de sítio
Séries temporais de aceleração
Comportamento dos solos sob carregamento cíclico
Modelo de Ramberg-Osgood
São Sebastião
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Symbols

Latin alphabet

\[ A \] – matrix used in the building of the tangential stiffness matrix

\( A_{\text{incident}} \) – Amplitude of incident wave at the interface between two layers in one-dimensional analysis

\( A_h \) – horizontal amplification ratio

\( A_{\text{refracted}} \) – Amplitude of refracted wave at the interface between two layers in one-dimensional analysis

\( A_{\text{reflected}} \) – Amplitude of reflected wave at the interface between two layers in one-dimensional analysis

\( A_{ij}\,(f,r) \) – factor due to the attenuation during the propagation path in generalized inversion scheme

\( A_v \) – vertical amplification ratio

\( AI \) – Arias intensity

\( a_0 \) – Rayleigh-damping coefficient proportional to the mass

\( a_1 \) – Rayleigh-damping coefficient proportional to the stiffness

\[ B \] – matrix used in the building of the tangential stiffness matrix

\[ C \] – tangential stiffness matrix

\( C_N \) – corrective factor in order to account the overburden pressure for the SPT test

\( c \) – viscous damping

\( D_{ij} \) – stiffness matrix

\[ D \] – stiffness matrix in PLAXIS

\[ D_{\text{total}} \] – total stress stiffness matrix

\[ D_{\text{effective}} \] – effective stress stiffness matrix
Symbols

\( de_{ij} \) – deviatoric part of the strain increment tensor

\( \{de\} \) – deviatoric strain increment vector

\( d\varepsilon_{ij} \) – strain increment tensor

\( \{d\varepsilon\} \) – strain increment vector

\( d\sigma_{ij} \) – stress increment tensor

\( \{d\sigma\} \) – stress increment vector

\( E[x(t)] \) – mathematical expectancy of time signal \( x(t) \)

\( E \) – energy of time signal \( x(t) \)

\( E \) – Young modulus

\( ER \) – Energy ratio

\( e \) – Napier’s number

\( e_{ij} \) – deviatoric part of the strain tensor

\( F(t) \) – excitation at the base of SDOF oscillator

\( F_A \) – factor concerning the geological age for the Otha and Goto relation

\( F_B \) – factor concerning the grain size for the Otha and Goto relation

\( F_{el} \) – elastic restitution force

\( f \) – linear frequency

\( f_0 \) – fundamental frequency

\( G \) – shear modulus

\( G^* \) – complex shear modulus

\( G_0 \) – initial shear modulus

\( G_S \) – secant shear modulus
Symbols

$G_t$ – tangent shear modulus

g – gravitational acceleration

$H(\omega)$ – transfer function

$H$ – soil layer thickness

$H$ – height of soil sample in Resonant Column test

$H_B$ – horizontal amplitude spectrum at the bedrock

$H_S$ – horizontal amplitude spectrum at the ground surface

$H_{SW}$ – horizontal amplitude spectrum of the Rayleigh-part of the wavefield

$(H/V)_{comb}$ – combined horizontal-to-vertical spectral ratio curve

$h(t)$ – unit impulse response

$Im$ – imaginary part

$PI$ – plasticity index

$I_p$ – mass moment of inertia of the soil sample at the Resonant Column test

$i$ – square root of -1

$i$ – discrete variable

$J_A$ – mass moment of inertia of the exciting mass at the Resonant Column test

$K$ – bulk modulus

$K$ – Stiffness of the active extremity of the Resonant Column test

$K_f$ – bulk modulus accounting the contribution of both ater and solid phase of the soil

$K_S$ – secant bulk modulus

$K_{Soil}$ – bulk modulus of the solid phase of the soil

$K_t$ – tangent bulk modulus
Symbols

$K_w$ – bulk modulus of water

$k$ – wavenumber

$k$ – SDOF elastic stiffness

$M$ – oedometric modulus

$M_0$ – Amplitude of the torsional moment at the Resonant Column test

$m$ – SDOF mass

$N$ – SPT blow count

$(N_i)_{60}$ – normalized SPT blow count

$n$ – discrete variable

$n$ – porosity

$p'$ – mean effective stress

$PS_v (T_n, \xi)$ – relative velocity pseudo-spectrum

$PS_i (T_n, \xi)$ – absolute acceleration pseudo-spectrum

$p(t)$ – sampling function

$Re$ – Real part

$R_X$ – autocorrelation function

$r$ – parameter of the Ramberg-Osgood model

$S_d (T_n, \xi)$ – relative displacement spectrum

$S_d$ – standard deviation of the logarithmic combination of horizontal-to-vertical spectral ratios

$SF$ – shape factor

$SI (\xi)$ – spectral intensity

$S_i (f)$ – factor due to the source in generalized inversion scheme
Symbols

$S_v (T_n, \xi)$ – relative velocity spectrum

$S_X (\omega)$ – power spectral density

$\tilde{S}_X (\omega)$ – unilateral power spectral density

$s_{ij}$ – deviatoric part of the stress tensor

$t$ – time

$T$ – period

$T$ – sampling period

$T_n$ – natural period

$U_y (f,r)$ – spectral amplitude in generalized inversion scheme

$u(t)$ – relative displacement of SDOF oscillator

$u_{ss} (t)$ – steady-state solution of SDOF dynamic equilibrium equation

$\dot{u}(t)$ – relative velocity of SDOF oscillator

$\ddot{u}(t)$ – relative acceleration of SDOF oscillator

$\ddot{u}_g (t)$ – ground motion acceleration at the base of SDOF oscillator

$V_B$ – horizontal amplitude spectrum at the bedrock

$V_P$ – pressure wave velocity

$V_S$ – shear wave velocity

$V_S$ – horizontal amplitude spectrum at the ground surface

$V_S^*$ – complex shear wave velocity

$V_{SW}$ – horizontal amplitude spectrum of the Rayleigh-part of the wavefield

$W_X (f)$ – experimental power spectral density
Symbols

$x$ – spatial coordinate

$x_1$ – spatial coordinate

$x_2$ – spatial coordinate

$x_3$ – spatial coordinate

$x(t)$ – time signal

$x_d[n]$ – discrete-time version of signal $x(t)$

$x_p(nT)$ – sampled version of signal $x(t)$

$X(\omega)$ – Fourier Transform of time signal $x(t)$

$y(t)$ – time signal

$Y(\omega)$ – Fourier Transform of time signal $y(t)$

$z$ – complex variable used as argument in $z$-transform

$z$ – depth at which the SPT blow count is made

$Z_j(f)$ – factor due to site response in generalized inversion scheme

Greek alphabet

$\alpha$ – impedance ratio

$\alpha$ – parameter of the Ramberg-Osgood model

$\alpha$ – parameter of Newmark-beta method

$\alpha_i$ – incidence angle of medium $i$

$\beta$ – parameter of Newmark-beta method

$\gamma$ – engineering shear strain

$\gamma(t)$ – shear strain time history
Symbols

\( \dot{\gamma}(t) \) – shear strain time derivative

\( \gamma(t) \) – shear strain amplitude

\( \gamma_{\text{assumed}} \) – shear strain assumed at the beginning of an iteration of the linear equivalent method

\( \gamma_{\text{a}} \) – shear strain at the point of unloading condition

\( \gamma_{\text{eq}} \) – equivalent shear strain

\( \gamma \) – shear strain at failure

\( \gamma_{\text{oct}} \) – octahedral shear strain

\( \epsilon \) – linear elastic shear strain threshold

\( \gamma_{\text{sat}} \) – saturated soil self-weight

\( \gamma_{\text{unsat}} \) – unsaturated soil self-weight

\( \chi \) – volumetric shear strain threshold

\( \Delta W \) – energy dissipation

\( \delta \) – shape factor

\( \delta(\Delta) \) – Dirac delta

\( \delta_{ij} \) – Kronecker delta

\( \epsilon_{ij} \) – strain tensor

\( \epsilon_{\text{e1}} \) – first invariant of the strain tensor

\( \epsilon_{\text{oct}} \) – octahedral volumetric strain

\( \epsilon_{\text{v}} \) – volumetric strain

\( \epsilon_{\text{l}} \) – principal strain
Symbols

\( \varepsilon_{ii} \) – principal strain

\( \varepsilon_{ii} \) – principal strain

\( \eta \) – viscosity associated to the Kelvin-Voigt model

\( \eta \) – second-order term used in the building of matrix \( [B] \)

\( \lambda \) – corrective factor in order to account the rod’s length for the SPT test

\( \lambda \) – wavelength

\( \hat{\lambda}_{x;k} \) – \( n^{th} \) order spectral moment of time signal \( X(t) \)

\( \mu_k \) – first-moment of time signal \( X(t) \)

\( \nu \) – Poisson coefficient

\( \theta(x(\omega)) \) – phase of the Fourier transform of signal \( x(t) \)

\( \theta_t \) – rotation at the top of the soil sample at the Resonant Column test

\( \rho \) – density

\( \sigma_k \) – second-moment of time signal \( X(t) \)

\( \sigma_{ij} \) – stress tensor

\( \tau \) – lag

\( \tau \) – shear stress

\( \tau(t) \) – shear stress time history

\( \tau_{c} \) – cyclic shear stress

\( \tau_f \) – shear stress at failure

\( \tau_{st} \) – static shear stress

\( \tau_{ss} \) – stead-state strength
Symbols

\( \xi \) – critical damping ratio

\( \zeta \) – hysteretic damping ratio

\( \xi_k \) – damping coefficient of the active extremity at the Resonant Column test

\( \Omega \) – central frequency

\( \Omega \) – discrete version of angular frequency \( \omega \)

\( \omega \) – angular frequency

\( \omega \) – resonant frequency of the Resonant Column system

\( \omega_n \) – natural frequency

\( \omega_c \) – cutoff frequency

\( \omega_M \) – Nyquist frequency

\( \omega_s \) – sampling frequency

\( \omega_1 \) – frequency corresponding to the first vibration mode

Acronyms

CPT – Cone Penetration Test

CSR – Cyclic Shear-stress Ratio

CSW – Continuous Surface Wave

CRR – Cyclic Resistance Ratio

DFT – Discrete-time Fourier Transform

FFT – Fast Fourier Transform

H/V – Horizontal-to-Vertical
Symbols

PGA – Peak Ground Acceleration

PGD – Peak Ground Displacement

PGV – Peak Ground Velocity

RSR – Reference-site Spectral Ratio

SASW – Spectral Analysis of Surface Waves

SDOF – Single Degree Of Freedom

SESAME – Site EffectS using AMbient Excitations

SCPT – Seismic Cone Penetration Test

SPT – Standard Penetration Test

STA/LTA – Short-Time Amplitude/Long-Time Amplitude

SWM – Surface Wave Method

USGS – United States Geological Survey
1. Scope

1.1. Introduction

For a long time that the scientific community has the notion that surface geology plays an important role in what ground shaking is concerned. Observations showing that there is difference in damage between buildings with foundations over rock and buildings with foundations over soil date as back as 1824 (Kramer, 1996). Several earthquakes over the last fifty years have shown the importance that local geological conditions play on earthquake damage, such as the Niigata 1964, Mexico City 1985, Loma Prieta 1989, Northridge 1994, Great Hanshin 1995 and Izmit 1999 earthquakes. Effects due to surface geology settings are usually named site effects.

Site-effect assessment has been a subject of several areas of the scientific community, namely Engineering Seismology, Engineering Geology and Geotechnical Earthquake Engineering. It is sometimes difficult to establish the thresholds in which area framework one is working under. The author believes that, in order to effectively analyze site effects, one must not bear in mind the mentioned thresholds, and must accept the contribution of all areas, considering that all the approaches to site-effect assessment are linked.

The present work is a paradigm of the latter statement. The use of acceleration time series in order to characterize ground motion has been a crucial tool in every single area of Earthquake Engineering; in Geotechnical Earthquake Engineering, research on site effects has been mostly linked to modeling soil behavior under cyclic loading, liquefaction and ground-motion amplification. In Engineering Seismology, the use of experimental techniques, mainly spectral ratios (whether concerning velocity or acceleration time series), in order to assess site effects has been one of the main research directions for the last twenty years.

Both approaches to site-effect assessment were used in the present work, which led to the existence of two main subjects. The first main subject of the present work was the development of a constitutive model in order to account soil behavior under cyclic loading. This was made under the framework of PLAXIS, one of the most used finite-element code in Geotechnical Engineering. The other key subject of this work was the use of acceleration time series in order to assess site effects, whether it concerned ground-motion characterization, one- and two-dimensional modeling of geological/geotechnical setting, and spectral ratios.
Chapter 1 – Scope

The present work was developed under the research project POCI/CTE-GEX/58579/2004, named “Strong Site Effects at São Sebastião volcanic crater”. São Sebastião, at Terceira Island, Azores, has shown anomalous seismic behavior, much greater amplification than its surrounding areas. Within the crater itself, there has also been evidence of different behavior for different sites. These facts made São Sebastião a perfect place for site-effect assessment. Several efforts have been made to understand São Sebastião’s behavior, from the scientific areas previously mentioned, and all of them were considered in this work.

1.2. Thesis outline

This thesis contains seven chapters:

- Chapter 1 – Scope;
- Chapter 2 – Characterization of seismic motion;
- Chapter 3 – Soil behavior under cyclic loading;
- Chapter 4 – Site effects;
- Chapter 5 – Implementation of the Ramberg-Osgood model in PLAXIS;
- Chapter 6 – Study on site effect at São Sebastião volcanic crater;
- Chapter 7 – Conclusions.

In Chapter 2, the most important tools used to characterize ground motion are presented, always in the perspective of the use of acceleration time series. A brief review of essential signals and systems theory is made, and several issues concerning acceleration time series processing are shown. Focus is made on time-domain and frequency domain parameters, as well as on hybrid characterization parameters.

Soil behavior under cyclic loading is reviewed in Chapter 3. The importance of shear strain value in soil behavior is underlined, and an overview of the most used models to account cyclic loading is made. A presentation of soil testing in order to assess soil behavior is made, focusing on the shear strain induced by the tests.

State-of-the-art concerning site effects is presented in Chapter 4. The different types of site effects are discussed, bearing in mind its influence on acceleration time series, either on time-domain and/or on
frequency-domain content. Numerical modeling issues of site effects are presented, as well as experimental site-effect assessment techniques.

Chapter 5 concerns the implementation of a specific model to analyze soil behavior under cyclic loading, the Ramberg-Osgood model, in PLAXIS finite-element code. The mathematical description of the model is thoroughly presented, as well as several issues concerning numerical implementation. A description of the subroutine needed to embed in PLAXIS is made. Finally, several validation tests are presented.

Chapter 6 regards site-effect assessment at São Sebastião volcanic crater. The timeline of research activities is followed, and presentation of all the characterization efforts previous to this thesis is made. Ground-motion characterization of a significant number of acceleration time series recorded at São Sebastião is made, comparing the obtained values of characterization parameters for different places within the crater, and for places outside the crater. One-dimensional modeling is made, and the obtained results are compared to experimental site-effect assessment techniques, namely reference-site spectral ratio and horizontal-to vertical spectral ratio. Boreholes and Resonant Column tests were made during the development of the thesis, and were used to build a two-dimensional model of the crater.

Finally, Chapter 7 contains the main conclusions of the thesis, and discussion is made on future works.
Chapter 1 – Scope
2. Characterization of seismic motion

2.1. Introduction

For one to detect an eventual site effect, one must not only check the different consequences regarding damage (normally, through Modified Mercalli Intensity), but also compare the seismic motion records at different sites. To do so, one must comprehend deeply the most essential tool for the characterization of seismic motion: the accelerogram.

The first subject of this chapter will be a brief description of the apparatus needed to record seismic motion, the accelerometer, referring its functioning and the type of records obtained from the signals and systems point of view: continuous-time signal, discrete-time signal and analog-to-digital conversion (sampling).

The next step on the characterization of seismic motion will be a more detailed analysis of seismic motion records, concerning the time domain and the frequency domain. Several concepts essential to understand the seismic motion will be introduced, such as response spectra, stochastic modeling, spectral moments, and hybrid characterization.

The characterization of seismic motion by itself may be an extremely useful tool in assessing the existence of ground motion amplification. This approach will be used in Chapter 6, when studying São Sebastião

2.2. Accelerometer

A ground-motion acceleration time signal acquisition is made by the apparatus known as accelerometer. The underlying principle concerning its functioning is one of a mechanical system of high damping and such a high resonance frequency that dynamic amplification of the system is unlikely for the usual type of excitation (seismic event). In what damping is concerned, the usual value for critical damping ratio for accelerometer rounds $\xi=60-80\%$; for resonance frequency, it is always higher than 25Hz. When subjected to ground-motion acceleration, the system mass oscillates, existing several ways of acquisition. As an example, in the forced-balanced accelerometers, the system mass displacement is sensed (by piezo-electrical or optical sensors), and an opposing force is applied to nullify the displacement. The magnitude of the stabilizing force is easy to obtain, and it is assumed that this force is equal to the force resulting from the ground-motion acceleration.
There are two types of data acquisition:

- Analog (continuous) acquisition;
- Digital (discrete) acquisition.

Figure 2.1 includes a schematic representation of the most usual analog accelerometers, the SMA-1:

Figure 2.1 - Analog Accelerometer (Kinemetrics SMA-1)

Despite being clearly an obsolete apparatus, there are still several of these accelerometers functioning, due to its robustness.

There are many recording media used in analog accelerometers, such as:

- Photographic paper
- Paper
- Analog cassette

Figure 2.2 presents a scheme concerning a digital acquisition accelerometer, with a piezo-electrical bending sensor:
In the case of digital accelerometers, the data is recorded, usually, to:

- RAM memory (Hard Disc Drive, memory cards);
- Digital cassette.

Nowadays, in order to enhance storage capacity and remote control, there are accelerometers with several types of connection (dial-up, satellite, IP) allowing data transmission.

The dynamic range associated to analog accelerometer sensors is much narrower than the one usual to digital accelerometer, as both analog and digital accelerometers have as upper recording acceleration a value around 1.5g to 2g, but the lower acceleration that analog accelerometers are able to record is, normally, 0.005g; for digital accelerometers, one may register accelerations as low as $10^{-6}$ g (Bard, 2006).

In order to signal the existence of a seismic event, there is an acceleration threshold from which the accelerometer records the event. To this threshold, one usually calls trigger acceleration. The main goal related to the trigger acceleration is to save storage capacity in continuous recording mode (it was absolutely critical when the accelerometric networks were analog).

Another technological aspect of major importance concerning accelerometer is the ability to record pre- and post-event acceleration. The fact that the time series is not tailored in a way that information concerning signal-to-noise ratio is or is not lost has an extreme importance in what data processing is concerned.
Data precision is another important aspect. Modern digital accelerometers may store data with precision to 24 bits, although most records have lower precision (16 bits and even 10 bits). Precision is crucial when measuring accelerations resulting from weak events, since low precision may lead to loss of information.

### 2.3. Signals and Systems

The cornerstone of the concepts presented along this chapter is the fact that the registered acceleration may be considered as an output signal of a mechanical system dependent on local geology and on the geotechnical parameters of the respective formations. For the different type of data acquisition, there are specific features concerning the records.

#### 2.3.1. Analog Records

Analog accelerometers that are still part of the actual accelerometric networks record physically the acceleration time signal. Along with the record itself, there is the generation of a pulse with a given time step. Hence, knowing the time pulse, one is able to convert the spatial record into a time record.

Under the signals and systems framework, analog time signals may be analyzed either in the time domain or in the frequency domain, as long as the system may be qualified as **Linear and Time Invariant** (LTI). The passage to the frequency domain is made, if the signal is periodic, expressing the latter as a linear combination of complex exponential (harmonic) signals with different angular frequencies, known as the **Fourier Series**. For non-periodic signals, the Fourier transform is applied:

\[
X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-i\omega t} \cdot dt
\]  

(2.1)

where \(x(t)\) is the time signal and \(\omega\) is the angular frequency. \(X(\omega)\) and \(x(t)\) form a **Fourier pair**. The Fourier transform may be seen as an extension of the Fourier Series. As the period tends to infinite, the harmonic components form a continuum.

When acceptable, analyzing the signal in the frequency domain presents many advantages. First and most important of all, it allows a much easier resolution of ordinary and partial differential equations, since these become algebraic equations. Besides that, it allows a complete characterization of the system by a single function, called **transfer function**, as shown in Equation 2.2:

\[
Y(\omega) = H(\omega) \cdot X(\omega)
\]  

(2.2)

where:
• \( X(\omega) \) - Fourier transform of input signal;
• \( Y(\omega) \) - Fourier transform of output signal;
• \( H(\omega) \) - Transfer function.

This kind of resolution approach is known as the complex response method. When applying this method, it becomes essential to understand the concepts of amplitude and phase. When applying the Fourier transform, as it implies the product of a real signal by a complex exponential, its result, \( X(\omega) \), for each value of the angular frequency, \( \omega \), may have both a real and an imaginary part. The absolute value, \(|X(\omega)|\), is its amplitude; its phase is expressed as the angle formed by the number’s representation on the complex plane and the real axis. Equation 2.3 and Equation 2.4 express both amplitude and phase:

\[
|X(\omega)| = \sqrt{\text{Re}(X(\omega))^2 + \text{Im}(X(\omega))^2} \quad (2.3)
\]

\[
\theta(X(\omega)) = \arctan\left(\frac{\text{Im}(X(\omega))}{\text{Re}(X(\omega))}\right) \quad (2.4)
\]

Amplitude and phase representations as functions of angular frequency are usually known as, respectively, Fourier amplitude spectrum and Fourier phase spectrum. These are essential tools for signal and system analysis. Concerning the transfer function, its amplitude is commonly referred to as the gain of the system; its phase is known as the phase shift of the system. Equation 2.5 and Equation 2.6 describe the system in terms of amplitude and phase:

\[
|Y(\omega)| = |H(\omega)| \cdot |X(\omega)| \quad (2.5)
\]

\[
\theta(Y(\omega)) = \theta(H(\omega)) + \theta(X(\omega)) \quad (2.6)
\]

For LTI systems, it is also possible to characterize the system by a single function in the time domain. the unit impulse function is a singularity (non-significant duration) such that:

\[
\delta(\Delta) = \begin{cases} 
\frac{1}{\Delta}, & 0 \leq t < \Delta \\
0, & \text{otherwise} 
\end{cases} \quad (2.7)
\]

for a small enough \( \Delta \), which leads to
An important notion is that the input signal may be described as a superposition of weighted shifted pulses (samples):

\[ x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau \]  

(2.9)

The response due to the input may be interpreted as a shifted and scaled version of one function, commonly referred to as *unit impulse response*, weighting the sampled input. Equation 2.10 describes the *impulse response method*:

\[ y(t) = \int_{-\infty}^{\infty} x(t) \cdot h(t-\tau) \cdot d\tau = x(t) * h(t) \]  

(2.10)

where:

- \( x(t) \) - Input signal;
- \( y(t) \) - Output signal;
- \( h(t) \) - Unit impulse response.

The integral that allows the determination of the response is referred to as *convolution integral*.

There is a relation between the complex response method and the impulse response method. If the output signal is determined in time domain by Equation 2.10, in the frequency domain the system is described by Equation 2.2, where \( x(t) \) and \( X(\omega) \), \( y(t) \) and \( Y(\omega) \), \( h(t) \) and \( H(\omega) \) are, respectively, Fourier pairs.

The use of analog records has always led to several difficulties in calculating integral-dependant parameters. There was also the problem associated to the excessive consumption of (physical) memory. Hence, as technology developed and as there was the possibility, the analog accelerometers have been replaced and their records digitally converted.

### 2.3.2. Digital Records

Characterizing a discrete time signal (or *time series*) presents many similar aspects to the characterization of continuous records, existing many parallel concepts. It is possible to analyze a
time series in the frequency domain applying the Discrete-time Fourier Transform (DFT), given by Equation 2.11:

\[ X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-i\omega n} \]  

(2.11)

Notice that, in spite of the discrete nature of the signal, the DFT is continuous. An important feature concerning the DFT is that, due to the periodicity inherent to the discrete complex exponential (harmonic) signals \( e^{i\omega n} = e^{i2\pi n} \), the DFT is always periodic, as it is a linear combination of discrete harmonic signals.

Discrete-time signal processing has suffered a major improvement with the implementation of the well-known FFT (Fast Fourier Transform) algorithm (Cooley and Tukey, 1965). Taking advantage of the periodic nature of the DFT, the FFT allows a much faster calculation of the latter.

There is also an obvious similarity between the convolution integral and the convolution sum for the time series, given by Equation 2.12:

\[ y[n] = \sum_{k=-\infty}^{\infty} x[n] \cdot h[n-k] = x[n] \ast h[n] \]  

(2.12)

It is possible to extend the concept concerning the relation between the complex response and unit impulse response methods, as the unit impulse response and the transfer function constitute a Fourier pair.

2.3.3. Analog-to-digital conversion

The use of digital accelerometers is based in the fact that, under certain conditions, a continuous-time signal (e.g., the acceleration time signal) may be represented, without any loss of information and with the possibility to be completely recovered, by samples equally spaced in time. The sampling is made through an auxiliary discrete-time signal, a periodic impulse train known as sampling function:

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \]  

(2.13)

where \( T \) is the sampling period. The sampling function may also be characterized by the sampling frequency, \( \omega_s = 2\pi/T \).

The sampled signal, \( x_p(t) \), is:
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\[ x_p(t) = x(t) \cdot p(t) \]  

(2.14)

The sampling theorem states the necessary conditions for which the continuous-time signal \( x(t) \) may be correctly characterized by the sampled signal \( x_p(t) \). For continuous band-limited signals, \( i.e., \) with \( X(\omega) = 0, |\omega| > \omega_M \), \( x(t) \) may be uniquely determined by its samples if \( \omega_s > 2\omega_M \). The continuous-time signal is sampled the impulse train. This impulse train is then processed through an ideal lowpass filter with constant gain \( T \) and cutoff frequency \( \omega_C \) greater than \( \omega_M \) and lesser than \( \omega_s - \omega_M \). \( \omega_M \) is referred to as the Nyquist frequency. Note that the ideal filter eliminates the periodic part in the discrete-time signal Fourier amplitude spectrum.

Applying an ideal filter has, obviously, a time-domain counterpart, as a filter is a transfer function. The unit impulse response associated to an ideal filter is given by Equation 2.15:

\[ h(t) = \frac{\omega_c \cdot T \cdot \sin(\omega_c \cdot t)}{\pi \cdot \omega_c \cdot t} \]  

(2.15)

Resulting in

\[ x_s(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \frac{\omega_c \cdot T \cdot \sin(\omega_c \cdot (t - nT))}{\omega_c \cdot (t - nT)} \]  

(2.16)

This kind of resolution, exclusively in the time domain, is referred to as band-limited interpolation. If the conditions concerning the sampling theorem are satisfied, there is no loss of information.

Disrespecting the sampling theorem’s conditions, through undersampling, leads to a phenomenon known as aliasing, consisting in a superpositioning of the periodic parts of DFT. The example in Figure 2.3, considering a sinusoidal continuous-time signal, shows the effect of aliasing.
After sampling, one can truly process a continuous-time signal, $x(t)$, through a discrete-time signal, $x_d[n]$, such that:

$$x_d[n] = x_p(nT)$$  \hspace{1cm} (2.17)

The process of, at first, sampling $x(t)$ to obtain $x_p(nT)$ and next, converting the latter into $x_d[n]$, is known as analog-to-digital conversion. In modern accelerograms, this conversion is made in situ. The obtained records are purely digital.

In the frequency domain, the relationship between the DFT’s of the sampled signal and of the digital signal are shown in Figure 2.4:

![Figure 2.3 – Effect of Aliasing](image)

![Figure 2.4 - Relationship between Fourier transforms of continuos signal (X), sampled signal (X_p) and discrete signal (X_d) (Oppenheim et al., 1997)](image)
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\( X_d(\Omega) \) is a frequency-scaled version of \( X_p(\omega) \) such that \( X_d(\Omega) = X_p(\omega \cdot T) \). This scaling effect also affects the transfer function. If the sampling theorem conditions are satisfied, the digital and the analog transfer functions are related by Equation 2.18:

\[
H_d(\Omega) = \begin{cases} 
H_c(\omega) & |\omega| < 2\omega_s \\
0 & |\omega| > 2\omega_s 
\end{cases}
\] (2.18)

In order to recover the response, one has to do the reverse process, referred to as digital-to-analog conversion. This process follows two steps. At first, the digital response is converted to an impulse train, such that \( y_d(nT) = y_d[n] \), with \( T \) equal to the sampling period. Next, an ideal lowpass filter with gain equal to \( T \) and cutoff frequency equal to \( \omega_s / 2 \) must be applied.

The overall system involving discrete-time processing of continuous-time systems is shown at Figure 2.5:

![Figure 2.5 - Discrete-time processing of continuous-time signal (adapted from Oppenheim et al., 1997)](image)

2.4. Data processing

The use of acceleration time series without any data processing may difficult the correct interpretation of the several aspects that matter in an engineering point of view. Accelerogram processing is mainly due to two phenomena. The first one implies the existence of non-zero mean acceleration considering the total duration of the time series. This leads to a detection of artificial signal energy for long periods, with severe consequences in time-domain integration. Bard (2006) states as possible causes for this effect hysteresis in the acquisition system, ground deformation and incorrect analog-to-digital conversion (aliasing). The second one concerns signal-to-noise ratio. Typically, the significant acceleration time series bandwidth concerning seismic motion has as higher interval boundary no more than 20Hz. If there are significant values of the Fourier amplitude spectrum over 20Hz, one may conclude that the signal-to-noise ratio is low. When such thing happens, the acceleration time series may be extremely difficult to analyze, especially in the time domain. Noise is classified as a broadband process, having significant amplitude along the amplitude spectrum.
There are many techniques in order to adjust the acceleration time series, either in the time domain and/or in the frequency domain. It is important to notice that analog records and digital records may have different issues, leading to different processing techniques.

One of the most usual techniques for the acceleration time series processing is the well-known baseline correction, or baseline adjustment. This technique had as its first main goal the elimination of the previously referred mean acceleration, as the existence of a long period artificial acceleration leads to unreasonable velocity and displacement time series. However, it was noted that baseline adjustment was also effective in removing low frequency noise (Boore and Bommer, 2005).

The technique consists of adjusting the acceleration time series along the acceleration axis. This adjustment is made normally analyzing the wrongful velocity and displacement time series, admit certain trends (there may be more than one trend) in those time histories, and, through differentiation, remove the trends on the acceleration record. There are several proposals regarding baseline adjustment procedures. Boore and Bommer (2005) refer the use of a technique first suggested by Iwan (1985), in which, after studying instrumentation, the authors recommend the adjustment of two different linear trends in the velocity time series, leading to two constant adjustments in the acceleration time series. The same authors also mention the application of a quadratic fit along the velocity time series for the event duration. For weak events, there may be the need to use more than one technique.

The existence of pre-event memory plays an important role on the baseline adjustment procedure, as one, having pre-event memory, knows the initial boundary conditions. The onset of the earthquake signal may not be easy to define, as the trigger acceleration may not define it (especially for weak-motion events). Hence, it is essential to do a detailed analysis of the record in order to define the exact moment when the signal starts.

Another issue to consider is the assumed boundary conditions for time-domain integration at the end of the record. It is usual to admit that, at the end of the record, both velocity and displacement are equal to zero. In what the displacement time series is concerned, this may not be necessarily true, as there may be residual displacements. However, for engineering purposes, residual displacements have no significant influence.

When applying baseline adjustment, one must take account that this technique has an unknown counterpart in the frequency domain. Therefore, it may be interesting to apply filters a priori, and use baseline adjustment only if needed (Bard, 2006). This procedure allows a better signal control.
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Filters are a fundamental tool for system control, and accelerogram analysis is no exception. A great advantage of applying filters is its objectiveness, as one knows exactly its consequences in the frequency domain. Another advantage is its versatility, as filters may remove low-frequency and high-frequency noise. When filters are applied, one must always be aware that there will be loss of information; the fundamental issue is evaluating if that information is of any importance in terms of signal characterization.

As mentioned in analog-to-digital conversion, a filter consists of a given system, with a given transfer function, diminishing the signal’s amplitude for a certain bandwidth. Filters may be classified as lowpass, highpass, and bandpass, controlling, respectively, the high-frequency content, the low-frequency content, and both high and low frequency content. Figure 2.6 shows the effect of filters applied to a signal:

![Effect of Filtering](image)

**Figure 2.6 - Effect of filtering**

Applying ideal filters (i.e., the pure cut of a given bandwidth) has non-desirable consequences in the time domain, originating a phenomenon referred to as **ringing**. Ringing leads to an oscillatory behavior of the filtered signal. This may be understood as an effect due to the inexistence of damping when an ideal filter is applied. All filters, in a certain measure, lead to ringing.

Concerning the application of highpass filters to acceleration time series, in order to remove low-frequency noise, the most important factor to consider may not be the kind of filter to apply, but a wise choice of the cutoff frequency to adopt (Boore and Bommer, 2005; Bard, 2006). There is no standard cutoff frequency; one must analyze the time series to choose the former. Another issue that one must keep in mind when applying filters is causality. A system is causal if it does not depend on future values. Boore and Akkar (2003) state that the use of causal filters distorts the time series in the
periods on engineering interest even for a cutoff frequency as low as 0.01Hz (as causal filters imply phase; acausal filters are phaseless). Applying acausal filters has an additional advantage: the velocity and displacement time series are less sensitive to the choice of the cutoff frequency when one applies acausal filters. Hence, applying acausal filters is preferable, in the case one has pre- and post-event memories. If these memories are not available, a zero padding operation may be made, but baseline adjustment may be needed afterwards.

The choice of the type of filter to apply to a time series is a trade-off between time-domain and frequency-domain consequences, respectively, settling time of the step response and band transition. In Engineering Seismology, Butterworth filters are the most used. Their main features are a wide transition band and no passband ripple, leading to a small settling time.

The transition bandwidth depends on the filter’s order, diminishing for increasing order. It is usual to apply fourth-order Butterworth filters. If one is to apply a \( n \)-th acausal filter, the latter is produced by \( 2 \cdot n/2 \)-th causal filters passing by the time series in opposite directions. Figure 2.7 shows the block diagram associated to acausal filters:

![Figure 2.7 - Block diagram associated to acausal filters](image)

The production of filters implies the use of the \( z \)-transform, instead of the DFT (filtering is easily processed for digital records). The \( z \)-transform is an extension of the DFT, allowing the analysis of some LTI systems that don’t comply the convergence conditions inherent with the DFT. The \( z \)-transform is described by Equation 2.19:

\[
X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} \quad \text{(2.19)}
\]

where \( z \) is a complex variable. Expressing \( z \) in polar form:

\[
z = r \cdot e^{i\omega} \quad \text{(2.20)}
\]

Equation 2.19 turns into:
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\[ X(r \cdot e^{i\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot (r \cdot e^{i\omega})^n = \sum_{n=-\infty}^{\infty} r^n \cdot x[n] \cdot e^{-i\omega n} \quad (2.19a) \]

From Equation 2.19a, one may conclude that the z-transform is the DFT of the sequence \( x[n] \) multiplied by a real exponential \( r^n \). The z-transform reduces to the DFT for \( r = 1 \). In the complex plane, the DFT is represented by the unit circle.

An area in the complex plane, considering all the complex values for which the series converge, normally defines convergence for the z-transform. This area is called the region of convergence (ROC). For finite duration signals (such as acceleration time series in seismic events), the ROC is the entire complex plane, except possibly \( z = 0 \) or \( z = \infty \).

The definition of the transfer function (Equation 2.2) may be extended to the variable \( z \):

\[ Y(z) = H(z) \cdot X(z) \iff H(z) = \frac{Y(z)}{X(z)} \quad (2.21) \]

An arbitrary transfer function, within the use of the z-transform, may be completely characterized by by what is usually known as the poles and the zeros. Describing the transfer function as a quotient between two polynomials, the roots of its numerator are its zeros, and the roots of its denominator are the poles. It is highly convenient to express the transfer function as a polynomial fraction, in order to easily obtain its zeros and poles.

For the production of a Butterworth filter, one must define:

- Normalized cutoff frequency (with respect to the Nyquist frequency).
- Filter order.

From this data, it is possible to describe the transfer function and, consequently, the filter is produced.

Another issue concerning acceleration time series processing is the phenomenon known as spectral leakage. When determining the frequency content of a discrete-time signal, using the DFT, one implicitly considers that the given time series is a period of a periodic waveform. If the cycles considered in the time series are not an integer, there is a discontinuity on the waveform, and an artificial amount of energy is generated. This effect is clearly understood considering a harmonic discrete-time signal (e.g., \( \sin(\omega t) \)). Instead of having a zero-valued DFT for frequencies other than \( \omega \), there are clearly non-zero values (therefore “leaking” energy to the other frequencies).
This phenomenon must be taken into account especially when one is only interested in a portion of the time series (as usual in seismic motion processing). To consider a given portion, one must apply a *time window*. This procedure consists of multiplying the original signal by an auxiliary time series, named the *window function*. Window functions are zero-valued, except for the time corresponding to the portion that is submitted. When applying time windows, one must be aware that there is always leakage. The choice of the adequate type of time window to apply depends largely on what is pretended and on the frequency content of the signal. If one wants to resolve two harmonic components of similar frequencies, one should consider *high-resolution* windows; if one wants to isolate weak harmonic components of different components in noisy signals, one should consider *high dynamic range* windows. The choice of the correct window is a tradeoff between resolution and dynamic range. In seismic motion records, the Tukey (co-sine tapering) window is widely used, as it allows a choice (depending on the percentage) between resolution and dynamic range. The most used when site-effect assessment is concerned is a 5% co-sine tapering window. It is a high-resolution window (in fact, almost rectangular).

### 2.5. Time-domain Characterization

Time-domain characterization alone is not the most valuable approach if one aims to characterize an acceleration time history. The latter is clearly influenced by the physical phenomena involved in the ground-motion generation, and its respective parameters: Magnitude (source), source-to-site distance (attenuation, propagation path) and site parameters (site effects).

The only parameters regularly used that one may immediately obtain from the time histories are the *peak ground values*. Peak ground values have been used to characterize ground motion since the onset of seismology, as these values are related to the amplitude and, in some way, should contain information about the damage (intensity) resulting from the seismic motion. At the beginning of accelerometer-based seismic analysis, there were indeed many leads to a strong correlation between Peak Ground Acceleration (PGA) and intensity. However, as the number of seismic records increased, great dispersion was noticed.

Peak values do not consider many facts that highly contribute to damage, such as effective duration, spectral content and the number of cycles existing in the acceleration time series. Nevertheless, PGA is the simplest parameter to obtain from an acceleration history, and it is widely used for a first evaluation of damage, as typical damaging earthquakes have PGA higher than 0.1g. PGA is often used for response spectra and rigid structures analyses.
As far as Peak Ground Velocity (PGV) and Peak Ground Displacement (PGD) are concerned, they have no better correlation with intensity than PGA.

2.6. Frequency-domain Characterization

Figure 2.8 presents the acceleration, velocity and displacement time histories for the 1986 San Salvador earthquake.

![Figure 2.8 – Time histories for 1986 San Salvador Earthquake](image)

Even having an initial glance at the time histories, one may say that the time histories have quite different frequency content. The acceleration time history is the signal with larger bandwidth, presenting high-frequency content (in terms of earthquake engineering). The bandwidth becomes narrower with time-domain integration.

2.6.1. Fourier amplitude spectrum

Passage to the frequency domain, as mentioned earlier, is made through the Fourier transform. As far as accelerograms are concerned, an essential tool for the frequency-domain analysis is the Fourier amplitude spectrum.

The typical shape of a Fourier amplitude spectrum may be described as follows:
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- Between null frequency and a given value, known as corner frequency, the amplitude spectrum increases quadratically. The corner frequency diminishes with increasing magnitude.

- The amplitude is nearly constant for an interval ranging from the corner frequency to a given frequency, usually 10-15Hz. This amplitude plateau widens for increasing seismic moment.

- After the plateau, the amplitude values decay.

An immediate analysis of the amplitude spectrum may allow the identification of the predominant period or frequency. This is especially true when the relevant bandwidth is narrow. Predominant frequency on the rock outcrop (i.e., not considering site effects) presents a strong dependency on magnitude and on source-to-site distance. The predominant frequency diminishes for increasing magnitude and for increasing source-to-site distance.

![Figure 2.9 – Variation of predominant period with distance and magnitude (in: Kramer, 1996)](image)

2.6.2. **Stochastic modeling**

The idea of modeling the seismic motion as a stochastic process has appeared early on earthquake engineering research. A stochastic process is such that each function value (the acceleration time history) must be considered the concretization of a random variable. Hence, describing the process implies statistical characterization, normally by its moments, i.e., its mean and its variance. As there is
a probability density function for every single time value, one has to consider joint probabilities. An essential concept throughout this section is the *autocorrelation function*. For two different instants, \( t_1 \) and \( t_2 \), the autocorrelation function, \( R_X \), corresponds to:

\[
R_X(t_1, t_2) = E[X(t_1) \cdot X(t_2)]
\]  
\[ (2.22) \]

Considering the lag as \( \tau = t_2 - t_1 \), the autocorrelation function is:

\[
R_X(t, \tau) = E[X(t) \cdot X(t + \tau)]
\]  
\[ (2.23) \]

For lag equal to zero, one has:

\[
R_X(t) = E[X(t) \cdot X(t)] = \sigma_X^2 + \mu_X^2
\]  
\[ (2.24) \]

For non-periodic processes, the autocorrelation function tends to zero as the delay increases.

There are two important notions when stochastic processes are concerned: *stationarity* and *ergodicity*. Stationarity regards the maintenance of statistical moments as the process develops. A first-order stationary process is such that all the probability density functions have the same mean; a second-order process maintains not only the mean but also the variance. Ergodicity regards the maintenance of the statistical moments for different realizations of the process. A \( n^{th} \)-order ergodic process is such that, for all the times the process occurs, the moments until the \( n^{th} \)-order are the same. This implies that a single realization allows the characterization of the process. Note that ergodicity implies stationarity. However, stationarity does not imply ergodicity.

### 2.6.2.1. Power Spectrum

For non-periodic finite-energy second-order stationary signals, the autocorrelation function has three important properties. The first property to consider is that the autocorrelation function has its maximum for lag equal to zero, *i.e.*, that the total energy of the signal corresponds to the maximum amplitude of \( R_X \). Second, the autocorrelation function is even. The last and most important property is the consequence of one being under the conditions of the *Wiener-Khinchin Theorem*. When this theorem is applicable, the autocorrelation function has a Fourier pair:

\[
R_X(\tau) = \int_{-\infty}^{\infty} S_X(\omega) e^{i\omega \tau} d\omega
\]  
\[ (2.25) \]

\[
S_X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_X(\tau) e^{-i\omega \tau} d\tau
\]  
\[ (2.26) \]
where $S_X(\omega)$ is referred to as the power spectral density. For lag equal to zero, one obtains:

$$R_X(0) = E[X^2] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_X(\omega) d\omega$$

(2.27)

One important property to consider is that the power spectral density is an even function. There are two other useful definitions for the power spectral density. The unilateral power spectral density, $\overline{S}_X(\omega)$, is such that:

$$\int_{-\infty}^{\infty} \frac{\overline{S}_X(\omega)}{\omega} d\omega = \int_{-\infty}^{\infty} S_X(\omega) d\omega$$

(2.28)

which, considering that $S_X(\omega)$ is even, leads to:

$$\overline{S}_X(\omega) = 2S_X(\omega)$$

(2.29)

The definition of experimental power spectral density, $W_X(f)$, is given by Equation 2.30:

$$\int_{-\infty}^{\infty} W_X(f) df = \int_{-\infty}^{\infty} S_X(\omega) d\omega$$

(2.30)

Considering that $\omega = 2\pi f$, one may say that (Azevedo, 1996):

$$W_X(f) = 4\pi \cdot S_X(\omega)$$

(2.31)

It is also possible to define the power spectrum density through the Parseval’s Relation, which states that the energy content may be obtained either in the time domain or in the frequency domain, which leads to:

$$\int_{0}^{T} x^2(t) dt = \frac{1}{2} \int_{0}^{\omega_M} |X(\omega)|^2 d\omega$$

(2.32)

where $\omega_M$ is the Nyquist frequency and $T$ is the signal duration. Considering the definition of power (energy per time), one must have:

$$\frac{1}{T} \int_{0}^{T} x^2(t) dt = \frac{1}{2T} \int_{0}^{\omega_M} |X(\omega)|^2 d\omega$$

(2.33)

and:
\[
\frac{1}{T} \int_0^T x^2(t)dt = \int_0^{\omega_u} S_x(\omega)d\omega
\]  
(2.34)

Equation 2.33 and Equation 2.34 lead to the following relationship between the Fourier amplitude and the power spectral density:

\[
S_x(\omega) = \frac{1}{2T} |X(\omega)|^2
\]  
(2.35)

The representation of the power spectral density in the frequency domain is referred to as the power spectrum, and it completely characterizes a stationary process. It is the most complete tool for seismic motion characterization, as it contains information about amplitude, frequency and duration of the seismic motion.

During the development of stochastic modeling in earthquake engineering, seismic motion was considered a stationary, zero-mean, Gaussian process. However, analyzing an accelerogram, one may see that seismic motion is clearly a non-stationary process, as there is a strong change of amplitude (variance) at the arrival of the S-wave train:

![Figure 2.10 - Acceleration time series of the February 12th 2007 Earthquake](image)

Nevertheless, the study of stationary zero-mean Gaussian processes has a great deal of interest, as non-stationary modeling is normally obtained from the former, existing several techniques to do so, such as filter processing or multiplication by a deterministic (modulation) function (Clough and Penzien, 2003; Kramer, 1996; Lin et al., 1997).
2.6.2.2. Spectral moments, central frequency and shape factor

Following the definition of power spectral density, one may define the concept of spectral moment. The $n^{th}$-order spectral moment is defined by Equation 2.36:

$$\lambda_{X,k} = \int_{-\infty}^{\infty} \omega^k S(\omega)d\omega = \int_{0}^{\infty} (2\pi f)^k W_X(f)df$$

(2.36)

A consequence of the Wiener-Khinchin theorem is that one is able to relate the derivative of processes through the spectral moments:

$$\begin{cases}
R_X(\tau) = \int_{-\infty}^{\infty} \hat{S}_X(\omega)e^{i\omega \tau}d\omega \\
R_X(\tau) = -\frac{d^2}{d\tau^2}R_X(\tau) = \int_{-\infty}^{\infty} \omega^2 S(\omega)d\omega \\
\Rightarrow S_X(\omega) = \omega^2 S_X(\omega)
\end{cases}$$

(2.37)

These considerations are valid for the following derivatives. Hence, one may write:

$$\lambda_{X,0} = \lambda_{X,2} \quad \text{and} \quad \lambda_{X,0} = \lambda_{X,2} = \lambda_{X,4}$$

(2.38)

The spectral moments are also useful to characterize the ground motion, via the power spectrum. Vanmarcke (1976) defined the central frequency, $\Omega$, as:

$$\Omega = \sqrt{\frac{\lambda_{X,2}}{\lambda_{X,0}}} = \sqrt{\frac{\sigma_X^2}{\sigma_X^2}}$$

(2.39)

The power contained in the signal tends to concentrate around the central frequency, as this parameter gives the expected rate of mean-upcrossings (Benacsiutti & Tovo, 2005). Note that the central frequency may not be coincident to the predominant frequency (especially for wider frequency distributions).

Another interesting parameter obtained using the spectral moments is the shape factor:

$$\delta = \sqrt{1 - \frac{\lambda_{X,1}^2}{\lambda_{X,2}\lambda_{X,0}}}$$

(2.40)

The shape factor is extremely useful to characterize the process bandwidth from the power point of view. For values close to one, the process is of broadband nature; values close zero translate a narrowband process.
2.7. Characterization using response spectra

Historically, the use of response spectra is intimately related to structural engineering. Response spectra are obtained considering the ground motion at the base of a single degree of freedom (SDOF) linear oscillator, which may be classified as a LTI system. The response of a SDOF linear oscillator is obtained considering the motion equation:

\[ m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + k \cdot u(t) = F(t) \]  

(2.41)

where:

- \( m \) - System mass;
- \( c \) - Viscous damping coefficient;
- \( k \) - Elastic stiffness;
- \( F(t) \) - Excitation at the base (input signal);
- \( u(t) \) - Oscillator’s displacement (output signal).

Considering the undamped natural frequency, \( \omega_n \):

\[ \omega_n = \sqrt{\frac{k}{m}} \]  

(2.42)

and the critical damping ratio, \( \xi \):

\[ \xi = \frac{c}{2 \sqrt{km}} = \frac{c}{2m \omega_n} \]  

(2.43)

Equation 2.41 becomes:

\[ \dddot{u}(t) + 2 \xi \omega_n \cdot \ddot{u}(t) + \omega_n^2 \cdot u(t) = \frac{1}{m} F(t) \]  

(2.41a)

When the input signal consists of a ground motion, one must consider two additional facts. First, the inertial force is proportional to the absolute acceleration. Second, the restitution forces are proportional to the relative displacement. As long as there is no external force, one may write:
\[ \ddot{u}(t) + 2\xi\omega_n \cdot \dot{u}(t) + \omega_n^2 \cdot u(t) = -i\ddot{u}_g(t) \]  

(2.44)

where \( u(t) \) corresponds to relative displacement and \( \ddot{u}_g(t) \) corresponds to ground motion acceleration. The resolution of the differential equation imposes determining the homogeneous (steady-state) solution and the particular (forced) solution. The steady-state solution is given by Equation 2.45:

\[ u_{ss}(t) = e^{-i\omega_n t} \left( A e^{i(\sqrt{1-\xi^2} \cdot \omega) t} + B e^{-i(\sqrt{1-\xi^2} \cdot \omega) t} \right) \]  

(2.45)

where \( A \) and \( B \) are obtained considering the boundary conditions.

For the particular solution, one must consider the seismic motion. Considering that the seismic motion is such that the Fourier transform converges (allowing harmonic characterization of the input signal), one may define the transfer function for this system, given in Equation 2.46:

\[ H = \frac{1}{(\omega_n^2 - \omega^2) + i \cdot 2\xi\omega_n \omega} \]  

(2.46)

The transfer function’s amplitude is:

\[ |H| = \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega_n \omega)^2}} \]  

(2.47)

And its phase is:

\[ \theta(H) = \arctan \left( \frac{2\xi\omega_n \omega}{\omega_n^2 - \omega^2} \right) \]  

(2.48)

One may also obtain the displacement time history using the unit impulse response. For this system the unit impulse response is:

\[ h(t) = \frac{e^{-i\omega_n t}}{\omega_n \sqrt{1 - \xi^2}} \cdot \sin \left( \omega_n \sqrt{1 - \xi^2} \cdot t \right) \]  

(2.49)

The relative displacement may be determined by the following convolution integral:
This particular formulation of the convolution integral is known as Duhamel integral.

After obtaining the relative displacement, one may obtain all the response spectra, as relative velocity, absolute acceleration and unit energy depend on relative displacement.

For a given spectrum, one studies the oscillator’s response to the ground motion, for several periods (or frequencies), for a given damping ratio. The most used value for the damping ratio is 5%.

### 2.7.1. Displacement spectrum

The displacement spectrum is a representation of the displacement time histories’ absolute value maxima for the different SDOF oscillator’s natural periods (or frequencies) and for a given critical damping ratio:

\[
S_d(T_n, \xi) = \max_t \| u(t, T_n, \xi) \| \tag{2.51}
\]

As stated in 2.6, the displacement time history bandwidth is narrower than for the other relevant time histories. As the response spectra are usually represented in a period vs. Spectral ordinate plot, the displacement spectrum tends to have its maximum values for higher periods, comparing to the other relevant spectra.

The typical shape of a displacement spectrum may be described as follows:

- From natural period equal to zero to a given value of natural period, the displacement spectrum increases. The increasing part of the spectrum widens for increasing magnitude. One may see it as a consequence of richer low-frequency content due to a lower corner frequency.

- Next, the displacement spectrum presents, approximately, a plateau, wider for increasing magnitude.

- Finally, for low- and average-magnitude events, there is decay in the displacement spectrum, converging to a given value. For high-magnitude events, this decay may not happen for significant periods in an engineering point of view \((0 < T < 10s)\).

Figure 2.11 presents displacement spectra for seismic events of increasing magnitude:
The displacement spectrum is the most sensitive tool to the application of signal processing, especially for periods longer than 10s. Theoretically, the displacement spectrum should converge to PGD as $T_n \to \infty$, since, for a flexible oscillator, the absolute value of the oscillator’s relative displacement should be equal to the ground displacement. However, due to the problems explained in 2.4, namely the existence of a mean acceleration not equal to zero, most times, for a displacement spectrum obtained from a “raw” acceleration time history this won’t happen. It may also not tend to PGD if one considers residual displacements. The simultaneous use of quadratic-fit baseline adjustment and an acausal Butterworth filter is the less sensitive technique in what long-period displacement spectrum ordinate is concerned (Boore and Akkar, 2003). Normally, for periods longer than 10s one has a plateau in the displacement spectrum, but the spectrum’s shape for long periods is not a consensual issue (Boore, 2004; Boore and Bommer, 2005).

The use of the displacement spectrum to characterize the seismic motion has gained greater importance, as recent developments in the structural design philosophy point out to a displacement-based seismic design (Priestley et al., 1996; Chopra and Goel, 2001; Priestley, 2006). Indeed, the fundamental action imposed to a structure is a displacement at its base, with consequences in terms of velocity, acceleration and energy (work). It allows the determination of the different maxima of the elastic restitution force, as $F_{El} = k \cdot u(t)$.
2.7.2. **Velocity spectrum and pseudo-spectrum**

The definition of velocity spectrum is perfectly similar to the definition of displacement spectrum: it is a representation of the absolute value maxima concerning the relative velocity time history, for different values of the oscillator’s natural periods, for a given critical damping ratio.

\[ S_v(T_n, \xi) = \max_t |\dot{u}(t, T_n, \xi)| \]  \hspace{1cm} (2.52)

The velocity time history may be obtained through the derivative of the Duhamel integral:

\[ \dot{u}(t) = -\int_0^t \ddot{u}_0(\tau) \cdot e^{-\xi \omega_n (t-\tau)} \cdot \cos(\omega \sqrt{1-\xi^2} (t-\tau)) d\tau + \xi \omega_n u(t) \]  \hspace{1cm} (2.53)

Just as the displacement spectrum, the velocity spectrum shape is sensitive the event’s magnitude.

The velocity spectrum does not have a direct physical counterpart on the oscillator’s response (Bilé Serra, 2005). It is usual to define the velocity pseudo-spectrum, obtained from the displacement spectrum by multiplying it by the undamped natural frequency for each period:

\[ PS_v(T_n, \xi) = \omega_n \max_t |u(t, T_n, \xi)| = \frac{2\pi}{T_n} \max_t |u(t, T_n, \xi)| \]  \hspace{1cm} (2.54)

The velocity pseudo-spectrum is directly related to the elastic potential energy of the oscillator.

2.7.3. **Absolute acceleration spectrum and pseudo-spectrum**

The absolute acceleration time history is obtained through the motion equation concerning the SDOF oscillator, considering that one already has the relative displacement and velocity time histories:

\[ \dddot{u}(t) = -2\xi \omega_n \dot{u}(t) - \omega_n^2 (t) \cdot u(t) \]  \hspace{1cm} (2.55)

Retaining the absolute maxima for different natural periods, for a given damping ratio, one is able to build the absolute acceleration spectrum. As it happens for the velocity spectrum, the absolute acceleration itself is not one of the most used tools. Instead, the absolute acceleration pseudo-spectrum is, undoubtedly, the most used tool when one uses response spectra to characterize a seismic motion. It is obtained, once again through the displacement spectrum:

\[ PS_a(T_n, \xi) = \omega_n^2 \max_t |u(t, T_n, \xi)| = \frac{4\pi^2}{T_n^2} \max_t |u(t, T_n, \xi)| \]  \hspace{1cm} (2.56)
One may justify the use of this pseudo-spectrum as a consequence of the initial steps on the development of earthquake engineering. The seismic motion (i.e., a displacement at the base), would be directly converted into statically equivalent restitution force acting at the oscillator considering:

\[ F_{el}(T_n, \xi) = k \cdot S_d(T_n, \xi) = m \cdot PS_a(T_n, \xi) \]  \hspace{1cm} (2.57)

The acceleration pseudo-spectrum is also the best spectrum to detect differences between acceleration time histories in terms of frequency content and oscillator’s amplification.

There are some remarkable values that one must consider when using the acceleration pseudo-spectrum. Theoretically, for natural period equal to zero (infinitely rigid oscillator), one must have absolute acceleration (and pseudo-acceleration) equal to PGA. For a completely flexible oscillator, i.e., with \( T_n \rightarrow \infty \), one must have pseudo-acceleration equal to zero, as there is no transmission of inertial force to the mass.

2.7.4. Demand Curves

The building of demand curves follows research efforts to characterize the structural response to a dynamic action in purely static terms (Bard, 2006; Pinho, 2006). It has become an essential tool in what building retrofitting studies are concerned.

When one considers the displacement spectrum and the acceleration pseudo-spectrum, one may describe the maximum acceleration as a function, not of the natural period, but of the maximum displacement. Therefore, one may obtain a representation \( S_d(\xi) \) vs. \( S_a(\xi) \), which, knowing the system mass, is directly convertible into a force-displacement curve:

One must determine the effective viscous damping of the system through iteration, considering the dissipative plateau of the capacity curve, which must be in agreement with \( \xi \).

2.8. Hybrid characterization

Some of the already presented tools to characterize the seismic motion could be named as “hybrid” (e.g., the power spectral density), as they contain information about peak values or frequency content. However, the following parameters do not have a clear relation to these topics, and must be presented individually.
2.8.1. Spectral Intensity

The first notion of spectral intensity was created by Housner. The underlying idea is that the frequency content where the velocity spectrum is rich matches the current natural frequencies of ordinary structures. In fact, there is a close relation between the velocity spectrum and the acceleration Fourier amplitude spectrum. There were also important leads to a close correlation between higher velocity content and damage. As a result of these two issues, Housner defined the spectral intensity:

\[ \text{SI}(\xi) = \int_{0.1}^{2.5} \frac{PS_v(T_n, \xi)}{T_n}dT_n \]  

(2.58)

2.8.2. Duration and Arias Intensity

There are several definitions concerning duration, that can be grouped as follows:

- Bracketed duration;
- Uniform duration;
- Energy-based duration.

The first two kinds of duration have a given acceleration threshold that defines the significant acceleration from which one starts to consider an effective duration. The difference between bracketed duration and uniform duration lies in the continuity of the effective duration. For the bracketed duration approach, one considers that effective duration lasts from the first exceedence of the acceleration threshold until the last exceedence of the latter; when considering uniform duration, the effective duration is the sum of the different time intervals where the acceleration is higher than the defined threshold.

Recalling the Parseval’s relation, the energy of a signal is:

\[ E = \int_{t_0}^{t_f} x^2(t)dt \]  

(2.59)

Where \( t_0 \) and \( t_f \) are, respectively, the onset and the end of the signal. The Arias intensity is directly associated to the energy of the acceleration time history:

\[ AI = \frac{\pi}{2g} \int_{t_0}^{t_f} a^2(t)dt \]  

(2.60)
There are several Energy-based definitions of effective duration. Husid (1969) first used the Arias intensity to define effective duration. The graphical representation of the evolution of the signal’s Arias intensity along time is referred to as the *Husid plot*. Figure 2.12 shows a typical Husid plot:

![Typical Husid Plot](image)

**Figure 2.12 – Typical Husid Plot**

Trifunac and Brady (1975) defined the effective duration as the time interval between 5% and 95% of the total Arias intensity at the end of the accelerometric record. This implies that the effective duration is closely related to the high-energy variation part of the signal. Trifunac and Westermo (1977) presented a similar definition. The effective duration, according to this definition, corresponds to the shortest time interval for which 90% of the signal’s energy is contained. This definition allows the contribution of second wave arrivals to the effective duration.

Energy-based definitions establish a criterion with direct physical meaning, both for weak motion and strong motion events.

There is a close relation between duration, magnitude and fault slip rupture. Duration is also related to damage.

**2.9. Concluding remarks**

The basics of signals and systems theory were presented, as these are a fundamental tool as far as seismic motion analysis is concerned. A close insight on signal processing techniques has been made, as it is of extreme importance when one is handling with acceleration time series.
Several parameters that allow characterizing the seismic were reviewed along this chapter. Focus was made not only on the parameters themselves, but also on what features of the seismic motion they are fit to characterize.

There are two main approaches to seismic motion analysis:

- **Time-domain analysis**, where the parameters most used to characterize the seismic motion are the peak values.

- **Frequency domain analysis**, where amplitude and power density spectra are the most important tools.

Both approaches are linked to each other, as showed by stochastic modeling. There are several parameters/techniques that allow considering both time-domain and frequency-domain features of seismic motion, being the response spectra a paradigm of such techniques.

Many of the presented parameters will be used to characterize seismic motion at São Sebastião, as one will see in Chapter 6.
3. Soil behavior under cyclic loading

3.1. Introduction

Evidence from major earthquakes, such as the Mexico, 1985; Loma Prieta, 1989; Northridge, 1994; and Kobe, 1995; has shown that soft alluvial deposits tend to amplify the ground motion, especially for lower frequencies (Pecker, 2006). However, one may not say in a straightforward manner that the existence of soft soil deposits corresponds to ground motion amplification. The soil response depends on the frequency content of the incident motion. The same author states that another factor essential to the soil response is the level of shaking induced by the earthquake. For different shaking levels, there are significant changes on the dynamic properties of the soil.

3.2. Soil behavior for very small shear strains

For very small shear strains, most soils respond according to the linear elastic model. Considering the conservation equation and Hooke’s law, one may relate the elastic properties of soils and wave propagation velocities. Let us consider that a wave is propagating only in one direction, and only inducing shear strain. Recalling Hooke’s law, one has (Equation 3.1):

$$\sigma_{ij} = \frac{E}{1+\nu} \epsilon_{ij} + \frac{E \nu}{(1+\nu)(1-2\nu)} \epsilon_{kk} \delta_{ij}, \quad i, j = 1, 2$$

(3.1)

If only shear strains are induced, $\delta_{11} = \delta_{22} = 0$, leading to:

$$\sigma_{12} = \sigma_{21} = \tau = \frac{E}{1+\nu} \epsilon_{12} = \frac{E}{2(1+\nu)} \gamma_{12} = G \cdot \gamma_{12}$$

(3.2)

Considering the dynamic equilibrium equation, as displacement derivatives in directions other than the propagation direction are null, one is led to:

$$\sigma_{ij} - \rho \cdot \ddot{u}_j = 0 \Rightarrow G \cdot u_{2,11} = \rho \cdot \ddot{u}_2 \Rightarrow \ddot{u}_2 - \frac{G}{\rho} u_{2,11} = 0 \Rightarrow \ddot{u}_2 - V_s^2 \cdot u_{2,11} = 0$$

(3.3)
where $V_s$ is the shear wave velocity and $\rho$ stands for the material density. As one may see, the dynamic and the mechanical properties are closely related. For a confined medium, the pressure wave velocity, $V_p$, may also be determined solving the dynamic equilibrium equation.

$$\sigma_{ij,j} - \rho \cdot \ddot{u}_j = 0 \Rightarrow M \cdot u_{l,11} = \rho \cdot \ddot{u}_i \Rightarrow \ddot{u}_i - \frac{M}{\rho} u_{l,11} = 0 \Rightarrow \ddot{u}_i - V_p^2 \cdot u_{l,11} = 0$$ (3.4)

As there is a relation between the bulk modulus, $K$, and the other elastic moduli, $M$ and $G$, one may determine the former:

$$K = M - \frac{4}{3} \cdot G = \left( \rho \cdot V_p^2 - \rho \cdot \frac{4}{3} V_s^2 \right) = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right)$$ (3.5)

This relation between dynamic and mechanical properties is the cornerstone of many tests for soil characterization, as one will show in 3.6.

### 3.3. Soil behavior from small to medium shear strains

However, even for small strains, soils exhibit energy dissipation and non-linear behavior, which cannot be modeled by the linear elastic model. It is usual to define a threshold, in terms of shear strain, for which the linear elastic model is no longer an acceptable tool, named as the linear cyclic threshold shear strain (Vucetic, 1994; Santos, 1999).

A simple mechanical system that allows one to consider energy dissipation of soils implies the parallel disposal of a spring and a dashpot, as seen in Figure 3.1. This system is known as the Kelvin-Voigt Model.
The constitutive relation of this model is given in Equation 3.6:
\[ \tau(t) = G \cdot \gamma(t) + \eta \cdot \dot{\gamma}(t) \]  
(3.6)

Considering a harmonic distortion, Equation 3.6 becomes:
\[ \tau(t) = G \cdot \gamma(t) + \eta \cdot \dot{\gamma}(t) = G \cdot \gamma_a \cdot e^{i\omega t} + i \cdot \eta \cdot \omega \cdot \gamma_a \cdot e^{i\omega t} = (G + i \cdot \eta \cdot \omega) \cdot \gamma_a \cdot e^{i\omega t} = (G + i \cdot \eta \cdot \omega) \cdot \gamma(t) \]  
(3.7)

This model leads to energy dissipation equal to:
\[ \Delta W = \int \text{Re}(\tau(t)) \cdot \text{Re}(d\gamma(t)) = \pi \cdot \eta \cdot \omega \cdot \gamma_a^2 \]  
(3.8)

This value alone doesn’t allow inferring the dissipation capacity, as nothing is known about elastic dissipation. Therefore, to account the latter, specific dissipation capacity or damping coefficient (analogy with SDOF oscillator) is defined as:
\[ \xi = \frac{1}{4 \pi} \cdot \frac{\Delta W}{W_{el}} = \frac{1}{4 \pi} \cdot \frac{\pi \cdot \eta \cdot \omega \cdot \gamma_a^2}{\frac{1}{2} \cdot G \cdot \gamma_a^2} = \frac{\eta \cdot \omega}{2 \cdot G} \]  
(3.9)

Energy dissipation under a cycle becomes:
\[ \Delta W = \pi \cdot \eta \cdot \omega \cdot \gamma_a^2 = \pi \cdot \frac{\xi}{\omega} \cdot 2 \cdot G \cdot \omega \cdot \gamma_a^2 = 2 \cdot \pi \cdot \xi \cdot G \cdot \gamma_a^2 \]  
(3.10)
Equation 3.9 indicates that energy dissipation should depend on the exciting frequency (i.e., soil response should depend on the loading rate). However, experimental evidence contradicts the above statement for frequencies of engineering interest. For cycles of constant amplitude (harmonic excitation), the soil response is characterized by a hysteresis loop. This implies that the damping is due to hysteresis (friction between soil particles), and that the damping coefficient is the parameter that characterizes energy dissipation, and not the viscosity. Hence, the Kelvin-Voigt Model is usually presented as:

\[
\tau(t) = (G + i \cdot \eta \cdot \omega) \gamma(t) = \left( G + i \cdot \frac{2 \cdot G}{\omega} \cdot \xi \cdot \omega \right) \gamma(t) = G \cdot (1 + i \cdot 2 \cdot \xi) \gamma(t) = G^* \cdot \gamma(t)
\]

(3.11)

where \( G^* \) is the complex shear modulus. As one may see from Equation 3.10, the effect of damping translates into phase. Equation 3.11 also leads to the definition of the complex shear wave velocity:

\[
V_s^* = \sqrt{\frac{G^*}{\rho}}
\]

(3.12)

A fundamental principle that allows the use of viscoelastic models in one-dimensional modeling of soft soil deposits is that the solutions for the dynamic equilibrium equation considering linear elastic behavior may be used, if one replaces the elastic shear wave velocity by the complex shear wave velocity. The former principle is known as the *Equivalence principle*.

The Kelvin-Voigt model was presented within the framework of one-dimensional analysis. Linear viscoelastic models may be generalized to two- or three-dimensional approaches. Under Lax’s principle, one may take Hooke’s law and expand it to allow energy dissipation, replacing the real bulk and shear moduli by their complex counterpart.

As it has been said, soil behavior under cyclic loading does not depend on the load rating. Figure 3.2 shows typical hysteresis loops for different shear strains.
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The larger the shear strain, the wider and flatter the hysteresis loop is. This leads to an increase of hysteretic damping and to a reduction of soil stiffness. It is usual to present this effect via secant shear modulus reduction and damping increase curve, such as those presented in Figure 3.3.

Figure 3.2 - Hysteresis loops for different shear strains (Pecker, 2006; adapted from Kramer, 1996)

The use of linear viscoelastic models alone doesn’t take into account these effects, as the soil properties are strain-independent. In order to surpass this shortcoming, and having as purpose the use of linear viscoelastic models, with all the presented features and equations inherent to these models, the Linear Equivalent Method was developed. This method will be discussed in 4.4.1.

Another approach used to model energy dissipation and stiffness reduction is to consider non-linear elastic models, coupled with the so-called Masing criterion. The use of these models has as its core the following definition: Hysteretic behavior of soils may be modeled by two curves, one relative to monotonic loading, known as the backbone or skeleton curve, and the other one relative to unloading/reloading conditions. Masing (1923) related both curves, under the following premises:
Chapter 3 – Soil behavior under cyclic loading

- The secant shear modulus for the strain (or stress) reversals is equal to the initial shear modulus;

- The shape of the unloading/reloading curve is equal to the backbone curve, but it is scaled up by a factor equal to two.

Mathematically, the Masing criterion is expressed by Equation 3.13, Equation 3.14 and Equation 3.15 (Santos, 1999):

\[
\tau = f(\gamma), \text{ for the backbone curve} \quad (3.13)
\]

\[
\frac{\tau - \tau_a}{2} = f\left(\frac{\gamma - \gamma_a}{2}\right), \text{ for the unloading curve} \quad (3.14)
\]

\[
\frac{\tau + \tau_a}{2} = f\left(\frac{\gamma + \gamma_a}{2}\right), \text{ for the reloading curve} \quad (3.15)
\]

The index \(a\) stands for the point of stress/strain reversal for the unloading curve. In the case of constant amplitude cycles, the stress/strain reversal for the reloading curve is perfectly symmetrical of the strain reversal for the unloading curve. Therefore, the reloading curve has as reference the strain reversal for the unloading curve. For closed cycles, one is also able to easily determine the damping coefficient.

\[
\xi = \frac{2}{\pi} \cdot \left[\frac{2}{\gamma_a} \cdot \int_0^{\gamma_a} f(\gamma) d\gamma - 1\right] \quad (3.16)
\]

For non-harmonic loading, the cycles no longer have constant amplitude and loading curves become far more complex. The two premises of the Masing criterion alone aren’t enough to describe the soil behavior. Kramer (1996) adds two premises in order to have a better description of the soil model:

- If the previous maximum shear strain (in absolute value) is overcome, the stress path follows the backbone curve.

- If the \(n\)-th stress/strain cycle is closed, the stress path may not surpass the path defined by the \((n-1)\)-th cycle.

Figure 3.4 contains the stress/strain path for an arbitrary loading.
One of the main advantages of the use of these models is that, for general loading, one is able to determine irreversible shear strains, i.e., the use of the extended Masing criterion allows one to consider plastic distortions via a non-linear elastic model. However, one must bear in mind that the use of this approach is only an approximation, as there isn’t a complete mathematical description of soil behavior, namely about volumetric behavior.

Next, one will present the most used models coupled with the Masing criterion.

### 3.3.1. Hyperbolic model

According to the hyperbolic model, the backbone curve is defined by Equation 3.17:

\[
\frac{d\tau}{d\gamma} = G_0 \left( 1 - \frac{\tau}{\tau_f} \right)^n
\]

(3.17)

where \( G_0 \) stands for the initial shear modulus, and \( \tau_f \) stands for the shear stress at failure. One may see clearly why the model is named hyperbolic: for increasing shear strains, the shear modulus asymptotically tends to 0. Another interesting feature is that the backbone curve is incrementally defined (hypoelastic model). Thus, Equation 3.17 defines the tangent shear modulus.

Normally, shear modulus reduction curves are strain-controlled; therefore, normally the backbone curve is defined as a function of the shear strain. Equation 3.17 is integrable:
\[
\frac{d\tau}{d\gamma} = G_0 \left( 1 - \frac{\tau}{\tau_f} \right)^n \iff d\gamma = \frac{1}{G_0} \left( 1 - \frac{\tau}{\tau_f} \right)^{-n} d\tau \iff \gamma + C = \frac{1}{n-1} G_0 \tau_f \left( 1 - \frac{\tau}{\tau_f} \right)^{1-n} \quad (3.18)
\]

Imposing as boundary condition that, for shear strain equal to zero, one has shear stress equal to zero, one may relate directly the shear stress and the shear strain, thus knowing the secant shear modulus. The stress/strain relation is the following:

\[
\gamma = \frac{\tau_f}{G_0} \left[ \left( 1 - \frac{\tau}{\tau_f} \right)^{1-n} - 1 \right] \quad (3.19)
\]

Kondner and Zelasko (1963), Duncan and Chang (1970) adopted the hyperbolic model for \( n=2 \). Hardin and Drnevich (1972) defined the reference shear strain, \( \gamma_r \), as:

\[
\gamma_r = \frac{\tau_f}{G_0} \quad (3.20)
\]

With this definition, one is able to determine the secant shear modulus reduction:

\[
\frac{G}{G_0} = \frac{1}{1 + \frac{\gamma}{\gamma_r}} \quad (3.21)
\]

The hysteretic damping coefficient inherent to the hyperbolic model coupled with the Masing criterion is given by Equation 3.22:

\[
\xi = \frac{4}{\pi} \left[ 1 + \frac{1}{1 + \frac{\gamma}{\gamma_r}} \right] \left[ \ln \frac{1 + \frac{\gamma}{\gamma_r}}{\frac{\gamma}{\gamma_r}} \right] - 2\pi \quad (3.22)
\]

The hyperbolic model presents as its main advantage the fact that it takes only two parameters to model the soil behavior: the initial shear modulus and the reference shear strain. However, it presents two important limitations. First, it is only when shear strains tend to infinity that no more strength may be mobilized. The second limitation concerns the fact that, for every possible case, the damping coefficient, for increasing values of shear strain, tends to a fixed value, equal to \( 2/\pi \).
In order to overcome the first issue of the model, Prévost and Keane (1990), proposed a modified hyperbolic model, allowing a plateau where no more shear strength may be mobilized.

The two shortcomings of the hyperbolic model lead to bad fitting between experimental curves and the model curves, especially concerning damping coefficient. One of main reasons that lead to the adoption of the Ramberg-Osgood model is linked to this fact.

### 3.3.2. Ramberg-Osgood model

Ramberg and Osgood (1943) first proposed a model with three parameters that would describe the stress/strain curves of aluminum-alloy and steel sheets. The first authors to use the Ramberg-Osgood model in soil modeling were Faccioli et al. (1973), in order to validate the shear modulus reduction curves first proposed by Seed and Idriss (1970) for sands. However, it was Idriss et al. (1978) who proposed the use of an adaptation of the Ramberg-Osgood model in order to obtain the shear modulus reduction. According to these authors, the stress/strain relation (backbone curve) follows Equation 3.23:

\[
\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} \left[ 1 + \alpha \left( \frac{\tau}{\tau_y} \right)^{\gamma^{-1}} \right] 
\]  

(3.23)
where \( \alpha \) and \( r \) are model (experimental) constants; and \( \gamma_y \) and \( \tau_y \) are reference values for shear strain and shear stress. The same authors adopted \( \tau_y = \tau_f \), i.e., the shear strength, and the reference strain, \( \gamma_y = \gamma_r \), as in the hyperbolic model:

This leads to:

\[
\frac{\gamma}{\gamma_r} = \frac{\tau}{\tau_f} \left[ 1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1} \right] \iff \tau = \frac{\gamma}{\gamma_r} \cdot \frac{\tau_f}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \iff \tau = \frac{\gamma \cdot G_0}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \tag{3.24}
\]

Knowing that the definition of secant shear modulus consists on the quotient between the shear stress and shear strain, one obtains:

\[
\tau = \frac{\gamma \cdot G_0}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \leftrightarrow \frac{G}{G_0} = \frac{1}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \iff \frac{G}{G_0} = \frac{1}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \tag{3.25}
\]

As it has already been mentioned when analyzing the hyperbolic model, most curves are expressed in terms of shear strains (strain-controlled). With this fact in mind, Equation 3.25 becomes:

\[
\frac{G}{G_0} = \frac{1}{1 + \alpha \left( \frac{\tau}{\tau_f} \right)^{r-1}} \leftrightarrow \frac{G}{G_0} = \frac{1}{1 + \alpha \left( \frac{\gamma}{\gamma_r} \cdot \frac{\tau}{\tau_f} \right)^{r-1}} \iff \frac{G}{G_0} = \frac{1}{1 + \alpha \left( \frac{G}{G_0} \cdot \frac{\gamma}{\gamma_r} \right)^{r-1}} \tag{3.26}
\]

As one may see, according to the Ramberg-Osgood model, it takes four parameters to define the secant shear modulus: the initial shear modulus, the reference shear strain, and the constants \( \alpha \) and \( r \).

The damping coefficient, according to this model, is equal to:

\[
\xi = \frac{2}{\pi} \frac{r - 1}{r + 1} \left( 1 - \frac{G}{G_0} \right) \tag{3.27}
\]

Equation 3.27 shows that the Ramberg-Osgood model implies an explicit relation between the secant shear modulus reduction and the damping coefficient increase. Santos (1999) refers that, for medium strains, it is reasonable to admit this assumption. Another feature that may be seen in Equation 3.27 is that is the parameter \( r \) that controls the energy dissipation inherent to a given soil. For increasing values of \( r \), the greater is the dissipation capacity. One must take into account that, as there is a linear
relation between the damping coefficient and the shear modulus reduction, one can foretell that the adoption of a certain value of $r$ also has its consequences in terms of shear modulus reduction. In fact, for low values of $r$, stiffness reduction, for the same reference strains and for the same $\alpha$, starts for lower strains and it happens more gradually. For high values of $r$, the shear modulus reduction occurs in the vicinity of the reference shear strain, with a steeper curve.

Figure 3.6 - Effect of parameter "r" in shear modulus reduction (Ishihara, 1996)

![Figure 3.6 - Effect of parameter "r" in shear modulus reduction](image)

Figure 3.7 - Effect of parameter "r" in damping ratio increase

The parameter $\alpha$ controls the shape of the modulus reduction curve. For the same reference shear strain and $r$, for increasing $\alpha$, one has shear modulus reduction for lower strains and the reduction of stiffness occurs in a smoother way.
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The parameters $\alpha$ and $r$ allow a much better fitting to experimental curves. This is, probably, the main reason why the Ramberg-Osgood is one of the most used models in geotechnical earthquake engineering. However, the Ramberg-Osgood model, according to the previous definition, shares a setback with the hyperbolic model: even for very large strains, one may mobilize a certain amount of strength. To surpass this issue, one defines the parameter $\alpha$ as a function of the reference shear strain (Ishihara, 1996).

$$\alpha = \frac{\gamma_f}{\gamma_r} - 1$$  \hspace{1cm} (3.28)

Where $\gamma_f$ is the shear strain at failure.

3.4. Medium to large strains

In fact, for an arbitrary loading, for a given shear strain threshold, the shear strain starts to be accompanied with volumetric strains (Figure 3.8).

![Figure 3.8 - Coupled volumetric and shear strains](image)

The soil behavior turns to be clearly non-linear. When volumetric strains start to play an important role, even for constant amplitude cycles, there is a change in shape of the hysteresis loop, as it becomes more inclined and has smaller area for an increasing number of cycles. An important fact concerning the coupling of volumetric and shear strains is that, as there is dilatation (or contraction), and, in most cases, during earthquakes, soils respond under undrained conditions (insufficient...
permeability for the loading rates imposed by earthquakes), pore-water pressure tends to vary. This may lead to severe consequences, as will be discussed in 4.3.1.

The threshold, in terms of shear strain, for which one is forced to consider volumetric behavior and, therefore, to use a full elastoplastic model, is usually known as the \textit{volumetric threshold shear strain} (Vucetic, 1994; Santos, 1999). The use of elastoplastic models implies that one must solve the differential equations of motion using a step-by-step procedure in the time domain, such as the Newmark method.

\textbf{3.5. Factors to be considered in soil behavior under cyclic loading}

As it has already been said, the soil behavior under cyclic loading is strongly dependent on the shear strain induced by ground shaking. Besides this, there are other factors that play a role in the dynamic properties of soils (Barros, 1997; Santos, 1999):\

- Effective stress normal to wave propagation path;
- Effective stress normal to the particle vibration direction;
- Void ratio;
- Plasticity Index;
- Saturation degree;
- Cementation.

Vucetic (1994) expressed the shear strain thresholds as functions of the Plasticity Index, as shown in Figure 3.9, where one may see that non-plastic soils, such as sands and some silts, present non-linear behavior for lower strains than plastic soils, namely clays (Santos, 1999).
By the definition of these thresholds, one has three domains where soil behaves distinctly. According to the same author, one must couple these domains of soil behavior with different modeling techniques.

<table>
<thead>
<tr>
<th>Cyclic shear strain amplitude, $\gamma$</th>
<th>Behavior</th>
<th>Elasticity and Plasticity</th>
<th>Cyclic degradation in saturated soils</th>
<th>Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very small $0 \leq \gamma \leq \gamma_s$</td>
<td>Practically linear</td>
<td>Practically Elastic</td>
<td>non-degradable</td>
<td>Linear Elastic</td>
</tr>
<tr>
<td>small $\gamma_s \leq \gamma \leq \gamma_v$</td>
<td>Non-linear</td>
<td>Moderately Elasto-plastic</td>
<td>Practically non-degradable</td>
<td>Equivalent linear</td>
</tr>
<tr>
<td>moderate to large $\gamma &gt; \gamma_v$</td>
<td>Non-linear</td>
<td>Elasto-plastic</td>
<td>degradable</td>
<td>Non-linear</td>
</tr>
</tbody>
</table>

In Chapter 4, one will present several methods (and respective models) to study the soil behavior when non-linear behavior becomes an important factor.

### 3.6. Soil characterization

From what has been discussed, soil behavior is strongly dependent on the shear strain imposed by the seismic event. Therefore, in order to characterize the properties of soils under cyclic loading, one must take into account this fact. Considering that from seismic events may result strains ranging from $10^{-6}$ to $10^{-2}$ (failure), this implies that, for a full description of soil behavior, more than one test has to be used, as every soil test imposes inherently a given strain and no soil test covers the strain range of
interest in an engineering point of view. This leads to the use of soil tests that are complementary and that must be used simultaneously, if one is to have the full characterization of soil behavior.

The fact that each soil test is associated to a given shear strain also has its implication on the soil modeling. For an instance, the use of soil testing in the very small-to-small shear strain range is normally linked to the use of viscoelastic models or non-linear elastic with Masing criterion models.

Beyond the difference on the imposed shear strain, there are other aspects that help to make a distinction between soil tests. Soil testing may directly measure a given property, or the given property may be determined via a correlation with a parameter directly obtained form the test. Another obvious distinction concerns field testing and laboratory testing.

Several soils tests will be briefly presented. These soil tests will be grouped mainly on the imposed shear strain, as this is the most important aspect of distinction between the different tests.

3.6.1. Soil testing in the very small to small strain domain

Soil testing in the very small strain domain takes advantage of the equations of wave propagation in a linear elastic medium, whether one is considering field or laboratory tests. One obtains an estimation of the propagation velocity of a given type of wave, which is directly related to the elastic parameters of the soil, as one discussed in Chapter 3. A clear distinction between the different tests concerns the type of wave (or, sometimes, energy source) used to assess the wished soil property, as one may determine the P-wave, S-wave or the Rayleigh-wave propagation velocity.

As far as field tests are concerned, the following are the most used:

- Refraction test;
- Reflection test;
- Surface wave method;
- Cross-hole test;
- Down-hole test.

Concerning the use of laboratory tests, the most used are the following:

- Resonant Column test;
• Bender Element test.

3.6.1.1. Tests made from ground surface

Both the refraction and the reflection tests require the use of an energy source (a hammer or explosives) and an alignment of geophones (devices that allow to measure velocity associated to ground motion) connected to a seismograph. The latter allows one to measure the time delay between the wave generation and the wave arrival at the geophones. The polarization of the propagating wave is associated to the used energy source. For an instance, the use of explosives or the use of a hammer hitting the ground perpendicularly to its surface leads to the generation of P-waves. For a hammer hitting transversely the ground surface, one generates S-polarized waves. The use of P-waves is admissible for sites where high water tables aren’t an issue to consider, as pressure waves also propagate on fluids. The use of S-wave wave presents as major setback the difficulty to obtain both a good S-polarization and sufficient energy, especially for larger distances. An important factor to consider in this test is until what depth one wants to obtain the dynamic/mechanical properties of the different formations. For the usual energy sources, one isn’t able to obtain information for depths above 30m.

These tests are based on the Snell’s law, which relates the wave propagation velocity of two media (1 and 2), according to Equation 3.29:

\[
\frac{V_1}{V_2} = \frac{\sin \alpha_1}{\sin \alpha_2}
\]  

(3.29)

where \(\alpha_1\) and \(\alpha_2\) stand for the incidence angle of media 1 and 2. When arriving at the layers’ interface, according to equilibrium and compatibility conditions in elastic media, there is reflection and refraction. The tests take advantage of these phenomena in distinct ways.

In the case of refracted waves, considering that the medium 1 is overlying medium 2, if the wave velocity of medium 2 is greater than the velocity of medium 1, as a consequence of Snell’s law, there is an incidence angle, \(\alpha_1\), known as critical incidence angle, for which \(\alpha_2 = \pi/2\). For this angle, the wavefront is modified, as wave propagation is made via the interface between the media 1 and 2, traveling with wave velocity corresponding to the media 2. From a given distance, \(x_{12}\), from the source, if one measures the time delay between the wave generation and the arrival, the measurement is no longer conditioned by what would be a direct propagation wave, but by the refracted wavefront (Figure 3.10).
Chapter 3 – Soil behavior under cyclic loading

In the reflection test, one takes advantage of waves reflected on the layers’ interface. One may determine the thickness of medium 1, as these are functions of the time arrival at the geophones and of the wave velocity of medium 1, which may be easily determined by the direct wave:

\[ t_{\text{arrival}} = \frac{\sqrt{x^2 + 4H^2}}{V_1} \]  

(3.30)

The geophone alignments for the refraction and reflection tests are disposed in different ways, as the tests are based on different waves, with different goals. In the refraction test, the spacing of the geophones and the length of the acquisition line is a direct function of the depth that one pretends to characterize. For the same acquisition line, the source is triggered in different positions. For the reflection test, the acquisition line “slides” with the source, and the space between geophones is narrower.

The refraction test, however, presents an important issue: if the stratigraphy is such that there are layers of lower wave velocity underneath other layers, the former are not detected, as one always admits increasing wave velocities along the depth.

The use of surface waves (namely, Rayleigh waves) was first developed to test the stationary vibration of a block foundation on a ground surface (Pecker, 2006). It was with the work of Nazarian and Stokoe (1984) that impact sources were first used to determine soil properties. These authors first used two acquisition devices and a seismograph. The obtained signals were transformed to the frequency domain. For surface waves, soils have what is called dispersive behavior, i.e., for different frequencies, soils have different wave propagation velocities. For each frequency, their phase shift is determined, allowing one to know the propagation time and velocity. Thus, one obtains the so-called experimental dispersion curve. The dispersion curve is the representation of the phase velocity as a
function of frequency. Assuming a given S-wave profile along the depth, one tries obtain the best fit the experimental and the theoretical dispersion curves.

![Dispersion Curves](image)

**Figure 3.11 - Fitting between experimental and theoretical dispersion curves (Pecker, 2006)**

There are several test procedures that have as background the propagation of surface waves. The procedure described in the previous paragraph is known as the *Spectral Analysis of Surface Waves* (SASW). The *Continuous Surface Wave (CSW)* procedure implies the use of a vibrating source, ranging different frequencies in order for one to obtain the dispersion curve. In order to characterize São Sebastião (Chapter 6), the *Surface Wave Method (SWM)* procedure (Strobbia, 2003) was used (Lopes, 2005). This procedure relies on multi-channel acquisition to overcome one of the setbacks of the SASW procedure: if only using two acquisition devices, one is only able to obtain an approximation of the fundamental mode. With multiple acquisition, the dispersion curve associated to higher modes may be determined. There are also several signal processing techniques in order to determine the phase shift between signals, such as the coherence function.

### 3.6.1.2. In-hole tests

These tests require, at least, the use of one borehole in which measuring devices are lowered, a PVC casing and grout to ensure a good coupling between the casing and the soil (Pecker, 2006).

In the down-hole test, the energy source is placed at the surface, in the vicinity of the borehole, and the measuring devices (geophones or hydrophones) are placed in the borehole. One may place only one device at several positions and repeat the impact, or one may have multi-acquisition, with several measuring devices. The last option is preferred, as the value obtained for the distance between measuring devices is more accurate, and the input signal is the same, which minimizes source effects.
on the measurements. The obtained wave velocities correspond to the wave profile along the borehole. This test has as one interesting feature: the possibility to sample layers of lower stiffness (if the spacing between measuring devices is small enough). Another aspect of extreme interest is that, as the test requires a borehole, one is able to easily relate the geological formations with the obtained wave velocities. However, it doesn’t allow to assess in-plane variations of wave velocity.

In the cross-hole, one uses two or more boreholes. In one of these boreholes, an energy source is placed inside the former (explosives or hammer), as the measuring devices are placed in the other boreholes. For each depth, the energy source is triggered, and the motion is recorded in a seismograph. Wave velocity is estimated via the direct wave. In this test, as in all others, one may obtain different types of wave polarization, depending on the source. In order to obtain the shear wave velocity profile, SV-polarized waves are used, as it is easier to generate shear waves with the vertical impact of a hammer (upwards or downwards).

In both these tests, the boreholes are supposed to be vertical. If not, the results obtained by in-hole tests should be corrected to take into account the bias.

### 3.6.1.3. Laboratory tests

In the resonant column test, one admits that soil behavior obeys the hysteretic Kelvin-Voigt model, and that the overall system behaves linearly. The system is lead to resonance via the tuning a given harmonic excitation. According to the Kelvin-Voigt model, one may relate the shear wave velocity with the soil sample stiffness and hysteretic damping. The soil sample stiffness varies with the imposed shear strain, leading to different resonance frequencies of system for harmonic excitations of different amplitudes. The resonant column apparatus scheme used in this thesis belongs to the Instituto Superior Técnico laboratory. The apparatus belongs to the Hardin type, in which one of the extremities is fixed, and the other one (the active extremity) is formed by a mass conditioned by a spring and a dashpot. It is this mass that allows one to excite the system. The excitation is made using electromagnetic coils. An accelerometer and a velocimeter are placed in the vicinity of the coils (the values obtained by these devices should be related, as the excitation is supposed to be harmonic). Figure 3.12 contains the apparatus scheme.
According to the one-dimensional wave propagation theory, and considering that the system is linear, if one knows the inertial properties of all the elements, one is able to determine the soil sample shear modulus. Figure 3.13 represents the resonant column system:
Santos (1999) obtained the analytical solution of this system. The shear modulus is determined via Equation 3.31:

\[
G \cdot I_p \cdot \frac{\omega}{\sqrt{\frac{G}{\rho}}} + \tan \left( \frac{\omega H}{\sqrt{\frac{G}{\rho}}} \right) \left( K - J_A \cdot \omega^2 \right) = 0
\]

where:

- \( I_p \) – mass moment of inertia of the sample;
- \( \rho \) – density of the sample;
- \( \omega \) - resonant frequency of the system;
- \( H \) – height of the sample.
- \( K \) – stiffness of the active extremity.
- \( J_A \) – mass moment of inertia of the exciting mass.

The hysteretic damping may be determined once one knows the shear modulus (Equation 3.32):

\[
\theta_0 = \frac{M_0 \cdot e^{i\alpha}}{G \cdot (1 + 2i \xi) \cdot I_p \cdot \frac{\omega}{\sqrt{\frac{G}{\rho} \cdot \sqrt{1 + 2i \xi}}} + K(1 + 2i \xi_k) - J_A \cdot \omega^2}
\]

\[
\tan \left( \frac{\omega H}{\sqrt{\frac{G}{\rho} \cdot \sqrt{1 + 2i \xi}}} \right)
\]

where:

- \( \theta_0 \) – Rotation at the top of the sample;
- \( M_0 \) – Amplitude of the torsional moment.
• $\xi$ – Damping coefficient of the active extremity.

As previously mentioned, this test is based on viscoelastic behavior of the soil sample. This means that the resonant column test is a useful tool not only for the very small strain domain, but for strains up to the volumetric strain threshold (typically, for shear strains not higher than $5 \times 10^{-4}$). In fact, the resonant column test is the most used tool to determine the dynamic properties of soil in laboratory environment. Another advantage associated to this test is the fact that it is non-destructable, which means that the same soil sample may be used to determine the soil properties at larger strains, if, for instance, a triaxial scheme is set in the same apparatus.

The bender elements test is fully based on one-dimensional wave propagation theory, within the scope of linear elastic behavior. At each of the extremities of a cylindrical soil sample (normally, inside a resonant column chamber), a piezokeramic transducer is placed. When electric current is applied to one of these transducers, it generates a given pulse, that will travel along the soil sample, until it reaches the other transducer. As one may understand, one of the transducers functions as energy source; the other one functions as receiver. As the name indicates, the standard piezokeramic transducer bends, inducing shear waves on the sample. One may have other polarizations, if the piezokeramic transducers function in a different way. Ferreira (2002), mentioned by Lopes (2005), refers the implementation of extender and compression transducers, inducing P-waves on the soil sample.

In order to determine the wave velocity, the simplest procedure consists on doing the ratio between the sample height and the time that the generated pulse takes to travel from the source to the receiver. There are also techniques in the frequency domain, namely analyzing the unwrapped phase spectrum and/or using the cross-correlation function between the input and the output signals.

### 3.6.2. Tests in the medium to large strain domain

In this strain range, one falls in the strain range of usual geotechnical tests, such as the Standard Penetration Test (SPT) and the Cone Penetration Test (CPT). The latter may be used in its “seismic” version (SCPT), in order to directly assess the wave velocity.

In the SPT test, one does not determine directly the shear wave velocity of the tested soil. Instead, one determines the SPT blow count, $N$, and corrects it in order to obtained the normalized value $(N_1)_{60}$, which is given by Equation 3.33, as it is in Eurocode 7 – part 2:

$$\left( N_1 \right)_{60} = C_N \cdot \lambda \cdot \frac{Er}{60} \cdot N$$  \hspace{1cm} (3.33)
where

- $C_N$ - Corrective factor in order to account the overburden pressure (sands only);
- $\lambda$ - Corrective factor in order to account the rod’s length;
- $E_r$ - Energy ratio (dependent of the used equipment).

There are several relations available in literature, depending essentially on the type of soil (grain size) and on the age of the geological formations. Lopes (2001) mentions that many of these were obtained in microzoning studies, and have a regional nature, being applied in other places with relative success. One of the most used relations was proposed by Ohta and Goto (1976; in Lopes, 2001). The shear wave velocity, $V_S$, is given by Equation 3.33

$$V_S = 69 \cdot (N_{60})^{0.17} \cdot z^{0.2} \cdot F_A \cdot F_B$$

(3.34)

Where

- $z$ - Depth at which the SPT blow count is made (m);
- $F_A$ - Factor concerning the geological age;
- $F_B$ - Factor concerning the grain size;

Table 3.2 and Table 3.3 give the values of $F_A$ and $F_B$.

<table>
<thead>
<tr>
<th>Geological Age</th>
<th>Holocene</th>
<th>Pliocene</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_A$</td>
<td>1.00</td>
<td>1.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Clay</th>
<th>Fine Sand</th>
<th>Medium Sand</th>
<th>Coarse Sand</th>
<th>Sandy Gravel</th>
<th>Gravel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_B$</td>
<td>1.00</td>
<td>1.09</td>
<td>1.07</td>
<td>1.14</td>
<td>1.15</td>
<td>1.45</td>
</tr>
</tbody>
</table>

The SPT test has also been used to assess liquefaction potential of the tested formations. As an example, Eurocode 8 – part 5 contains a plot for determining the liquefaction potential as a function of SPT results.

Similarly to the SPT test, one doesn’t determine directly the shear wave velocity from the CPT results. The shear wave velocity is usually expressed as a function of the cone resistance, $q_c$, and of...
the grain size. The CPT has essentially used in geotechnical earthquake engineering, not to determine the shear wave velocity, but to assess the liquefaction potential.

3.7. Concluding remarks

An insight concerning soil behavior under cyclic loading was made, focusing on different issues. At first, it was clearly shown that the shear strain imposed to soils by the seismic motion is a key factor when one pretends to analyse seismic response of soils. Within the shear-strain-range overlook, evidence was shown that modeling techniques and the strain range imposed to the soil are intimately connected. Three distinct zones have been clearly identified:

- For very small shear strains, one may consider that soils behave according to the linear elastic model, as non-linear strains are negligible.
- For small strains, hysteretic damping starts to play a role; models that allow to account damping, such as viscoelastic models or non-linear elastic with Mains criterion are used.
- For medium-to-large strains, volumetric strains become significant, and the use of full non-linear constitutive relations is required.

Accordingly, the concept of strain thresholds has been presented.

The use of non-linear elastic models coupled with the Masing criterion allows one to overcome this latter setback of the linear viscoelastic models. However, both the hyperbolic and the Ramberg-Osgood methods present a setback, as these models, even for very large strains, lead to a certain mobilization of strength.

There is also a close relation between soil tests used to determine dynamic properties of soils and the strain range imposed by them. In Chapter 6, many of the tests mentioned in the present chapter will be used in order to help in the site-effect assessment of São Sebastião.
4. Site effects

4.1. Introduction

The effect of surface geology on seismic ground motion has been one of the leading research directions in the past decades in Engineering Seismology and in Earthquake Geotechnical Engineering. As discussed in Chapter 2, acceleration time series at the surface clearly shows dependency on source, directivity, or on distance (propagation path), but many effects cannot be explained by these factors. They are the so-called site effects.

It is essential at this point to discuss the different aspects concerning local site effects. First, one will present effects that are directly linked to the ground motion, referred to as ground-shaking effects, namely the effects due to soil behavior under cyclic loading and topographic effects, always on the site effects point of view. Next, one will briefly present induced effects.

Deeper discussion will be made on the numerical modeling, focusing on the well-known linear-equivalent method and on non-linear modeling, whether it concerns one-dimensional, two-dimensional or three-dimensional analyses.

The different experimental site-effect assessment techniques will be presented. Even though one must always bear in mind that the present work is focused on the use of acceleration time series on the determination of site effects, state-of-the-art techniques that aren’t based on acceleration time series will be presented.

Site-effect assessment is an essential tool to do the microzoning of a given area, i.e., to subdivide a given area into sectors with similar behavior with respect to a given set of parameters.

4.2. Ground-shaking (direct) effects

Direct site effects are intertwined with wave propagation, with amplification (or de-amplification) for certain frequencies, according to the mechanical properties of the geological formations near the surface, and/or according to the topography.

These effects are due essentially to the following physical phenomena (Bard, 2006):

- Response of mechanical systems;
Chapter 4 – Site effects

- Non-linear behavior of soils;
- Wave trapping;
- Focusing of waves;
- Diffraction of waves.

The first two phenomena are the cornerstone of site-effect assessment of soil deposits; the last three are directly linked to topographic effects.

As one may conclude from what was mentioned above, signals and systems theory is essential to the study of site effects, as well as wave propagation theory. For the different type of direct site effects, the most important features concerning these theories will have a closer insight.

First, the effects of soil behavior under cyclic loading on the ground motion will be discussed, focusing on the issues such as energy dissipation, strain dependency, behavior thresholds and constitutive models. After that, one will present the main features concerning topographic effects.

4.2.1. One-dimensional amplification of ground motion

Amplification in soil deposits, for a simplified one-dimensional approach, may be explained considering that the geological layers right underneath the surface constitute a linear mechanical system, with a given transfer function. This may be valid for very low strains. An important remark that must be made at this point is that, when one considers this approach valid, one implicitly considers that the soil response is mainly caused by SH-waves propagating vertically on a half-space, horizontally layered. The first assumption may be justified by refraction along the propagation path, as the wave velocity tends to decrease for more superficial layers.

If the geological and geotechnical scenario is such that it may be modeled considering a superficial layer over a half-space, the fundamental frequency, i.e., the lowest frequency for which occurs major (indeed, the greatest) amplification is approximately obtained by Equation 4.1:

$$f_0 = \frac{V_s}{4H}$$  \hspace{1cm} (4.1)

The value of the transfer function associated to a superpositioning of soft soil layers is strongly dependent on the impedance ratio or impedance contrast between the layers. The impedance ratio between two layers (layer 1 and layer 2) is defined by Equation 4.2:
\[ \alpha = \frac{\rho_1 \cdot V_{S1}^*}{\rho_2 \cdot V_{S2}^*} \]  

(4.2)

Where \( \rho_1 \) and \( V_{S1}^* \) stand for the density and complex shear wave velocity of layer 1. The damping coefficient also plays a role in the value of the transfer function, not shifting, however, the fundamental frequency of the system.

The response of a multi-layered medium may be obtained via the complex response method. The transfer function is obtained considering reflections and refractions of a SH-polarized wave propagating upwards, with the impedance ratio playing a fundamental role on these phenomena. At each layer’s interface, the reflected and the refracted amplitude of the harmonic shear wave are direct functions of the impedance ratio, of incident amplitude and of the layer, as may be seen in Equation 4.3 (Bilé Serra, 2005):

\[
\begin{bmatrix}
A_{\text{incident}} \\
A_{\text{reflected}}
\end{bmatrix} = \frac{A_{\text{refracted}}}{2} \begin{bmatrix}
1 + \alpha & 1 - \alpha \\
1 - \alpha & 1 + \alpha
\end{bmatrix} \begin{bmatrix}
e^{-\frac{\alpha\omega t}{V_s}} \\
e^{-\frac{\alpha\omega t}{V_s}}
\end{bmatrix} \begin{bmatrix}
1 + \alpha & 1 - \alpha \\
1 - \alpha & 1 + \alpha
\end{bmatrix}^{-1}
\]

(4.3)

Note that, considering that this approach is valid, one may relate the seismic motion at the top and at the base (usually named seismic bedrock) inverting the transfer function of the system. In fact, the process of, knowing the acceleration time series at the top, obtaining the acceleration time series at the seismic bedrock is a fundamental tool on site-effect assessment. This process is named the deconvolution of the signal. In Chapter 6, this process will be used to model site effects at São Sebastião.

An important note concerning this kind of analysis is that one admits that the induced shear strain is harmonic. This statement, normally, is not true, leading to the need to use the Fourier Transform (for digital signals, normally, using the FFT algorithm).

4.2.2. Influence of soil behavior under cyclic loading on ground shaking

The difference of behavior as soil non-linearity plays a more important role has, of course, consequences in the soil response to seismic motion. A major consequence is linked to the fact that stiffness degrades for increasing shear strains. Recalling what has been exposed in Chapter 3, as stiffness degrades, the shear velocity of the soil decays, leading to a fundamental frequency shift towards lower frequencies. As the energy dissipation increases, for the fundamental frequency, amplification for the vibration modes gets less significant. This effect may be checked via the records’
characteristics. The Fourier amplitude spectrum is an obvious tool, as there are clear consequences in terms of frequency content. If possible to obtain, transfer functions are the best tool to assess the differences in soil response. The influence of soil behavior under cyclic loading is clearly noted on the registered PGA. Figure 4.1 allows the comparison between PGA obtained at soft soil sites, where the effect of soil non-linearity is strongly felt, and the PGA obtained at rock sites.

![Figure 4.1 - Relation between PGA at rock site and PGA at soft soil site (adapted from Bard, 2006)](image)

As one may see, for increasing strong motions, the ratio between PGA at soft soil site and PGA at rock site tends to decrease (if the soil behavior were to be elastic, this ratio would remain constant).

### 4.2.3. Topographic effects

Topography’s influence on acceleration time series leads mainly to the following situations (Oliveira et al., 2006; Bard, 2006):

- Amplification at the top of hills and de-amplification at its base;
- Amplification at basins or valleys, as well as when subsurface topography plays an important role;
- Lateral discontinuity of geological layers (fault system).

Effects concerning the amplification at the top of hills do not have a strong theoretical background yet. Bard (2006) states that the amplification may be due to focusing of waves, with reflection on the side slopes playing an important role. According to the same author, there’s a connection between
hills’ width and the frequency for which there is major amplification. There is also evidence of a connection between hills’ shape and the verified amplification.

Considering elastic behavior and dynamic equilibrium, one is able to relate frequency and spatial contents of waveforms. Equilibrium is given by Equation 4.4, for each of the components of the displacement field (Bilé Serra, 2005):

\[ V^2 \cdot \nabla^2 \left( u(x_1, x_2, x_3, t) \right) - \left( \bar{u}(x_1, x_2, x_3, t) \right) = 0 \]  

(4.4)

Where \( V \) stands for wavefront velocity. For propagation of a plane wave, applying separation of variables, the homogeneous part of the solution will be of the kind given by Equation 4.5:

\[ u(x_1, x_2, x_3, t) = X_1(x_1) \cdot X_2(x_2) \cdot X_3(x_3) \cdot T(t) \]  

(4.5)

where \( t \) stands for time and \( x_1, x_2 \) and \( x_3 \) stand for the three spatial dimensions. This leads to:

\[ \frac{T''(t)}{T} = V^2 \left( \frac{X_1''(x_1)}{X_1} + \frac{X_2''(x_2)}{X_2} + \frac{X_3''(x_3)}{X_3} \right) = \omega^2 \]  

(4.6)

where:

- \( \omega \) – Frequency [T\(^{-1}\)]
- \( V \) – Wavefront velocity [LT\(^{-1}\)]

The wavelength is defined as:

\[ \lambda = \frac{2\pi \cdot V}{\omega} = \frac{V}{f} \]  

(4.7)

and has [L] dimension. The wavelength represents the length of a complete wavecycle.

Major amplification at the top of hills occurs when the wavelength is equal to the hill’s width at the base. Knowing the wavefront velocity, one is able to determine the respective frequency. This effect has more significance for the horizontal component, and the amplification factor may range from two to ten (Bard, 2006).

The influence of hills’ shape is considered using the shape factor, defined in Equation 4.8:
\[ SF = \frac{h}{l} \]  

(4.8)

where:

\begin{itemize}
  \item \( h \) – height of the hill;
  \item \( l \) – width of the hill divided by two.
\end{itemize}

Amplification normally increases for increasing shape factor, \( i.e., \) for hills with steeper slopes. The site effect is notorious both in Peak Ground Acceleration and in spectral ratio considering the Fourier spectra at the top and at the base of the hill.

It is essential to consider subsurface topographic effects in basin-like geological settings. The impedance contrast between the basin edges profoundly alters the soil response comparing with the response one would expect if a one-dimensional response were to be valid. Diffraction and wave trapping are the main causes that help to explain the difference in the amplification.

Diffraction plays a role converting body waves into surface waves. Somerville \textit{et al.} (2002) state that diffraction at the basin edge leads to the presence of Love waves in the horizontal component of the ground motion that are parallel to the edge; and to the presence of Rayleigh wave in the horizontal component that is perpendicular. This effect increases for deeper basins, and has repercussions in terms of the significant duration of the motion. According to the same authors, for deeper basins, one may expect the significant duration to increase.

Bard and Bouchon (2000), mentioned by Roten \textit{et al.} (2004), used the Aki-Larner technique to simulate a 2D sine-shaped basin.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{response_of_a_2d_sine_shaped_basin}
\caption{Response of a 2D sine-shaped basin (Roten \textit{et al.}, 2004; adapted from Bard and Bouchon, 2000)}
\end{figure}
For two-dimensional structures, the SH-wave portion of the ground motion is insufficient to explain the amplification ratio. The P-wave part of the ground motion, as usual, plays an essential role on the vertical acceleration felt at the surface; however, it also induces ground motion in the horizontal direction (in the plane of the basin structure), with maximum amplification occurring between the edge and the center of the basin. As for the SV-wave portion, it amplifies ground motion horizontally in the plane of the basin structure especially at the center. This portion also amplifies vertical motion between the center and the edge of the basin. As one may infer from Figure 4.2, there are also consequences in terms of phase.

As a consequence, Bard (2006) mentions that for basin-like geological settings, there may appear a second amplification peak, normally for frequencies ranging from 2.0Hz to 5.0Hz. The spectral ratio is also much more scattered. The same author, referring to the Grenoble case, where the impedance contrast is quite significant (very hard bedrock, contrasting with thick lacustrine deposits), obtained the experimental spectral ratio shown in Figure 4.3:

![Experimental spectral ratio obtained at HATZ site, at Grenoble (adapted from Bard, 2006)](image)

**Figure 4.3 - Experimental spectral ratio obtained at HATZ site, at Grenoble (adapted from Bard, 2006)**

The spectral ratio not only allows one to infer that amplification occurs to a much broader band, but that there is clearly a second amplification peak at about 2.0-3.0Hz.

In the same line, Chávez-García *et al.* (1999), studying the Parkway basin at New Zealand, show that the superposition of one-dimensional and two-dimensional resonance is impossible to dissociate in the frequency domain, as one-dimensional resonance peak will be contaminated with laterally propagating waves.
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The impedance contrast between the edges and the layers within the basin, along the basin shape, leads to a more efficient wave trapping, with significant vertical and lateral reverberations. In terms of acceleration time series, the latter is prone to exhibit late wave arrivals, constructive interferences and tends have a much more scattered nature. There may appear a shift of the fundamental frequency towards higher frequencies.

The basin shape is a fundamental issue concerning the choice of the right model to study site effects. For wide, shallow basins, one-dimensional modeling at the center of the basin is admissible, while at the edge two-dimensional effects may start to play an important role. For deep basins, a one-dimensional approach may not be suitable.

4.3. Induced effects

Ground shaking in certain soils and/or certain geotechnical structures may lead to severe consequences on the building stock and/or on vital infrastructures that may not be explained directly by it, but by effects that are triggered by it. They are the so-called induced effects.

When one is referring to induced effects, one normally is considering the following phenomena:

- Liquefaction;
- landslides.

4.3.1. Liquefaction

Liquefaction most commonly occurs with seismic motion in medium-to-loose sand deposits, for shear strains beyond the volumetric strain threshold. The phenomenon is due to the fact that the loading rate is such that pore pressure may not be dissipated during earthquake shaking (i.e., even for sands, one may consider that one has undrained conditions).

In contractive sands, the shear strain associated by ground shaking induces compaction. However, as one is in undrained conditions, the volumetric strain is null and pore pressure develops. The increase in pore pressure leads to a reduction of the mean normal effective stress, $p'$. In some cases, according to Terzaghi’s effective stresses principle, the mean normal effective stress may reduce drastically to residual values, and inter-particle friction may not be mobilized. When this occurs, one may say that one has liquefaction. An important remark concerning liquefaction is that, as the volumetric and distortional behavior are intertwined, when liquefaction occurs, the shear modulus decreases drastically, which has severe consequences as far as deformation is concerned (Finn, 1993).
In terms of constitutive behavior, considering a given soil deposit, one may have a given initial static shear stress, \( \tau_{ST} \). Under cyclic loading, considering a given stress value, \( \tau_{cy} \), for contractive sands, for a given number of cycles, strain softening occurs, with a massive drop of resistance to a residual value, know as steady-state strength, \( \tau_{SS} \). Liquefaction may be triggered if (Finn, 1993):

\[
\tau_{ST} > \tau_{SS} \tag{4.9}
\]

or

\[
\tau_{ST} + \tau_{cy} > \tau_{SS} \tag{4.10}
\]

Dense sands have dilative behavior, and under cyclic loading, deformation results from stiffness reduction as the number of load cycles increase, and pore pressure builds up. In order to compensate the increase of pore pressure, dilation occurs. This means that effective stresses do not suffer a massive drop under cyclic loading. This type of behavior is known as cyclic mobility.

There are soils that are between these two extremes, and exhibit a hybrid behavior. For these soils, usually, strain softening occurs (as would happen in liquefaction), but, for a given strain, there is a regain of strength, and soil behavior may be explained by cyclic mobility.

From what as been described, the number of cycles plays an important role in liquefaction and cyclic mobility. Therefore, the seismic event’s magnitude and duration necessarily have consequences in terms of liquefaction triggering.

Another major factor in liquefaction is grain-size distribution and the void ratio (i.e., the spatial disposal of particles). Medium-to-loose sands and silty sands, poorly graduated, are the most liquefaction-vulnerable soils. The presence of fine-grained particles, for soils of similar strength, increases the steady-state strength of the soil (residual strength), mking it more difficult for soils to liquefy. However, even fine-grained soils may liquefy. Wang (1979), based in chinese earthquakes experience, established criteria for liquefaction to occur in plastic fine-grained soils, based on the Atterberg limits, water content and clay particles content. For soils with the same void ratio, for increasing fines content, there is a decrease in cyclic strength (Troncoso, 1990; in Finn, 1993).

Another essential aspect to consider is the initial stress state, along with the geological history, as, not only dilative and contractive behavior is stress-state controlled, but there may also exist an initial static shear stress. Seed et al. (1985) proposed to normalize the shear stress in terms of the overburden pressure, introducing the concept of cyclic shear-stress ratio, CSR.
There are several liquefaction assessment techniques, largely based in field testing, namely using SPT and CPT/CPTU results. Seed et al. (1985) expressed the cyclic resistance ratio, CRR (the strength counterpart of CSR), as a function of the normalized SPT blow count, \((N_{1})_{60}\), and the fines content, for \(M=7.5\) event. One may say, according to this technique, that liquefaction occurs if CSR>CRR. In Eurocode 8, a safety factor equal to 1.25 is used. In order to duly account the effect of grain-size distribution and void ratio, \((N_{1})_{60}\) is corrected to what is known as the equivalent clean sand \((N_{1})_{60}\). If one is considering event of different magnitude and/or with a different number of cycles, another corrective factor has to be used when one is determining CRR. In order to consider the possibility of an initial shear stress (e.g. for embankments), CRR should also be corrected.

In the case of the CPT/CPTU test, Seed and De Alba (1986) proposed that the CRR should be determined as a function of the cone resistance, and of the fines content (similarly to the SPT-based technique. The soil classification chart proposed by Robertson (1990) may be used to determine the fines content. Robertson and Wride (1997) proposed a technique based not only on the cone resistance, but also on the shaft friction (Jeremias et al., 2007).

4.3.2. Landslides

Landslides occur during seismic events due to many causes, such as inertial forces, liquefaction, pore pressure build-up. These causes lead to the loss of equilibrium. Coelho (1991), mentioned by Lopes (2001), states that landslides occur in the following situations:

- Potentially unstable slopes, where the seismic event only enhances an already on-going failure process;

- Statically stable slopes, where the seismic event completely changes the force balance, leading to failure.

Failure surfaces for the first situation don’t present significant differences, as for the second situation the former may exhibit awkward shapes.

Landslides are strongly dependent on magnitude and geological history of potentially unstable slopes. Of course, soil strength also plays a decisive role. Keffer (1984), mentioned by Kramer (1996), presents a classification, based on the type and speed of movement, water content of the slope, internal fragmentation and failure surface depth.

A first approach to landslide assessment in a mechanical point of view is the Newmark’s sliding block method, that consists of a pseudo-static analysis, where equilibrium is checked, using an acceleration
threshold for which there is block movement (normally, a scaled value of regulatory PGA). If the acceleration time history has values that exceed the threshold, displacement occurs. Using time-domain integration, one may determine the displacement time history. More advanced numerical techniques, such as finite-element or finite-difference methods, may be used.

In terms of predicting and preventing landslides, geological, topographic, hydrological survey is fundamental. In fact, landslide assessment presents many similar approaches to microzoning, as will be discussed in 4.6.

4.4. Numerical modeling of non-linear soil behavior

As one may understand from what has been exposed, one must account several issues, such as the geometrical definition of the model, the topographic features of the site to be modeled, the geological and geotechnical setting, and soil non-linearity, in order to obtain a coherent and valid model that allows one to explain the occurrence of amplification at a given site. There are inherent complexities in terms of wave propagation in the geological and geotechnical setting that are extremely difficult to properly model, citing, for example, what may be defined as “seismic bedrock”, or what is, effectively, the edge of a subsurface basin structure. These two features, alone, may alter entirely the waveform, with consequences in the ground shaking. Also, geotechnical (seismic) parameters such as the shear wave velocity variation along the depth may be difficult to obtain, especially if soil deposits’ depth exceeds 30m. When this occurs, typical geophysical campaigns may face some difficulties. Even the event’s magnitude plays a role, as for stronger motions non-linear effects become increasingly important. Another thing that one must always bear in mind is that, the more complex the model is, the greater need of geological and geotechnical characterization.

There are several modeling techniques that take into account, with more or less accuracy, many of the referred issues. Focus will be made, at this point, on the soil behavior modeling, considering models that may model strains beyond the elastic strain threshold, to the volumetric strain threshold, where non-linear behavior is absolutely fundamental on the study of the ground motion.

The choice of the non-linear soil modeling may, sometimes, be coupled with geometric consideration. A clear example of this case is the linear equivalent method.

4.4.1. Linear equivalent method

The basic idea of the linear equivalent method is that non-linear soil behavior may be described by a linear viscoelastic model, as long as the shear modulus and the hysteretic damping coefficient are not constants, but instead functions of the shear strain amplitude induced by the ground motion. The
method has its origin on the works of Idriss and Seed (1968). These authors first proposed the use of the hysteretic Kelvin-Voigt model, introduced in Chapter 3. The development of the SHAKE algorithm by Schnabel et al. (1972) led to a widespread use of this method, at least for a first approach to the study of strong-motion amplification at a given site.

One of the main characteristics of this method is that, as the shear strain induced by the ground motion alters the shear modulus and the hysteretic damping, and the latter parameters are the ones that define the induced shear strain (via the constitutive relation), one is led to an iterative procedure in order to have matching shear strain, shear modulus and hysteretic damping. An important remark that must be made at this point is that, for all iterations, one has constant shear modulus and hysteretic damping coefficient, as one is considering a linear model.

This procedure is done for each of the soil layers considered in the model. An important issue concerning this method is that, in order for one to have a more detailed acceleration (or shear stress, or shear strain) profile along the soil column, one must consider several soil layers, as, for each adopted layer, only for the mid-point in terms of layer’s thickness one has the time series.

Another issue of extreme importance is the fact that, as has already been mentioned, one normally is dealing with non-harmonic transient signals. If one doesn’t take into account this fact, the use of the linear equivalent method leads to an overestimation of the strain induced by the ground motion. Comparing a harmonic signal with the same maximum shear strain (amplitude of shear strain) as one that is not, the former carries much more energy that the latter, especially for transient signals as one may see in Figure 4.4:

Figure 4.4 - Harmonic signal and non-harmonic transient signal with the same maximum shear strain

Kramer (1996) and Ordoñez (2006) refer that it has been empirically found a ratio between the named equivalent shear strain and the maximum shear strain. The ratio usually ranges from 0.50 to 0.70. The more uniform the shear strain histories are, the higher the ratio is. In SHAKE, for an instance, the
default value of this ratio is 0.65. The equivalent shear strain is determined, in this method, for all the considered layers, for their mid-point in terms of layer’s thickness. The equivalent shear strain is duely taken into account in the method’s convergence, as, for each layer, the convergence criterion is expressed as a function the equivalent shear strain, as one may see in Equation 4.11:

$$\frac{\gamma'_{eq} - \gamma'_{\text{assumed}}}{\gamma'_{eq}} \leq 5\%$$  \hspace{1cm} (4.11)

If convergence is not met, $\gamma'_{\text{assumed}} = \gamma'_{eq}$ and the iterative procedure continues. The full flow chart of the convergence method is shown in Figure 4.5:

![Flow chart of convergence method](image)

**Figure 4.5 - Linear equivalent method flow chart**

There are many advantages associated to the use of this method. Probably, the main advantage is its formal simplicity (even though, even for this method a complex geotechnical characterization should be made). Another aspect of extreme importance is that, as this method is of general use, it has been widely tested, allowing one to say that, if the method’s premises are fulfilled, one is obtaining a reasonable estimation of the ground response at the site.
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The use of this method is limited, in its original formulation, to cases where one-dimensional may be valid, as this method is built on the same assumptions that led to the definition of the viscoelastic model.

One important limitation concerns the fact that this method does not take into account volumetric strains. Hence, this method should not be used in cases where the volumetric shear strain threshold is overcome. As the volumetric threshold is overcome, there are irrecoverable strains; as the model is linear, it doesn’t allow one to determine these strains.

Another important issue concerning the original SHAKE algorithm formulation (hysteretic Kelvin-Voigt model) is that energy dissipation is well modeled, but the stiffness is overestimated. This is due to the fact that, in the frequency domain, the shear modulus’ amplitude is greater than its real value. Recalling Equation 3.10, one may see why that happens.

\[
G' = G \cdot (1 + i \cdot 2 \cdot \xi) \Rightarrow |G'| = G \sqrt{1^2 + (2 \xi)^2} = G \sqrt{1 + 4 \xi^2}
\]

(4.12)

For low hysteretic damping, the complex stiffness doesn’t differ much of the real stiffness. However, for hysteretic damping values over 25%, the former statement is not true (one may argue, not without reason, that, for this damping, normally one already has irrecoverable strains, making the use of the linear equivalent method unacceptable). Figure 4.6 shows the relation between the complex shear modulus and the real shear modulus for different hysteretic damping.

![Figure 4.6 - Increase of stiffness as a function of damping, according to the hysteretic Kelvin-Voigt model](Bardet et al., 2000)

Idriss and Sun (1992), taking into account this fact, modified the SHAKE algorithm and built a new version, SHAKE91. In this algorithm, the complex shear modulus follows:
\[ G^* = G \left( \left[ 1 - 2 \cdot \xi^2 \right] + i \left[ 2 \cdot \xi \cdot \sqrt{1 - \xi^2} \right] \right) \Rightarrow |G^*| = G \sqrt{\left[ 1 - 2 \cdot \xi^2 \right]^2 + \left[ 2 \cdot \xi \cdot \sqrt{1 - \xi^2} \right]^2} = \]

\[ = G \sqrt{1 - 4 \cdot \xi^2 + 4 \cdot \xi^4 + 4 \cdot \xi^2(1 - \xi^2)} = G \sqrt{1 - 4 \cdot \xi^2 + 4 \cdot \xi^4 + 4 \cdot \xi^2 - 4 \cdot \xi^4} = G \] (4.13)

As one may see, the complex shear modulus’ amplitude, for this model is equal to the real shear modulus. However, energy dissipation is underestimated according to this model.

\[ \Delta W = 2 \cdot \pi \cdot G \cdot \xi \cdot \sqrt{1 - \xi^2} \cdot \gamma_a^2 \] (4.14)

Comparing with Equation 4.9, one may see that energy dissipation is lower than the real by a factor equal to \( \sqrt{1 - \xi^2} \). For hysteretic damping coefficient within the linear equivalent method validity frame, this factor doesn’t lead to gross underestimation of the energy dissipation (Figure 4.7).

![Figure 4.7 - Decrease of normalized energy dissipation as a function of damping, according to the SHAKE91 model (adapted from Bardet et al., 2000)](image)

Dormieux and Canou (1990), mentioned by Pecker (2006), proposed a model that allows one to correctly model both the real stiffness and energy dissipation. The viscoelastic model obeys the following constitutive relation.

\[ G^* = G \sqrt{1 - \xi^2} + i \cdot \xi \] (4.15)

In spite of the Dormieux model allowing the correct modeling of both the stiffness and the energy dissipation, the use of the SHAKE and SHAKE91 algorithms and coupled tools is much more
frequent (especially, the use of SHAKE91 model). In the present work, the model associated to SHAKE91 will be used in Chapter 6.

4.5. Experimental site-effect assessment techniques

The previous sections focused on the theoretical background that allow one to comprehend and to model site effects. At this section, one will introduce the experimental techniques used to determine the effect of near-surface geology on the seismic response of the site. The first approach to assess the presence of site effects is to observe the difference and contrast of damage after an earthquake (Oliveira et al., 2006). With these macroseismic observations, and with the use of damage maps, if data from previous seismic events are available, one may detect patterns in ground motion.

Another tool, used as a first approach to determine the possible presence of a site effect is to analyze strong-motion records at the site on study. If the obtained signal presents special features, such as higher PGA, larger significant duration or large amplification for a given frequency, comparing to other sites, for similar epicentral distances, there is evidence of site effects.

Most experimental site-effect assessment techniques focus on spectral ratios. All of these techniques allow the determination of the fundamental frequency, in order to determine, essentially, the fundamental frequency at a given site, and to have a notion on the amplification ratio, i.e., these techniques give an approximation of the transfer function associated to a given site. To obtain these ratios, weak-to-moderate motion records and microtremors have been used; for most techniques, both sources are used (sometimes, the same technique appears with slightly different name, depending on the type of energy source used). Each source of energy has its own issues, which will be presented.

Another aspect that one has to bear in mind is the fact that the present work focuses on acceleration time series; therefore, the description of the site-effect assessment techniques is always accelerometer-oriented. Nevertheless, certain techniques are exclusively based on, or are most applied using, seismometers. In these cases, mention to the use of seismometers will be made.

4.5.1. Reference-site Spectral Ratio (RSR) and inversion schemes

The Reference-site Spectral Ratio, or Standard Spectral Ratio, was first introduced by Borcherdt (1970). Conceptually, this technique is quite simple: if one has a rock outcrop site at the vicinity of the site to assess, as the former should not be influenced by site effects, if one calculates the ratio between the Fourier amplitude spectrum of latter and the Fourier amplitude spectrum of the former (the so-called reference site), one should obtain approximately the transfer function of the site.
This technique presents as advantage, along with its formal simplicity, the fact that it doesn’t require heavy signal processing (in fact, only Fourier transforms are needed). If the premises of this technique are met, i.e., if one has a reference station where there are no site effects, this technique should give the best estimation of site effects at a given station.

However, this method presents a major setback: it may be extremely difficult to find a reference site. Steidl et al., (1996) state that, in some geological settings, this happens indeed. According to the same authors, another issue that makes this technique prone to error is the fact that even some rock sites may have site effects, as happens, for an instance, when topography intervenes in the site response (this is especially important, typically, in valleys).

In order to exclude the possibility of site effects in rock sites, in dense arrays, this technique has been improved using the Generalized Inversion Scheme technique.

Andrews (1986) proposed, for network arrays with a significant amount of stations, an inversion scheme in order to isolate effects of the source, propagation effects and site effects. The natural logarithm of the spectral amplitudes may then be expressed, for an event, $i$, at a recording site, $j$, as a function of frequency, $f$, and hypocentral distance, $r$:

$$\ln U_j(f,r) = \ln S_i(f) + \ln A_{ij}(f,r) + \ln Z_j(f)$$

(4.16)

where $S_i(f)$ is the factor due to the source, $Z_j(f)$ is the factor due to site response, and $A_{ij}(f,r)$ is the factor referent to the attenuation during the propagation path. Several studies use this technique, with different inversion procedures (Chávez-Garcia et al., 1999; Parolai et al., 2004; Sokolov et al., 2004). These schemes normally allow for one to have a clear reference station, as multiple information is crossed.

The use of both the reference-site spectral ratio and generalized inversion schemes present as the most important issue the fact these techniques require the simultaneous use of at least two recording apparatus (either seismometers or accelerometers). In dense recording arrays, this doesn’t pose a problem; however, when one has limited resources, this fact may play an essential role. This was the reason that non-reference techniques became so popular, in particular the use of the Horizontal-to-Vertical spectral ratio (H/V).

4.5.2. Horizontal-to-Vertical Spectral Ratio

Despite being firstly introduced by Nogoshi and Igarashi (1971), in order to study the wave composition of microtremors, it was Nakamura (1989) the first author to use the H/V technique
having as purpose the estimation of the amplification factor at a given site. The technique was first
developed using microtremors.

According to Nakamura (2000), for a geological setting consisting of a soft soil deposit over a half
space, the horizontal amplitude spectrum, $H_s$, and vertical amplitude spectrum, $V_s$, may be expressed
as:

$$H_s = A_h \cdot H_B + H_{SW} \quad (4.17)$$

$$V_s = A_v \cdot V_B + V_{SW} \quad (4.18)$$

where $A_h$ and $A_v$ are, respectively, the horizontal and vertical amplification ratios of vertically incident
body waves; $H_{SW}$ and $V_{SW}$ are, respectively, the horizontal and vertical amplitude spectra of the
Rayleigh-wave part of the wavefield; and $H_B$ and $V_B$ are, respectively, the horizontal and vertical
amplitude spectra at the bedrock. According to what has been stated, the horizontal-to-vertical
spectral ratio would be:

$$H / V = \frac{H_s}{V_s} = \frac{A_h \cdot H_B + H_{SW}}{A_v \cdot V_B + V_{SW}} = \frac{H_B}{V_B} \frac{A_h + \frac{H_{SW}}{H_B}}{A_v + \frac{H_{SW}}{H_B}} \quad (4.19)$$

The fundamental premise of this technique states that, at the bedrock, the horizontal and vertical
spectra are the same, i.e., $H_B = V_B$. This has been experimentally verified by Nakamura, using
microtremors at a borehole; Lermo & Chávez-García (1993) link an usual uncertainty value lesser
than or equal to two, current to spectral ratios, to this premise. Bearing the fundamental premise in
mind, one sees that the wavefield content, according to Equation 4.28, may change the physical
meaning of the horizontal-to-vertical spectral ratio. If the body wave contribution at the bedrock were
to carry more energy than the surface wave part at the station, the horizontal-to-vertical spectral ratio
should correspond approximately to the SH transfer function, as one usually has $A_v \approx 1$ (for the SH
fundamental mode, for usual Poisson coefficient values, one doesn’t have vertical amplification).

Nakamura initially justified the validity of this technique with multiple refraction of SH-wave.
However, this assumption has been a subject of deep discussion within the scientific community over
the last fifteen years, with no consensus being met. The composition of microtremors isn’t a
consensual theme too. Lermo and Chávez-García (1994) were the first authors to consider that the
horizontal-to-vertical spectral ratio could be explained by the peak of Rayleigh wave ellipticity,
comparing simple numerical models to the horizontal-to-vertical spectral ratios obtained at three sites
in Mexico. The Rayleigh ellipticity implies that, around S-wave fundamental frequency, the vertical component of ground motion is null, as one passes from retrograde to prograde motion. Lachet and Bard (1994) further went on the numerical modeling, backing the notion that this technique should be explained by the peak of ellipticity. Bard (1998) and the SESAME team (2004) have made many efforts in order to study the composition of microtremors and their applicability in the study of site effects. These studies point out that the horizontal-to-vertical spectral ratio peak at the fundamental frequency may be explained by several parts of microtremors wavefield, but mainly by the ellipticity peak, especially for geological setting with impedance contrast greater than 2.5. Nakamura (2000), however, maintains that the technique is due to multiple reflection of the SH waves, arguing that, for the fundamental frequency, the Rayleigh-wave part of microtremors doesn’t carry significant energy.

Lermo and Chávez-García (1993) expanded the horizontal-to-vertical spectral ratio using earthquake records, obtained both by seismometers and accelerometers. These authors have determined the spectral ratio for the S-wave part of ground motion. The adoption of the S-wave windows of ground motion was justified by these authors with two lines of argument. First, analyzing data from Mexico City, concerning the 1985 Mexico Earthquake; there was evidence that, even for sites of great horizontal amplification, the vertical component of displacement had the same character and similar amplitudes, i.e., the vertical component of ground motion mainly contains features only concerning source and propagation effects. Second, using Aki-Larner’s method, for a simple stratigraphic model given by Seed et al. (1988) for SCT station at Mexico City, several incidence angles of ground motion were considered. For every case, the horizontal-to-vertical spectral ratio clearly identified the transfer function peaks, although the amplification values were not correct. There was also evidence that the amplification ratio determined by the technique was strongly dependent on the incidence angle.

The use of horizontal-to-vertical spectral ratio has as enormous advantage the fact that it doesn’t require a reference site. This is especially important in low-to-moderate seismicity zones, where there aren’t network arrays. The fact that it demands only one station makes this technique attractive in an economical point of view.

This technique, applied to microtremors, allows the identification of the fundamental frequency of a given site; when ground motion records are used, the horizontal-to-vertical spectral ratio permits the identification of other vibration modes. An important advantage of the use of horizontal-to-vertical spectral ratio using ground motion records is that one is able to determine the presence of topographic effects, for simple topographic settings (Chávez-García et al., 1996, 1999; Sokolov et al., 2004).

The application of this technique using both energy sources present as shortcoming the erroneous estimation of the amplification ratio; this is especially true when using microtremors. In some
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geological settings, such as sites where subsurface topography is relevant, major amplification doesn’t occur for the fundamental frequency; therefore, the use of microtremors as energy source may be useless or even misleading as far as amplification is concerned.

When using accelerometric stations, the use of microtremors is highly discouraged, as accelerometers’ sensitivity isn’t enough for ambient vibration levels (SESAME team, 2004). Within the scope of using accelerometric stations, weak-to-moderate ground motion records are the most used energy sources when horizontal-to-vertical spectral ratio is to be applied. This statement falls in line with the already mentioned works by Chávez-García et al. (1996, 1999) and Sokolov et al. (2004).

When using accelerometric stations, reference-site and horizontal-to-vertical spectral ratios (along with generalized inversion schemes) are the most used methods to assess site effects. Nevertheless, as already mentioned, the author believes that it is important to briefly present the state-of-art techniques, even if the use of acceleration time series is not considered. This is clearly the case of microtremor array measurements.

4.5.3. Microtremor array measurements

Microtremor array measurements consist of the synchronized recording of ambient vibration at different places. This technique relies on signal processing and stochastic modeling of ground motion. The most used procedures to interpret the results of array measurements are the frequency-wavenumber or f-k method (Lacoss et al., 1969), and the Spatial Autocorrelation method, usually known as the SPAC method, introduced by Aki (1957).

The use of array measurements is based on the assumption that microtremors are mainly composed of surface waves (Wathelet, 2005). For this kind of waves, soils have what is called dispersive behavior, i.e., for different frequencies, soils have different wave propagation velocities. The main goal concerning these arrays is to determine the dispersion curve (presented in 3.6.1.1). In order for one to have information about phase velocity, one must have both time and spatial measurements. The dispersion curve allows one to determine the \( V_s \)-wave velocity profile under the recording station via an inversion method approach.

The f-k method uses stationary records of microtremors. As usual records have transient parts, these are removed, using time windows (co-sine tapering). As stations have a fixed spatial setting and have simultaneous records, one is able to express the signal as the combination of harmonic signals, dependent of time and position; \( t, x \) and \( y \). The Fast Fourier Transform is applied on the signals, in order for one to do the analysis in the frequency domain. Using the 2D-FFT, for each frequency, one has horizontal plane waves propagating at a given direction (described mathematically by two
wavenumbers) with a given velocity. These waves arrive at the different stations of the array with different time arrivals, leading to a given phase shift. If several waves have similar propagation properties, there is constructive interference, leading to a peak in the array output. The quotient between the output and the spectral power is called the semblance function. When the semblance function has its maximum in the wavenumbers plane, one has a valid estimation of the velocity and azimuth of the propagating waves across the array (Wathelet, 2005), allowing the determination of the dispersion curve.

The SPAC method is directly based on the concepts of stochastic modeling of ground motion introduced in Chapter 2. Once again, stationary windows are preferred, and tapering is essential. This method links the autocorrelation function to phase velocities. If the signals obtained at two different stations are narrowly band-passed filtered around a given frequency, $f_0$, the ratio between the autocorrelation functions at each of the station may be related to a 0th-order Bessel function, mathematically dependent on $f_0$, the distance between the stations and the dispersion curve.

The use of microtremor array measurements presents as main advantage the overcoming of several setbacks of the horizontal-to-vertical spectral ratio, allowing to have a theoretical background and, at the same time, to have a much more detailed description of the amplification properties of the site under study (in fact, one may consider that, more than focusing on the amplification, this technique focuses on a detailed characterization of the geological setting). Again, the use of microtremors doesn’t require a powerful energy source, which is very useful in places of low-to-moderate seismicity.

However, the theoretical background of array measurements is cumbersome, and the technique requires heavy data processing. One must also have several stations recording at the same time, which may turn this technique unattractive in an economical point of view.

### 4.6. Microzoning

Seismic zoning concerns to the process of subdividing a given region of similar behavior regarding a given set of seismic parameters (Oliveira et al., 2006). Zoning procedures constitute a fundamental tool for seismic risk assessment, as they allow one to predict, with a certain degree of uncertainty, the most vulnerable sites. Microzoning concerns the subdividing of a small area, such as a city.

Microzoning is made using acceleration time series’ parameters presented in Chapter 2. The parameters most used in order to distinguish site behavior of different places are the following (Oliveira et al. 2006):
• Macroseismic intensity increase of a given site relatively to reference site in the vicinity;

• Increment of peak ground acceleration at a given site relatively to a reference site;

• Predominant period or frequency of ground motion;

• Transfer function of a given site;

• Site-specific response spectrum.

Usually, in order for microzoning to be an effective tool in site-effect prediction, a large amount of information has to be gathered, not only concerning geological/geotechnical features, but also in terms of topography, hydrology, soil occupation. Therefore, microzoning studies are very often accompanied with survey works. In microzoning studies, geographical information systems became a generalized tool.

There are several guidelines for zoning and microzoning, such as the one provided ISSMGE (1999). The zoning procedures, of course, depend on the pretended scale. In the case of microzoning studies (at a municipality level), it is usual to use maps with a scale equal to 1:500 (Oliveira et al., 2006). For examples of microzoning, see Teves-Costa and Senos (2004), Jahfari et al. (2004).

4.7. Concluding remarks

A review of the most important concepts concerning the state-of-the-art knowledge about site effects was made. Distinction was made between direct and induced site effects, linking these kinds of effects with acceleration time series features, both in the time and frequency domains. Special attention was paid to the consequences of topographic effects in ground motion.

Discussion was made over the most used tools to model site effects (for strains lower than the volumetric shear strain). The linear equivalent method was thoroughly analyzed. Its main advantages consist of:

• Its formal simplicity;

• The large experience of the geotechnical community in the use of this tool.

The linear equivalent method has two major setbacks:

• The most usual formulation of the method (SHAKE algorithm) misfits the real stiffness or the energy dissipation;
• One can’t obtain irrecoverable strains, as the method is within the elasticity framework.

The linear equivalent method will be used in Chapter 6. The Ramberg-Osgood model is the starting point of all the work explained in Chapter 5.

Experimental site-effect techniques were presented. Focus was made essentially in reference-site spectral ratio and horizontal-to-vertical spectral ratio, as these techniques may be applied for acceleration time series. These techniques will be also used in Chapter 6.
5. Implementation of the Ramberg-Osgood model in PLAXIS

5.1. Introduction

One of the main goals of this thesis is the implementation of a calculation routine in a commercial finite-element method, allowing dynamic analyses on the very small-to-small strain range. For this purpose, the implementation of a tri-dimensional constitutive relation based on the Ramberg-Osgood model was made, using the commercial program PLAXIS. The latter is one of the most used programs in the geotechnical community and, at the time the routine was developed, it hadn’t models that allowed one to account hysteretic behavior for very small-to small shear strains. The Ramberg-Osgood model coupled with the Masing criterion, as mentioned in Chapter 4, is one of the most used tools in dynamic analyses.

First, the mathematical description of the constitutive relation will be made, focusing on aspects such as the incremental formulation, the assemblage of the tangent stiffness matrix as a function of the secant stiffness and all the values needed to do so.

Next, the implementation in PLAXIS will be thoroughly described, both on the programing and on the mathematical point of view. All the necessary steps to implement a constitutive model in PLAXIS will be shown. Comments will be made concerning numerical issues and data storage needed for the routine to work correctly.

Finally, simple validation examples will be presented. An element under simple shear will be subjected to several loading conditions and one will model a given soil column and will compare the results of a one-dimensional modeling using SHAKE2000.

5.2. Mathematical description of the fundamental (“backbone”) constitutive relation

The mathematical description of the adopted formulation closely follows what is described in Chen and Mizuno (1990), concerning the isotropic non-linear incremental (i.e., tangential) formulation based on the secant bulk modulus, $K_S$, and the secant shear modulus, $G_S$. In the developed model, the secant shear modulus was considered to be a function of the octahedral shear strain, $\gamma_{oc}$, according to the Ramberg-Osgood model. The octahedral shear strain is described by Equation 5.1:
The octahedral shear strain is intimately linked with the principal strains, as the octahedral plane is defined by the latter. The octahedral plane is such that its normal makes equal angles with the principal strains, defining the octahedral line. The octahedral fiber has as normal strain (according to Cauchy’s law) a value equal to the mean of the principal strains:

$$\varepsilon_{oct} = \frac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$

(5.2)

The octahedral shear strain is a scalar measure of the shear deformation on a given point (Maranha, 2005), and is closely related to the second invariant of the strain tensor (just as happens with the deviatoric stress, $q_s$, for the stress tensor).

Considering what has been mentioned, the secant shear modulus used in the routine is given by Equation 5.3:

$$G_s = \frac{G_0}{1 + \alpha \left( \frac{G_0}{G_r} \cdot \frac{\gamma_{oct}}{\gamma_r} \right)^{r-1}}$$

(5.3)

Where $G_0$, $\alpha$, $r$ and $\gamma_r$ have the meaning presented in 4.4.2.2.

One considered isotropic linear elastic behavior in terms of volumetric strain, therefore the tangent bulk modulus, $K_r$, is equal to the secant bulk modulus, $K_s$. The secant constitutive relation is the following:

$$\sigma_{ij} = K_s \cdot \varepsilon_{kk} + 2 \cdot G_s \cdot \varepsilon_{ij}$$

(5.4)

where $\varepsilon_{kk}$ is the first invariant of the strain tensor, and equal to the volumetric strain, $\varepsilon_v$. One may also separate the normal effective stress, $p'$, and the deviatoric part of the stress tensor, $s_{ij}$:

$$p' = K_s \cdot \varepsilon_v$$

(5.5)

$$s_{ij} = 2 \cdot G_s \cdot \varepsilon_{ij}$$

(5.6)
where $e_d$ is deviatoric part of the strain tensor. One may opt to describe the constitutive relation in octahedral terms:

\[ p' = \sigma_{\text{oct}} = 3 \cdot K_s \cdot \varepsilon_{\text{oct}} \]  \quad (5.7)

\[ s_{ij} = 2 \cdot G_S \cdot e_{ij} \iff s_{ij} \cdot s_{ij} = 4 \cdot G^2_S \cdot e_{ij} \cdot e_{ij} \iff 3 \cdot \tau_{\text{oct}}^2 = \frac{3}{4} \cdot 4 \cdot G^2_S \cdot \gamma^2 \iff \]

\[ \tau_{\text{oct}}^2 = G^2_S \cdot \gamma^2 \Rightarrow \tau_{\text{oct}} = G_s \cdot \gamma_{\text{oct}} \]  \quad (5.8)

where $\tau_{\text{oct}}$ is the octahedral shear stress. In order for one to have an incremental formulation, one must determine the derivatives:

\[ \frac{d\sigma_{\text{oct}}}{d\varepsilon_{\text{oct}}} = \frac{d(3 \cdot K_s \cdot \varepsilon_{\text{oct}})}{d\varepsilon_{\text{oct}}} = 3 \cdot \left( \frac{d(K_s)}{d\varepsilon_{\text{oct}}} \cdot \varepsilon_{\text{oct}} + \frac{d(\varepsilon_{\text{oct}})}{d\varepsilon_{\text{oct}}} \cdot K_s \right) = 3 \cdot \left( \frac{dK_s}{d\varepsilon_{\text{oct}}} \cdot \varepsilon_{\text{oct}} + K_s \right) \Rightarrow \]

\[ \Rightarrow d\sigma_{\text{oct}} = 3 \cdot \left( \frac{dK_s}{d\varepsilon_{\text{oct}}} \cdot \varepsilon_{\text{oct}} + K_s \right) \cdot d\varepsilon_{\text{oct}} \iff d\sigma_{\text{oct}} = 3 \cdot K_t \cdot d\varepsilon_{\text{oct}} \]  \quad (5.9)

\[ \frac{d\tau_{\text{oct}}}{d\gamma_{\text{oct}}} = \frac{d(G_s \cdot \gamma_{\text{oct}})}{d\gamma_{\text{oct}}} = \frac{d(G_s)}{d\gamma_{\text{oct}}} \cdot \gamma_{\text{oct}} + \frac{d(\gamma_{\text{oct}})}{d\gamma_{\text{oct}}} \cdot G_s = \frac{d(G_s)}{d\gamma_{\text{oct}}} \cdot \gamma_{\text{oct}} + G_s \Rightarrow \]

\[ \Rightarrow d\tau_{\text{oct}} = \left( \frac{dG_s}{d\gamma_{\text{oct}}} \cdot \gamma_{\text{oct}} + G_s \right) \cdot d\gamma_{\text{oct}} \iff d\tau_{\text{oct}} = G_t \cdot d\gamma_{\text{oct}} \]  \quad (5.10)

where $K_t$ and $G_t$ are, respectively, the tangent bulk and tangent shear moduli. The volumetric/hydrostatic part of the constitutive relation is easily expressed as a function of the strain tensor:

\[ dp' = 3 \cdot K_t \cdot d\varepsilon_{\text{oct}} = K_t \cdot d\varepsilon = K_t \cdot \delta_{ij} \cdot d\varepsilon_{ij} = K_t \cdot \delta_{kl} \cdot d\varepsilon_{kl} \]  \quad (5.11)

To do the same with the deviatoric part is somewhat more complicated. The deviatoric stress increment may be determined differentiating Equation 5.6:
Chapter 5 – Implementation of the Ramberg-Osgood model in PLAXIS

\[ ds_{ij} = 2 \left( \frac{dG_s}{de_{ij}} \cdot e_{ij} + G_s \right) \cdot de_{ij} \]  

(5.12)

One may write the increment of the octahedral shear strain, \( d\gamma_{oct} \), as a function of \( de_{rs} \) (sum of the deviatoric part of the strain tensor):

\[ d\gamma_{oct} = 4 \cdot \frac{e_{rs}}{3 \cdot \gamma_{oct}} \cdot de_{rs} \iff de_{rs} = d\gamma_{oct} \cdot \left( 4 \cdot \frac{e_{rs}}{3 \cdot \gamma_{oct}} \right)^{-1} \]  

(5.13)

Thus, Equation 5.12 becomes:

\[ ds_{ij} = 2 \left( \frac{dG_s}{de_{ij}} \cdot e_{ij} + G_s \right) \cdot de_{rs} \cdot \delta_{ij} \cdot \delta_{jk} = 2 \left( \frac{dG_s}{d\gamma_{oct} \cdot \left( 4 \cdot \frac{e_{rs}}{3 \cdot \gamma_{oct}} \right)} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} = \]

\[ = 2 \left( \frac{dG_s}{d\gamma_{oct} \cdot \left( 4 \cdot \frac{e_{rs}}{3 \cdot \gamma_{oct}} \right)} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} \]  

(5.14)

Taking advantage of Equation 5.10, one may write:

\[ ds_{ij} = 2 \left( \frac{dG_s}{d\gamma_{oct} \cdot \left( 4 \cdot \frac{e_{rs}}{3 \cdot \gamma_{oct}} \right)} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} \iff \]

\[ \iff ds_{ij} = 2 \left( \frac{G_t - G_s}{\gamma_{oct}} \cdot \frac{4}{3 \cdot \gamma_{oct}} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} \iff \]

\[ \iff ds_{ij} = 2 \left( \frac{4 \cdot \left( G_t - G_s \right)}{\gamma_{oct}^2} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} = \]

\[ = 2 \left( \eta \cdot e_{rs} \cdot e_{ij} + G_s \cdot \delta_{ij} \cdot \delta_{jk} \right) \cdot de_{rs} \]  

(5.15)

where

\[ \eta = \frac{4}{3} \cdot \frac{G_t - G_s}{\gamma_{oct}^2} \]  

(5.16)
In order for one to have a constitutive relation, one must express the latter as a function of the strain increment tensor. Thus, Equation 5.15 becomes:

\[ ds_{ij} = 2(\eta \cdot e_{rs} \cdot e_{ij} + G_S \cdot \delta_{sr} \cdot \delta_{ij}) \cdot de_{rs} = \]

\[ = 2(\eta \cdot e_{rs} \cdot e_{ij} + G_S \cdot \delta_{sr} \cdot \delta_{ij}) \left( \delta_{ik} \cdot \delta_{jk} - \frac{1}{3} \delta_{rs} \cdot \delta_{kl} \right) \cdot de_{kl} = \]

\[ = 2 \cdot (\eta \cdot e_{rs} \cdot e_{ij} \cdot \delta_{ik} \cdot \delta_{jk} - \frac{1}{3} \delta_{rs} \cdot \delta_{kl} \cdot \eta \cdot e_{rs} \cdot e_{ij} + G_S \cdot \delta_{sr} \cdot \delta_{js} \cdot \delta_{ik} \cdot \delta_{jk} - \]

\[ - \frac{1}{3} G_S \cdot \delta_{sr} \cdot \delta_{js} \cdot \delta_{ik} \cdot \delta_{kl}) \cdot d\varepsilon_{kl} = \]

\[ = 2 \left( \eta \cdot e_{kl} \cdot e_{ij} - 0 + G_S \cdot \delta_{ik} \cdot \delta_{jl} - \frac{1}{3} G_S \cdot \delta_{ij} \cdot \delta_{kl} \right) \cdot d\varepsilon_{kl} \quad (5.17) \]

Merging Equation 5.11 and Equation 5.17, one has the full constitutive relation in the index notation:

\[ d\sigma_{ij} = 2 \left[ \left( \frac{K_t}{2} - \frac{G_S}{3} \right) \delta_{ij} \cdot \delta_{kl} + G_S \cdot \delta_{ik} \cdot \delta_{jl} + \eta \cdot e_{kl} \cdot e_{ij} \right] \cdot d\varepsilon_{kl} \quad (5.18) \]

In the matrix form, the constitutive relation may be written as:

\[ \{d\sigma\} = [C_t] \cdot \{de\} \quad (5.19) \]

where \([C_t]\) is the tangential stiffness matrix. This matrix may be expressed as the sum of two parts:

\[ [C_t] = [A] + [B] \quad (5.20) \]

Matrix \([A]\) is given by Equation 5.21:
This matrix results from the sum of the first two parts of Equation 5.18, and it has a similar shape to the isotropic linear elastic stiffness matrix, but with $G_S$ and $K_t$ replacing $G$ and $K$. As one may see, the tangential stiffness matrix is strain-dependent, as the secant shear modulus is a function of the actual strain state (via $\gamma_{oct}$). Matrix $[B]$ is given by Equation 5.22:

$$[B] = 2 \cdot \eta \cdot \{e\} \cdot \{e\}^T$$

(5.22)

where $\{e\}$ is the deviatoric part of the strain tensor, expressed as a vector. This part of the tangential stiffness matrix is clearly dependent on the actual strain state and it presents two main features. In one hand, this matrix is clearly unsymmetrical, as the deviatoric part of the strain state may have all of its components different from 0. On the other hand, matrix $[B]$ depends of the second-order term $\gamma_{oct}^2$ (via $\eta$), which may lead to matrix entries with values that may seen awkward at a first glance. When one is dealing with a shear modulus decay, $\eta$ assumes a negative value.

An important remark underlying the adopted formulation must be made at this point. This formulation (non-linear elastic models with secant moduli) is based on the isotropic linear elastic formulation. If one defines the secant moduli as functions of the octahedral strains, one guarantees that the adopted formulation is path-independent and, therefore, integrable (Chen and Mizuno, 1990). According to these authors, this is so because one is implicitly relating the strain and stress invariants, which leads to the existence of strain energy function.

An important issue that is inherent to the adopted formulation is the independence between volumetric and distortional behaviors, as the formulation is a direct adaptation of the isotropic linear elastic formulation. For medium to large strains, the mentioned behaviors are clearly intertwined, making the present formulation inadequate for this strain range.
5.3. Definition of the extended Masing criterion

As mentioned in 3.3, the extended Masing criterion, for a one-dimensional model, is made of the following premises (Kramer, 1996):

- The secant shear modulus for the strain (or stress) reversals is equal to the initial shear modulus;
- The shape of the unloading/reloading curve is equal to the backbone curve, but it is scaled up by a factor equal to two.
- If the previous maximum shear strain (in absolute value) is overcome, the stress path follows the backbone curve.
- If the $n$-th stress/strain loop is closed, the stress path may not surpass the path defined by the $(n-1)$-th loop.

For the present formulation, the mentioned premises were fulfilled only when distortional behavior was concerned. Despite having a three-dimensional meaning, Equation 5.3 is formally similar to Equation 3.25, which falls in the definition of a backbone curve, developed from a direct relation between the octahedral shear stress, $\tau_{oct}$, and the octahedral shear strain, $\gamma_{oct}$ (recall Equation 5.8). Therefore, the application of the extended Masing criterion was made exactly as one would do for the one-dimensional case, but relating the octahedral values of stress and strain.

In order for one to be able to apply the extended Masing criterion in terms of octahedral shear strain to the adopted incremental formulation, several issues had to be tackled, mainly concerning the detection of octahedral shear strain reversals, unloading and reloading situations, and maintenance of the stress path envelope.

First, for the octahedral shear strain reversals in the backbone to be detected, comparison had to be made in each step of calculation between the previously calculated octahedral shear strain, and the one determined for the current step if one is in the backbone curve. If there were to be a decrease in terms of octahedral shear strain, then there was a strain reversal. The detection of strain reversals when the strain path is in the backbone curve implied the storing the previous maximum octahedral shear strain in the strain history for each step. This maximum is usually known as the backstress, for stress-controlled descriptions. Recalling Figure 3.4, the backstress would correspond to the stress at point $a$. 
The scale factor in unloading/reloading situations was made adapting Equation 5.3. Assuming a maximum octahedral shear strain, $\gamma_a$, according to the second premise of the Masing criterion, one has a scale factor in the unloading/reloading curve equal to 2. This means that one may say the octahedral shear strain used to determined the secant shear modulus in the unloading/reloading curve is half the real value. As one may see in Equation 5.23, this last statement may be replaced by the following equivalent statement: the reference shear strain used to determine shear modulus in the unloading/reloading curve is twice the real value.

$$G_S\left(\frac{\gamma_a - \gamma_{oct}}{2}\right) = \frac{G_0}{1 + \alpha \left[\frac{G_S}{G_0} \frac{\gamma_a - \gamma_{oct}}{2 \cdot \gamma_r}\right]^{t-1}} \iff$$

$$\iff G_S\left(\frac{\gamma_a - \gamma_{oct}}{2}\right) = \frac{G_0}{1 + \alpha \left[\frac{G_S}{G_0} \left(\frac{\gamma_a - \gamma_{oct}}{2 \cdot \gamma_r}\right)\right]^{t-1}} \quad (5.23)$$

This result proved to be important in the proposed incremental formulation. As matrix $[B]$ depends on the actual deviatoric strain tensor quadratically, it would be difficult to implement the scale factor to the stiffness matrix via the octahedral shear strain. Using this result, both matrices $[A]$ and $[B]$ are easily determined, without any interference with the strain tensor.

The third and fourth premises of the extended Masing criterion were the most difficult to implement. Both premises imply that, in order for one not to surpass the previously defined hysteretic loops, one must store the octahedral shear strain where the strain reversal occurs, whether it concerns the backbone curve or the previous cycles. As the adopted formulation is incremental, and, therefore, depends on the deviatoric part of the strain tensor, $\{e\}$, via matrix $[B]$, one must also store the independent components of the strain tensor. These issues are conceptually easy to understand: the soil needs to have a kind of “memory” in order to retake the previous stress/strain path. In order to fulfill the fourth premise, one must be able to store large amounts of information concerning the stress/strain reversal. The present considerations were extremely important in terms of the implementation in PLAXIS. In fact, for the created subroutine, one assumed that there would be a maximum of fifteen within the first one.
5.4. Implementation in PLAXIS

5.4.1. Introduction

PLAXIS Version 8 allows a given user to implement any constitutive models in this finite-element program if the user follows certain procedures to describe the constitutive model in programming terms. These procedures must be implemented in a given programming language via a subroutine. In the present case, Digital Visual Fortran 6.0 was used, which is highly recommended, because of compiling-compatibility issues, and because PLAXIS provides a number of useful subroutines in Fortran concerning algebraic and matricial operations that are not properly compiled by more recent compilers. After programming the subroutine, the latter must be compiled into a Dynamic Link Library (DLL) and then added to the directory where PLAXIS is installed.

The user is supposed to provide information for PLAXIS calculation program to be able to determine the current effective stress, time and state variables. In order to do so, PLAXIS is supposed to provide the user the previous values of effective stress, time and state variables and also the strain and time increments.

These remarks and the following are largely based in what is described in PLAXIS manual (Brinkgreve et al., 2004). Nonetheless, practical remarks concerning the implementation of the adopted formulation will be made. The source-code of the subroutine lies in Annex A of the present work.

5.4.2. Subroutine header and general structure

In order for PLAXIS to recognize the DLL, the user-defined subroutine must have a fixed header, where 31 variables are defined. Several of these variables constitute what one will now on call switches, which allow the calculation program to distinguish different situations, such as loading conditions (drained or undrained), stress dependency of the model, time dependency of the model, symmetry of the stiffness matrix, total or incremental formulation, plasticity (or non-linearity, as will be discussed ahead). These switches are 4-byte integer variables, and most of them are boolean variables, i.e., these variables constitute “true”/“false” statements, with “1” being equal to “true”, and “0” equal to “false”. The most important variable in terms of subroutine functioning is a switch (not a boolean one) named “IDTask”.

“IDTask” controls what in the PLAXIS manual is named tasks. These tasks control the necessary milestones which the calculation needs to reach to correctly obtain the current stress tensor and state variables. There are six tasks, which are following:
Chapter 5 – Implementation of the Ramberg-Osgood model in PLAXIS

- Task #1: Initialization of state variables (IDTask = 1);
- Task #2: Calculation of the constitutive stress tensor (IDTask = 2);
- Task #3: Creation of the effective stiffness matrix (IDTask = 3);
- Task #4: Definition of the number of state variables (IDTask = 4);
- Task #5: Definition of main characteristics of the stiffness matrix (IDTask = 5);
- Task #6: Creation of the elastic stiffness matrix (IDTask = 6).

Each of these tasks are summoned by the calculation program. An important remark that must be made at this point: the subroutine didn’t summon the tasks in numerical order, *i.e.*, when IDTask is equal to 2, it doesn’t mean that, in the calculation process, the calculation of the stress tensor is the second task to be executed. These tasks will now be presented.

### 5.4.3. Tasks #1 and #4

In fact, two of the mentioned tasks are only needed to have the initial conditions of the very first calculation step. The first task to be summoned is task #4. In this task, the user provides the number of state variables inherent to the constitutive model. This allows the program to know the dimension of the array where the state variables of the model are to be stored. After executing task #4, the initial values of the state variables in the fictitious previous step are generated in task #1, according to the constitutive model, and the 8-byte real-number array, \( StVar0 \) (*“Real(8), Dimension” in Fortran*), is filled. If nothing is declared in this first step, PLAXIS assumes that \( StVar0 \) is an array of zeros. This was the case of the present formulation.

For calculation steps other than the first one, PLAXIS copies the values of the determined (current) state variable array, \( StVar \), to the array \( StVar0 \), in order to execute the next calculation step.

An important remark must be made at this point. Between calculation phases in PLAXIS, state variables may be reset if one replaces the defined material by another one. This may be made without losing the data concerning the stress state in the soil. This procedure is important in terms of initial stress state generation, especially for finite-element models where there are vertical or lateral discontinuities.
5.4.4. Tasks #3, #5 and #6

After the generation of the previous state variable array, \( StVar0 \), task #3 is summoned. In this task, the effective stiffness matrix is generated. There are two approaches in terms of stiffness matrix generation, with consequences in the numerical method used to obtain a non-linear solution. In one hand, one may generate the linear elastic stiffness matrix, which means that, when calculating the constitutive stresses in task #2, there is the need to explicitly define the stresses, and comparison the stress obtained by the elastic stiffness matrix and the ones obtained using the constitutive model is needed (the equivalent nodal stresses are obtained by the difference between the elastic stresses and the constitutive ones); on the other one, one may define the tangential stiffness matrix in task #3, without the need to define constitutive stress, as they may be obtained via the product of matrix \([D]\) and the increment strain array, \( \{d\varepsilon\} \). The first approach implies the use of the modified Newton-Raphson method; the second approach implies that the full Newton-Raphson method is used. The calculation program checks the balance between the equivalent nodal stresses and the external loads using the elastic work as a parameter. For this purpose, the calculation program uses the stiffness matrix defined in task #6. For the first approach, the matrix is exactly the same as the one in task #3.

The original models in PLAXIS follow the first approach, and, despite the fact that one has adopted an incremental formulation, so was the case of the user-defined subroutine under analysis. This was due to the fact that, during task #3, one isn’t able to summon the state variables array of the previous step, \( StVar0 \). One may argue that, creating an external storage file, where one would store the values needed to the formulation explained in 5.4.2, i.e., the strains associated to each strain reversal and respective deviatoric strain vector, one would be able to create a tangential stiffness matrix. However, one preferred to store these values as state values, and proceed to constitutive stress correction in task #2. Storing the reversal strains and the deviatoric strains as state variables within PLAXIS had another advantage: one may use the output module of PLAXIS to study the values at each of the calculation steps.

The adoption of this approach had consequences in two other tasks. First, task #5, where one controls the numerical attributes of the stiffness matrix defined in task #3, one declared a symmetrical, stress-independent, time-independent, and secant stiffness matrix (this is done using four boolean switches with value equal to zero, namely the variables \( NonSym, iStrsDep, iTimeDep \) and \( iTang \)). Second, task #2 became the most important task in the subroutine.

Next, for one to be able to correctly model undrained behavior, one must give the conditions to assemble the total stiffness matrix as a function of the effective stiffness matrix. This was done using the procedure as defined by Naylor et al. (1981). The bulk modulus of water, \( K_W \), is one of the input
variables of the user subroutine \((\text{BulkW})\). PLAXIS generates the total stiffness matrix according to Equation 5.24:

\[
[D]_{\text{total}} = [D]_{\text{Effective}} + K_f
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where \(K_f\) is the bulk modulus accounting the contribution of both the fluid and the solid phases of the soil. It is defined as a function of the bulk modulus of water, the bulk modulus of the solid phase, \(K_{\text{soil}}\), and the porosity, \(n\), and it is given by Equation 5.25:

\[
K_f = \frac{1}{n} + \frac{1-n}{K_W + K_{\text{soil}}}
\]

Taking into account that the bulk modulus of the water is much greater than the one of the solid phase, for current values of porosity, one may say that \(K_f \approx K_W\). This assumption is made within the PLAXIS framework. If no value is declared, the default value in PLAXIS for the bulk modulus of water is the following:

\[
K_W = 100 \cdot \frac{D_{11} + D_{22} + D_{33}}{3}
\]

For the built subroutine, one adopted a bulk modulus of water that leads to an initial value of Poisson coefficient equal to 0.495.

PLAXIS is able to recognize that one considering undrained conditions using, once again, a switch, named \(\text{IsUndr}\).

### 5.4.5. Task #2

This task is the cornerstone of the subroutine. It is where the constitutive relation is, in fact, implemented in PLAXIS. The first action to do in this task was to declare the properties inherent to the constitutive model. These properties are stored in an array (vector) of 50 entries, and they were the following, according to the adopted formulation:
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- The initial (elastic) shear modulus, \( G \) (stored in the first entry of the array \( \text{Properties} \));
- the initial tangent bulk modulus, \( K \) (stored in the second entry of the array \( \text{Properties} \));
- the reference strain, \( g_{\text{ref}} \) (stored in the third entry of the array \( \text{Properties} \));
- the \( \alpha \) parameter of the Ramberg-Osgood model, \( \alpha \) (stored in the fourth entry of the array \( \text{Properties} \));
- the \( r \) parameter of the Ramberg-Osgood model, \( exp_r \) (stored in the fifth entry of the array \( \text{Properties} \)).

In order to correctly model the increase of pore pressure under undrained conditions, one must specify that, for undrained conditions (i.e., if \( \text{IsUndr}=1 \)), the volumetric strain increment, \( dEpv \), leads pore pressure build-up. The pore pressure build-up is named \( dSwp \) in the subroutine, and it is determined using Equation 5.27:

\[
dSwp = \text{BulkW} \cdot dEpv
\]  

(5.27)

After these initial actions declared in task #2, one started to define the constitutive stress, according to the adopted formulation. This was made using several switches, that controlled the calculation flow.

One of these switches consists of the variable \( ipl \). This variable, according to PLAXIS manual, allows the program to recognize if one is in elastic \( (ipl=0) \) or elastoplastic calculations \( (ipl \neq 0) \). In fact, what this switch controls is the numerical procedure in order to determine the constitutive stresses. If one states \( ipl=0 \), with the non-linear elastic model, the stress correction isn’t made. In contrast, if \( ipl \neq 0 \), the modified Newton-Raphson method is triggered, even if elastic calculation is made. In the case under study, one adopted \( ipl=1 \), as there was place to stress correction.

The second switch was user-defined, and it was named \( caso \). This was the main switch of the subroutine, as it defined the load case, and, therefore, controlled the application of the Masing criterion and the storage and erasing of \( StVar \). For each of the loading cases, \( caso \) assumed a given value. These were:

- Null loading or loading on the backbone curve: \( caso = 1 \);
- Unloading concerning the \( n^{th} \) hysteresis loop: \( caso = 2n \);
- Reloading concerning the \( n^{th} \) hysteresis loop: \( caso = 2n+1 \).
As one has already mentioned earlier, one admitted a maximum of fifteen hysteresis loops. Therefore, one adopted 32 as the maximum value of caso. If this were to be exceeded, the subroutine would result in elastic calculation, which, for a typical transient seismic record, wouldn’t be far from the truth, as there would have to be more than fifteen displacement reversals during the seismic event.

The maximum value of caso was also important in terms of the definition of the arrays StVar and, consequently, StVar0. As one adopted the value 32, the maximum components of the strain tensor needed to be stored are 192. The storage of variables was made according to the following procedures:

- In terms of components of the strain tensor, $dEps$, for an integer variable $i$, varying from 1 to 6, $StVar(6 \cdot (caso - 1) + i) = dEps(i)$;
- In terms of octahedral shear strain, $g_{oct}$, $StVar(200 + caso) = g_{oct}$.

The octahedral shear strain, $g_{oct}$, was determined using a subroutine defined by the author, named GrandezasD. In this subroutine, one determines:

- The array (vector) of components of the deviatoric strain tensor, $defdist$;
- The octahedral shear strain of the previous step, $g_{oct_0}$;
- The octahedral shear strain of the current step, $g_{oct}$;
- The octahedral shear strain increment, $d_g_{oct}$;

The source-code of this subroutine is very compact, as it is fully based on algebraic operations. It is in Annex A of this work.

Caso also defines a slave switch named $ki$. The latter intervenes in the erasing of components of $StVar$ when one passes from the $n$-th loop to the $(n-1)$-th loop.

The load case was determined considering, as a premise, that there was an increase in the octahedral shear strain for the assumed value of caso. If such premise was true, the subroutine proceeded to determine the real stiffness matrix according to the adopted formulation. If not, one added 1 to the assumed value of caso, and repeated the procedure in terms of determining the state variables.

The real stiffness matrix was determined using a specific subroutine named CalculoD, having as input parameters the Ramberg-Osgood model parameters and the output parameters of subroutine
and returning the real stiffness matrix, $D_{ne}$. For values of caso other than 1, the reference shear strain was multiplied by 2, as described in 5.3.

The subroutine $CalculoD$ has embedded another subroutine, named $secante_RO$, that determines the secant shear modulus according to Equation 5.3, via the one-dimensional application of the Newton-Raphson method. One opted for this numerical method as it was the one that, comparing to the fixed-point method, exhibited the best numerical robustness. The source-code of $secante_RO$ is also in Annex A.

Having the value of the secant shear modulus, the next step within $CalculoD$ was to determine matrices $[A]$ and $[B]$. Matrix $[A]$, having the secant shear modulus, was made using Equation 5.21. In order to calculate matrix $[B]$, one had to numerically estimate the derivative of Equation 5.3 in terms of the octahedral shear strain. This fact has as a consequence that, in PLAXIS, one should adopt small load increments in order to have good numerical precision. The source-code of $CalculoD$ is also in Annex A.

Finally, having the real stiffness matrix, $D_{ne}$, one determines the array with the components of the stress increment, $dSig$, and the array with the component of the current stress tensor, $Sig$. These algebraic operations were done using subroutines $MatVec$ and $AddVec$, which are provided by PLAXIS.

Figure 5.1 contains the flow chart containing all the steps in task #2. As all the other subroutines, the subroutine $User_Mod$ is in Annex A.
Figure 5.1 – Flow chart concerning task #2
5.5. Validation of the subroutine

5.5.1. Introduction

In order to validate the subroutine, four tests were made, concerning not only the subroutine itself, but also a component of it, namely the subroutine *secante_RO*. The mentioned tests were the following:

- For the subroutine *secante_RO*, one should obtain shear modulus reduction curves similar to the ones showed in Figure 3.6. For the same value of $r$, for increasing $r$, one should obtain stiffness reduction for lower normalized shear strains. If one follows a quadrature rule, one should be able to have a similar value of hysteretic damping as the one obtained using Equation 3.25;

- An element subjected to an imposed distortional displacement according to an harmonic function should, for the backbone curve, follow the formulation indicated in 5.2 and should imply an hysteresis loop for the complete load cycle;

- An element subjected to an imposed distortional displacement according to a linear combination of four harmonic functions should, for the backbone curve, follow the formulation indicated in 5.2 and should obey to all the premises of the extended Masing criterion;

- Considering a two-dimensional plane-strain model in PLAXIS with a width much larger than its height and lateral absorbent boundaries, for the central part of the model, one should obtain similar results in PLAXIS and in SHAKE2000.

The results of these tests will be presented in the following paragraphs.

5.5.2. Validation of *secante_RO*

The validation of *secante_RO* was done compiling an executable file having as input the Ramberg-Osgood model parameters, and having as output two vectors, one corresponding to the normalized shear strain, and the other one with the secant shear modulus with respect to the shear strain of equal entry. These vectors were determine for linear increments of normalized shear strain equal to 0.001, ranging from $10^{-3}$ to $10^1$, which lead to vectors with 10000 entries. Figure 5.2 contains the data concerning the tested curves.
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Figure 5.2 – Normalized shear modulus reduction curves obtained using *secante_RO*, for the same value of $\alpha$

Comparing with Figure 3.6, one may see that the curves obtained using *secante_RO* as calculation kernel are similar with the ones obtained by Ishihara (1996).

In the following validation procedure, one tested the influence of $\alpha$. This was done assuming a constant value of $r$ equal to 3.0. The obtained results are shown in Figure 5.3.

Figure 5.3 – Normalized shear modulus reduction curves obtained using *secante_RO*, for the same value of $r$

As one expected, for increasing values of $\alpha$, stiffness reduction occurs for lower normalized shear strains.

The last validation test concerning *secante_RO* was made estimating numerically the value of hysteretic damping. This was done calculating, for a given shear strain, the work for the determined stiffness reduction law and comparing it with the matching elastic value, as states Equation 3.8. After
determining that value for each shear strain, the obtained curve was compared with the theoretical curve given by Equation 4.23. This was done for three sets of values:

- $\alpha = 50, \ r = 2.0$;
- $\alpha = 50, \ r = 2.5$;
- $\alpha = 50, \ r = 3.0$.

Numerical integration was made in order to determine the work associated to the Ramberg-Osgood curve. The results are shown in Figure 5.4, Figure 5.5, and Figure 5.6.

Figure 5.4 – Comparison between theoretical and numerical curves, $\alpha = 50$ and $r = 2.0$

Figure 5.5 – Comparison between theoretical and numerical curves, $\alpha = 50$ and $r = 2.5$
The results show that subroutine *secante_RO* follows the Ramberg-Osgood model, as the superposition between the curves is very good (there is a misfit for $r=2.0$, due to the initialization of the numerical integration procedure).

### 5.5.3. Element under pure distortional cyclic loading – single harmonic signal

In order to test the adopted formulation in PLAXIS, one created a model corresponding to an element under pure distortional cyclic loading. Figure 5.7 contains an image of the model.

Figure 5.6 – Comparison between theoretical and numerical curves, $\alpha=50$ and $r=3.0$

The adopted model corresponds to a square with a width equal to 10m, with fixed displacements at the base, and fixed vertical displacements at the lateral boundaries. At the top, a harmonic load is applied.
displacement was prescribed to the model. In terms of material properties, one considered a massless material (in order, at this time, not to account with inertial effects), with the following parameters:

- \( G = 10000 \text{kPa}; \)
- \( K = 26000 \text{kPa}; \)
- \( \gamma_{ref} = 1 \times 10^{-3}; \)
- \( \alpha = 50; \)
- \( r = 2.5 \)

The prescribed displacement boundary condition was imposed in a dynamic calculation phase, and followed the function shown in Figure 5.8.

![Harmonic signal](image)

**Figure 5.8 – Displacement multiplier in dynamic calculation phase: single harmonic signal**

The prescribed displacement at the input phase was equal to 0.001m, which means that the maximum shear strain imposed to the model was equal to \( 1 \times 10^{-4} \). The calculation phase obeyed the Newmark-beta method, according to the linear acceleration procedure.

At the end of the calculation phase, the shear strain and shear stress in the element were constant within all of its area, as one would expect in pure shear state (Figure 5.9 and Figure 5.10). In the case of the shear strain, it may seem this last statement is not true. Differences are due to numerical precision issues. The maximum value of the shear stress was \( 6.3 \times 10^{-1} \text{kPa} \)
A control point was chosen in order to obtain information concerning the constitutive behavior of the element. The shear strain/shear stress curve that resulted from the calculation is in Figure 5.11.
Figure 5.11 – Shear strain/shear stress curve obtained at the end of the calculation phase for a harmonic signal

As one may see, for the case of a harmonic loading, the implemented subroutine clearly obeys the first two premises of the Masing criterion. Another feature that the subroutine correctly models is the non-linear behavior.

In order to check if the constitutive relation was correctly modeled, comparison was made with the results obtained using the already validated subroutine secante_RO, considering as input value the shear strain imposed in the plane-strain model. Figure 5.12 shows the two curves, and the linear elastic behavior for the same initial shear modulus.

Figure 5.12 – Comparison between the curves obtained via PLAXIS and secante_RO
One may think, at a first glance, the subroutine would not be functioning as it should. As shown in Figure 5.12, the comparison was biased. This was due to the following: the octahedral shear strain, in plane-strain for a two-dimensional pure-shear analysis, doesn’t assume the same value as the applied shear strain.

Considering pure shear loading in plane-strain, with a shear strain equal to 1, the octahedral shear strain is the following:

$$\gamma_{oct} = \frac{2}{3} \sqrt[6]{\left(\frac{\gamma}{2}\right)^2} = \frac{2}{3} \sqrt{\frac{3}{2}} \gamma^2 = \frac{\sqrt{6}}{3} \gamma = 0.816 \cdot \gamma$$ \hspace{1cm} (5.28)

In the pretended comparison, the octahedral shear strain is approximately equal to $8.16 \times 10^{-5}$, leading to a normalized shear strain equal to $8.16 \times 10^{-2}$. For this value of octahedral shear strain, the secant modulus estimated using the vector created in order to do the analysis in 5.5.2 and doing linear interpolation, was approximately 6310.2 kPa. Recalling the maximum shear stress shown in Figure 5.10, $6.306 \times 10^{-1}$ kPa, one may determine the secant shear modulus in PLAXIS model using Equation 5.29.

$$G_s = \frac{\sigma_{xy}}{\gamma_{xy}} = \frac{6.306 \times 10^{-1} kPa}{1 \times 10^{-4}} = 6306 kPa$$ \hspace{1cm} (5.29)

As one may see, the subroutine is obeying the constitutive relation as defined in 5.2. The constitutive relation leads to a slightly stiffer model when comparing to the standard one-dimensional Ramberg-Osgood model.

### 5.5.4. Element under pure distortional cyclic loading – linear combination of four harmonic signals

In order to test the subroutine for more complicated loading situations, a different displacement multiplier was applied to the model used in 5.5.3, maintaining the same material properties, the geometric definition and the same mesh. A linear combination of four harmonic signals, each with different frequency from the others, was used to obtain the displacement multiplier evolution with time. The linear combination had a period equal to 250s. In order to assess the subroutine behavior when the loading condition translates in a return to the backbone curve, one admitted a time history of 500s. Between 250s and 500s, one applied a scale factor equal to 2 with respect to the function between 0s and 500s. Another feature that one pretended to assess in this test was the stress state at the end of the calculation phase, for a final shear strain equal to 0. The applied function is shown in Figure 5.13.
Figure 5.13 – Displacement mutliplier in dynamic calculation phase: linear combination of four harmonic signals

The calculation phase obeyed, once again, the Newmark-beta method, according to the average-acceleration procedure. The shear stress/shear strain curve at the end of the calculation phase is shown in Figure 5.14.

Figure 5.14 - Shear strain/shear stress curve obtained at the end of the calculation phase for a linear combination of four harmonic signals

One may see that the subroutine obeyed all the extended Masing criterion premises. There is a closed loop (with inner loops) between \(-2.5 \times 10^{-4}\) and \(2.5 \times 10^{-4}\). The return of the stress path to the backbone curve happened when the shear strains surpassed the value \(2.5 \times 10^{-4}\). Having as purpose showing that the subroutine followed the backbone curve after the first hysteresis loop was closed, one isolated the curve at two parts:
• Between 0,0s and 11,0s;

• Between 253,5s and 261,0s;

The result of this comparison is shown in Figure 5.15.

![Linear combination of 4 harmonic signals](image)

**Figure 5.15 – Comparison between two parts of the shear stress/shear strain curve for a linear combination of four harmonic signals**

The shear stress/shear strain curve doesn’t show any discontinuity, as it would if it didn’t obey the third premise of the extended Masing criterion.

Returning to Figure 5.14, the shear stress/shear strain curve exhibits one of the features due to the extended Masing criterion, namely the fourth premise. The hysteresis loop defined by the condition of unloading after $5,0 \times 10^{-4}$ constitutes an envelope concerning other loops. Another feature that is due to the adopted formulation is that, at the end of the calculation phase, one has residual stress.

From what has been exposed, one may conclude that the subroutine was functioning properly, as long as inertial effects are not considered. The next test implied a full dynamic calculation, in order to test the subroutine at a model setting of the kind for which it was developed.

### 5.5.5. Comparison between calculations in PLAXIS and in SHAKE2000

In order to test the subroutine in terms of seismic-motion modeling, one introduced a two-dimensional model in PLAXIS that, at least at a part of it, may be compared to a one-dimensional soil column. Figure 5.16 shows the adopted model.
For the central area of the model, for an imposed acceleration at the base of the model, one should obtain a result similar to one obtained at a one-dimensional analysis, as lateral effects are not significant. The model has a height of 10m and a width of 250m. A single material was considered, having the following properties:

- \( G = 20000 \text{kPa}; \)
- \( K = 200000 \text{kPa}; \)
- \( \gamma_{\text{ref}} = 1 \times 10^{-2}; \)
- \( \alpha = 50; \)
- \( r = 2.5; \)
- \( \gamma_{\text{unsat}} = \gamma_{\text{sat}} = 20 \text{kN/m}^3. \)

The standard numerical integration procedure in PLAXIS admits a Newmark-beta algorithm with numerical damping. One preferred to proceed with the calculation with an average-acceleration version, and with a Rayleigh-type damping equal to 2% adjusted to the first and third vibration modes. The adjustment was made following Equation 5.30 and Equation 5.31.

\[
a_0 = \frac{5}{3} \cdot \xi \cdot \omega_1 \quad (5.30)
\]

\[
a_1 = \frac{1}{3} \cdot \frac{\xi}{\omega_1} \quad (5.31)
\]

where \( \omega_1 \) stands for the angular frequency corresponding to the first vibration mode.

The fact that one prescribed a given self-weight meant that PLAXIS would perform a full dynamic calculation. The prescribed displacement at the base followed a well-known acceleration time series, obtained at Gilroy #1 array, for the 1989 Loma Prieta earthquake.
The one-dimensional analysis was done using the commercial program SHAKE2000, with an equivalent shear strain ratio equal to 0.65. A soil column of 10m was considered, with underlying bedrock with a shear stiffness 100 times greater. The stiffness reduction and material damping curves were compliant with the adopted Ramberg-Osgood parameters. The scale factor between the octahedral and the plane shear strains, as demonstrated with Equation 5.28, was duly accounted, dividing the shear strain that served as input for the program based on $\text{secante}_\text{RO}$ by $\frac{\sqrt{6}}{3}$. The mentioned curved are shown in Figure 5.17 and Figure 5.18.

![Figure 5.17 – Stiffness reduction curve used in SHAKE2000](image)

![Figure 5.18 – Damping curve used in SHAKE2000](image)

In order to compare the results obtained by both models, one retrieved the acceleration and the shear stress time series at different depths. These depths were:
• 0m (at surface);
• 1m;
• 3m;
• 7m.

In terms of acceleration time series, the results are shown in Figure 5.19, Figure 5.20, and Figure 5.21.

![Figure 5.19 – Acceleration time series by both models at the surface](image1)

![Figure 5.20 – Acceleration time series by both models for a depth equal to 1m](image2)

![Figure 5.21 – Acceleration time series by both models for a depth equal to 7m](image3)
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There are slight differences between the two models. The acceleration time series obtained using PLAXIS exhibit less damping, especially at the end of the time series. This may be due to the fact that, in PLAXIS, the prescribed displacement boundary doesn’t allow vertical displacements, which means that PLAXIS models implicitly account for rigid bedrock. For real situations where one doesn’t have rigid bedrock, such as the model in Chapter 6, one must determine an equivalent radiation damping. In terms of PGA, SHAKE2000 leads to slightly higher values. Nonetheless, the overall results of both programs in terms of acceleration time series are pretty similar.

The obtained shear strain time series are shown in Figure 5.22 and Figure 5.23.

![Figure 5.22 – Shear strain time series by both models for a depth equal to 1m](image1)

![Figure 5.23 – Shear strain time series by both models for a depth equal to 7m](image2)

There is an interesting feature concerning the shear strain time series obtained using PLAXIS. There is evidence that at the end of the calculation process, the value of the shear strain isn’t equal to 0, which is in agreement with the adopted formulation.

Comparing the shear strain time series obtained by both programs, one may conclude that SHAKE2000 leads to globally higher strains. Once again, times series obtained using both calculation programs may be considered similar, and PLAXIS exhibits less damping.
In order to conclude the comparison, the transfer function relating the acceleration at the surface and at the bedrock was calculated for both programs. Figure 5.24 contains the transfer function

![Transfer function comparison between PLAXIS and SHAKE2000](image)

**Figure 5.24 – Transfer function for both programs**

The models exhibit the same vibration modes. Comparing the level of amplification, PLAXIS exhibits higher amplification values for the vibration modes. This falls in line with the previous results, which showed that PLAXIS leads to less damping. Another remark that may be made is that PLAXIS amplifies much more than SHAKE2000 for high frequencies.

### 5.6. Concluding remarks

Considering what has exposed, one may consider that the Ramberg-Osgood model was successfully implemented in PLAXIS.

First, a full theoretical constitutive model was exposed, showing all the details. Important remarks concerning eventual numerical implementation were done at this point.

Issues concerning the extended Masing criterion were also shown, focusing not only on theoretical aspects, but mainly on the consequences in terms of programming implementation of all the premises.

The implementation within the PLAXIS framework was thoroughly analyzed, as all the steps concerning the subroutine needed to define a soil model were described. The role that each of the components within the subroutine play was pointed out. This was done especially for the calculation tasks and for extremely important variables, such as caso. The source-code is in Annex A.
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A series of validation tests were made. Considering those tests, one believes that the overall results are quite satisfactory, and that they fall in line with what was pretended with the model.

An important remark must be made at this point. Despite the three-dimensional nature of the adopted constitutive relation, one only tested it for two-dimensional situations, as all the calculations in this work were in plane-strain. The three-dimensional behavior of the subroutine is untested.

The implemented subroutine will be used in Chapter 6 in the two-dimensional model of São Sebastião volcanic crater.
6. Study on site effects at São Sebastião volcanic crater

6.1. Introduction

The present work was developed within the framework of a research project, which led to several efforts in order to determine the causes of site effects at São Sebastião volcanic crater. The mentioned efforts, of course, were made at several stages, and they were adapted to certain features within the crater that were given by, at the time, preliminary results. Therefore, it seems of the utmost importance that the presentation of all the research activities described in the present chapter follows the real timeline.

First, one will present general information concerning São Sebastião, its geological setting, and the reason that led to the onset of research activities related to São Sebastião. Discussion will be made on possible geological formation processes of the crater, and their consequences in terms of site effects, following closely the work by Santos et al. (2007).

Next, the first efforts to characterize São Sebastião will be reviewed. Focus will be made on the first boreholes made at the crater and on geophysical tests, especially on the surface wave tests made by Lopes (2005).

These efforts weren’t sufficient in order to build a model that fully explained amplification at São Sebastião. Having as starting point all the gathered information concerning São Sebastião at the time, several site-effect assessment techniques were used in order for one to understand the reasons that lead to strong site effects at the crater.

These techniques served as effective tools in optimizing new characterization tests. The link between these tests and the results of the previously mentioned site-effect assessment techniques will be presented, based on the work of Lopes et al. (2008). The mentioned tests, at the end, were essential for the definition of two-dimensional models of the crater.

6.2. Background. Geological setting

The main goal of the present work concerned site-effect characterization of São Sebastião volcanic crater, which Sebastião is located in Terceira island, in the Azores archipelago.
Figure 6.1 - Location of São Sebastião area: a. Central Group of the Azores Archipelago; b. Terceira Island (in: Santos et al., 2007)

Figure 6.2 contains the epicenter of most important seismic events at the archipelago

Figure 6.2 - Schematic epicentral map for some of the main tectonic events in the central group of the Azores archipelago (modified from: Madeira & Brum da Silveira and Nunes et al.): G - Graciosa; T - Terceira; Sj - São Jorge; Fa - Faial; P - Pico; DJc - D. João de Castro bank

There is historical evidence that São Sebastião volcanic crater exhibits stronger ground motion than its surrounding areas, for similar epicentral distances. The first strong evidence of this anomalous behavior consisted of the isoseismic map concerning the January 1\textsuperscript{st} 1980 earthquake, shown in Figure 6.3.
There was also evidence of site effects with a more local expression, as the damage concerning the building stock of the village of São Sebastião presented significant spatial variation. Figure 6.4 contains the damage distribution along São Sebastião for the previously mentioned seismic event.

These facts led to the installation of a permanent accelerometric station at Escola, in the vicinity of the most damaged areas. During a given period, there were two temporary accelerometric stations, at Junta and Misericórdia. Their location is also shown in Figure 6.4.
São Sebastião crater has an average diameter of 1100 m. The detailed volcanological map of São Sebastião and surrounding area was made by Nunes (2000). A simplified version of the map is presented in Figure 6.5.

Figure 6.5 - Simplified volcanological map of the São Sebastião area (in: Santos et al., 2007; adapted from Nunes, 2000)

Figure 6.5 shows that the geological formations of the area have mainly basaltic composition. Nunes (2000) showed that the northern side of the crater rim cuts basaltic lava flows while in the southern part a lahar deposit is identified. The eastern side of the crater shows a scoria cone (Monte das Cruzes) and inside (SE side) there is an outcrop of basaltic lavas of the same nature as the ones that appear on the NW-W side. The depression is filled mostly by slope deposits of different nature and composition and by fluvial deposits.

The genesis of São Sebastião volcanic crater is not a consensual issue. Montesinos et al. (2003) proposed a pit crater structure for the area while Madeira (2005) agreed with the idea proposed by Lloyd and Collis (1981) of a phreatomagmatic nature. The latter authors had proposed this genetic nature for São Sebastião because such a large crater could only be formed by a very explosive event as the phreatomagmatic explosions. Madeira (2006), mentioned by Santos et al. (2007), justifies that the lahar deposit is a phreatomagmatic flow deposit, resultant from the formation of the crater and couldn’t be associated to other source because it doesn’t appear anywhere else in the surrounding area.

The phreatomagmatic volcanoes, usually designated by maar, are low standing volcanoes with very wide bowl-shaped craters that can range from a few hundreds of meters to about 3 km. They result
from the contact of basaltic magma with water (the water level, an aquifer or the sea water) that produces a blast of fine-grained particles and a steam explosion. As they usually form holes in the surface, afterwards they get filled with water, forming lakes (Cas and Wright 1987, Francis 1993, Fisher et al. 1997).

The understanding of the geological nature and evolution of the crater is highly dependent on the volcano-stratigraphic relation between the different geological materials and is very important for the comprehension of the site effects occurred in the village.

6.3. Site-characterization data prior to site-effect assessment using acceleration time series

The first data of interest in a site-effect assessment point of view concerning São Sebastião was a deep borehole (182 m) made in 1979, in the SE limit of the crater, for geothermal prospection (Lloyd and Collis, 1981). This borehole, for the first 34 m; detected the presence of soft materials, crossing a shallow deposit of brown sandy clay soil (4 m) followed by 30 m of fluvial/pyroclastic deposits described as “a mixture of angular to sub-rounded feldspar crystals, weathered fragments of scoria and aphyric basalt, minor volcanic glass and fine grained ash”. After these first layers, the borehole intersected many intercalations of aphyric basaltic lavas, porphyritic basaltic lavas and weathered basaltic scoria. For depths between 82 m and 136 m no cuttings were obtained (no recovery), which may be due to very weathered basaltic scoria and/or weathered pyroclastic deposits.

After the 1980 earthquake, the geological and geophysical information gathered on São Sebastião resulted mainly from a research project dedicated to the study of seismic hazard and risk in the central group of the Azores archipelago (PPERCAS Project). The most important works concerning site-effect assessment made in the framework of PPERCAS were three boreholes within the crater (Malheiro, 1998), the volcanological map presented before (Nunes, 2000), noise acquisition (Senos et al., 2000) and gravimetric measurements (Montesinos et al., 2003).

The mentioned boreholes were made with a rotation drill and had as objective a detailed characterization of the filling deposits of the crater (Malheiro, 1998). Results were not the best as the recovery ratio was very low (considering all the boreholes, the individual recovery ratio was less than 15%); the main causes are related to the type of materials, to the presence of a near-surface water table, and to the water injection used as a circulation fluid for the drill. For the first 6 to 7 m, mainly silty to clayey with rock fragments and sand were detected. Below 20 m the materials are mainly sand and rock fragments with a silty to clayey matrix.
A total of 130 noise measurements acquisitions of 10 minutes each were made in a quadrangular grid with 120 m between measurement points (Senos et al., 2000). The results pointed to a predominant frequency between 1 Hz and 3 Hz. The higher damage occurred in the areas where the predominant frequency is about 2 Hz.

Besides the volcanological map of the area, the more detailed study on São Sebastião that resulted from the PPERCAS project was the work by Montesinos et al. (2003). This work comprehended not only a description of the geological materials, but also a gravimetric study of the area. 334 gravity sites were acquired in São Sebastião and surroundings for this study, with spacing equal to 200 m in the regional study and equal to 50 m in the urban zone, inside the crater.

The results were obtained by a method of gravity inversion, constrained with the interpretation of the geology of the area to help removing inadequate density. From the regional study it is important to notice some low density contrasts (Figure 6.6) that the authors associated mainly with the infillings of the crater and with the scoria cones in the vicinities. Noticeable is that the only low density contrast at a depth of about 500 m is inside the crater area. The mentioned authors related the anomaly with the genesis mechanism of the crater.

![Figure 6.6 – Deep horizontal sections of the density contrast 3-D regional model for the area of São Sebastião and surroundings, for -100 and -500 m (UTM coordinates in meters) (in: Santos et al., 2007; adapted from Montesinos et al., 2003)](image)

In the detailed study of the area those authors present 3 horizontal sections and 3 W-E cross-sections (Figure 6.7) located where were placed the strong motion stations that recorded the earthquakes that will be discussed further on.
The shallower of the horizontal sections crosses at \( z=100 \text{ m} \) (above the sea level), which is almost 50 m below the surface of the crater (\( \approx 150 \text{ m} \)). Information on the first 50 m appears only in the cross-sections. In general two main high-density bodies were identified. The authors associated these bodies with the presence of the basaltic lava flows that form the crater wall and define the collapsed area. The mentioned bodies are similar in all horizontal sections of the model seeming to characterize very steep walls that associate with the collapsed mechanism proposed. The negative density contrast present in the inner area corresponds to the post-generation soft sediments and infillings of the crater. Also, the conduit system is filled with breccia-type deposits, which are also identified as negative density contrast bodies (Montesinos et al., 2003).

After that work, some seismic data in the São Sebastião crater were acquired, mainly by the Surface Wave Method (SWM) (Lopes, 2005). The seismic data is composed by 14 multichannel surface wave lines to determine an approximate shear wave velocity for the different materials in the area and also to get an idea on the distribution and thickness of the fillings within the crater. For the velocities 3 main groups were identified: for the basalts from the north area the shear wave velocity is around 1000 m/s; for the pyroclastic and scoria deposits from the Cruzes cone in the east the values varies between 250-450 m/s increasing with depth; and finally for the fillings of the crater the values are much smaller and between 90 and 200 m/s also increasing progressively with depth.

As the one-dimensional information obtained from the SWM was distributed in the area, Lopes (2005) made some maps to plot the velocity distribution at different depths. The isolines maps (Figure 6.8) show that in the area of Misericôrdia and Junta the shear wave velocity near surface (\( z<50\text{m} \)) is higher than in the Escola area, justifying the differences observed in damage. Besides, high velocities in the western area of the crater were measured and the total thickness of the fillings was not possible.
to determine due to the low velocity of the superficial materials and intrinsic limitations of the method.

There seems to be a relationship between the damage distribution shown in and the $V_S$ distribution in depth. Comparing Figure 6.4 and Figure 6.8, one sees that most of the cases where damage was more severe are located over areas with lesser shear velocity for a 2m depth. The results concerning the western part of the crater led Santos et al. (2007) to consider the possibility of the existence of a lava tongue. Considering this hypothesis valid, the outcrop of intermediate lava that appears in the SE limit may be the end of the lava tongue that flowed into the crater and outcrops in that area due to its lower altitude relatively to the surroundings. This altitude corresponds approximately to the altitude in which this lava flow appears in the middle of the sediments of the crater, corresponding to the base of the crater at the moment that the lava flow occurred.

### 6.4. Site-effect assessment techniques using acceleration time series

#### 6.4.1. Introduction

The main source regarding the site-effect assessment at the time was a set of accelerometric records obtained at the village of São Sebastião during two seismic crises in the Azores Archipelago. The first of the mentioned crises lasted from June 1997 to October 1997; the second crisis lasted from July 1998 to October 1998. During the first crisis, all the previously mentioned three digital accelerometric stations were placed at the village of São Sebastião, namely, Escola, Junta and Misericórdia. For the
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second crisis, the only active station was the one at Escola site. The acceleration time series obtained for these crises were analyzed and the following procedures for site-effect assessment were used:

- Characterization of the different seismic records
- 1-D modeling;
- H/V spectral ratio;
- Reference-site spectral ratio (RSR);

These subjects will be thoroughly analyzed in the present chapter, always in the perspective of comparing the techniques at the three sites, in order to explain the different amplification inside the crater.

6.4.2. Characterization of the different seismic records

During the mentioned crises, several tenths of seismic motion records were obtained at S. Sebastião. The accelerometric devices placed at S. Sebastião were, for the Junta and Misericórdia sites, 12-bit 3-component stations. These stations were as triggered both for a STA/LTA ratio range from 15dB of 20dB or for an acceleration greater than 2mg. These stations have an internal memory equal to 256K. For the Escola site, during the 1997 crisis only, a more recent 16-bit digital-acquisition station was placed, with an internal memory equal to 1024K (Oliveira et al., 1997). The events concerning the second crisis were only recorded, inside the crater, at Escola site. There is also a record at Praia da Vitória that was subjected to analysis. The orientation (azimuth) of the stations’ horizontal components, for the 1997 events, is shown at Table 6.1:

<table>
<thead>
<tr>
<th>Station</th>
<th>Longitudinal</th>
<th>Transversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escola</td>
<td>N175E</td>
<td>N85E</td>
</tr>
<tr>
<td>Junta</td>
<td>N117E</td>
<td>N27E</td>
</tr>
<tr>
<td>Misericórdia</td>
<td>N273E</td>
<td>N183E</td>
</tr>
</tbody>
</table>

Between 1997/10/03 and 1997/10/09, the stations at Junta and Misericórdia were removed, and the Escola station was replaced by the one formerly placed at Junta. The new station was oriented directly to N-S orientation (i.e., the transversal component has as azimuth N0E).

The first issue concerning the different records was signal-to-noise ratio. As most seismic events, during the mentioned crises, could be classified as weak-motion events (with magnitude lesser than 4.0), a great number of records were useless as far as the considered site-effect techniques are concerned. Therefore, these records haven’t been subjected to further analysis.
Table 6.2 contains the date, magnitude and recording stations for all the records taken into account for site-effect assessment. All the events were numbered, in order to ease interpretation of the obtained results.

<table>
<thead>
<tr>
<th>Date (UTC)</th>
<th>Magnitude</th>
<th>Locations</th>
<th>Event Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997/07/15 21:00</td>
<td>4.5</td>
<td>Escola, Junta, Misericórdia</td>
<td>1</td>
</tr>
<tr>
<td>1997/08/27 18:50</td>
<td>4.9</td>
<td>Escola, Junta, Misericórdia</td>
<td>2</td>
</tr>
<tr>
<td>1997/07/07 20:07</td>
<td>4.5</td>
<td>Escola, Junta, Misericórdia</td>
<td>3</td>
</tr>
<tr>
<td>1997/10/03 08:58</td>
<td>3.7</td>
<td>Escola, Junta, Misericórdia</td>
<td>4</td>
</tr>
<tr>
<td>1998/07/09 05:19</td>
<td>5.7</td>
<td>Escola</td>
<td>5</td>
</tr>
<tr>
<td>1997/07/07 19:39</td>
<td>4.1</td>
<td>Escola, Junta</td>
<td>6</td>
</tr>
<tr>
<td>1998/07/09 05:28</td>
<td>--</td>
<td>Escola</td>
<td>7</td>
</tr>
<tr>
<td>1998/07/09 05:59</td>
<td>--</td>
<td>Escola</td>
<td>8</td>
</tr>
<tr>
<td>1998/07/09 06:10</td>
<td>4.3</td>
<td>Escola</td>
<td>9</td>
</tr>
<tr>
<td>1997/10/09 18:14</td>
<td>3.3</td>
<td>Escola</td>
<td>10</td>
</tr>
<tr>
<td>1997/10/11 12:30</td>
<td>3.6</td>
<td>Escola</td>
<td>11</td>
</tr>
<tr>
<td>1997/06/27 04:40</td>
<td>5.6</td>
<td>Escola</td>
<td>12</td>
</tr>
<tr>
<td>1997/06/27 10:43</td>
<td>3.7</td>
<td>Escola</td>
<td>13</td>
</tr>
<tr>
<td>1997/06/27 17:24</td>
<td>3.5</td>
<td>Escola</td>
<td>14</td>
</tr>
<tr>
<td>1997/06/27 22:29</td>
<td>4.5</td>
<td>Escola</td>
<td>15</td>
</tr>
<tr>
<td>1997/06/27 22:32</td>
<td>4.4</td>
<td>Escola</td>
<td>16</td>
</tr>
<tr>
<td>1997/06/27 23:53</td>
<td>4.9</td>
<td>Escola</td>
<td>17</td>
</tr>
<tr>
<td>1997/06/29 03:54</td>
<td>4.4</td>
<td>Escola</td>
<td>18</td>
</tr>
<tr>
<td>1997/06/29 04:15</td>
<td>3.7</td>
<td>Escola</td>
<td>19</td>
</tr>
<tr>
<td>1997/06/29 04:23</td>
<td>4.5</td>
<td>Escola</td>
<td>20</td>
</tr>
<tr>
<td>1997/06/29 04:59</td>
<td>3.8</td>
<td>Escola</td>
<td>21</td>
</tr>
<tr>
<td>1997/06/29 22:28</td>
<td>4.7</td>
<td>Escola</td>
<td>22</td>
</tr>
<tr>
<td>1997/06/30 04:57</td>
<td>4.5</td>
<td>Escola</td>
<td>23</td>
</tr>
<tr>
<td>1997/07/03 05:18</td>
<td>--</td>
<td>Junta, Misericórdia</td>
<td>24</td>
</tr>
<tr>
<td>1997/07/04 18:12</td>
<td>3.7</td>
<td>Junta</td>
<td>25</td>
</tr>
<tr>
<td>1997/07/12 02:08</td>
<td>4.4</td>
<td>Junta</td>
<td>26</td>
</tr>
<tr>
<td>1997/10/11 12:29</td>
<td>--</td>
<td>Misericórdia</td>
<td>27</td>
</tr>
</tbody>
</table>

As one should note, in spite of existing three different stations at the crater, only six seismic events were recorded by more than one station.

An important issue concerning these records was long-period noise, as the events’ magnitude was almost always lower than 5.0. Hence, time series processing techniques were needed. The adopted procedure closely follows the standard procedure of USGS:

1. The pre-event time was used to determine a mean.

2. This mean was removed from the whole record.
3. The mean-corrected acceleration time series was numerically integrated (Trapezoidal rule), in order to obtain the velocity time series.

4. A second-order polynomial curve was fitted to the velocity time series. The polynomial was such that, for the onset of the earthquake, its value was null.

5. The derivative of the former polynomial curve was removed from the acceleration time series.

6. An acausal 4\textsuperscript{th}-order Butterworth high-pass filter, with cut-off frequency equal to 0.04Hz, was applied to the acceleration time series.

The use of an acausal filter was due to the reasons explained at 2.4. The adopted cut-off frequency was the lowest allowing that the velocity and displacement time series (determined via Newmark-beta method, $\alpha = 1/2$, $\beta = 1/6$) would not be sensitive to the cut-off frequency, although it was possible there to be some residual displacements. Note that this procedure is applied to the whole waveform, and window functions were not considered.

After the data processing, the first step concerned the use of FFT to compare the frequency content of the different records was the sampling rate, as the former assumes different values for signals digitally converted using different sampling rates. There were records with a sampling frequency equal to 100Hz (namely, at Junta and Misericórdia) and others having 200Hz as sampling frequency. In order not to loose information, the records with a sampling frequency equal to 100Hz were oversampled to 200Hz using linear interpolation.

For all the records, the following ground motion parameters were obtained:

- Significant duration
- Arias intensity (AI)
- Peak Ground Acceleration (PGA)
- Fourier amplitude spectrum
- Power spectrum
- Central frequency
- Shape factor
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The first parameter to be determined was the significant duration of the events, using the definition of Trifunac and Brady (1975). For this purpose, numerical integration was made using the trapezoidal rule. As the latter definition depends on the time series’ energy, one may obtain directly the Arias intensity. The Husid plot (AI vs. time) was a fundamental tool for the pretended comparison between different records for the same event. In order to determine the evolution of energy along the event’s duration, a function in MATLAB was developed.

In terms of time-domain characterization, the adopted parameter was the PGA. The value was determined for each acceleration component.

MATLAB was used in order to determine the FFT of all records, as this programming language has a direct command to determine it. Also in the frequency domain, a MATLAB function was developed aiming the calculation of the power spectrum, spectral moments, and, subsequently, the central frequency and shape factor. When determining the frequency-domain parameters, a 30s 5% co-sine tapering window was applied (except for event #5, where the window’s length was 40s). This was due to two facts. First, the significant duration normally didn’t exceed this value; second, having the same length and sampling rate, these time windows would allow direct comparison of the FFT of all records.

The characterization of the different seismic motion records was made regarding the following aspects:

- Characterization of records at Escola;
- Characterization of records at Junta;
- Characterization of records at Misericórdia;
- Comparison between characteristics of records for the same event at different stations

For event #5, comparison was made relatively to the Praia da Vitória station, as this station is placed outside the crater and, in certain conditions, may be used as a reference site (as will be discussed ahead). In order to compare the results, the longitudinal and transversal components of the different stations were rotated to N-S and E-W directions.
6.4.2.1. Characterization of records at Escola

According to the described procedure, in terms of significant duration, the obtained results are shown in Table 6.3, where one may also find the determined magnitude (Nunes et al., 2004). The significant duration was obtained for the horizontal components.

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S duration (s)</th>
<th>E-W duration (s)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22,480</td>
<td>19,400</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>28,995</td>
<td>32,065</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>29,585</td>
<td>21,440</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>18,765</td>
<td>21,885</td>
<td>3.7</td>
</tr>
<tr>
<td>5</td>
<td>37,565</td>
<td>25,155</td>
<td>5.7</td>
</tr>
<tr>
<td>6</td>
<td>25,725</td>
<td>19,850</td>
<td>4.1</td>
</tr>
<tr>
<td>7</td>
<td>23,375</td>
<td>23,715</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>39,330</td>
<td>19,690</td>
<td>--</td>
</tr>
<tr>
<td>9</td>
<td>42,415</td>
<td>42,415</td>
<td>4.3</td>
</tr>
<tr>
<td>10</td>
<td>12,895</td>
<td>10,105</td>
<td>3.3</td>
</tr>
<tr>
<td>11</td>
<td>17,630</td>
<td>16,965</td>
<td>3.6</td>
</tr>
<tr>
<td>12</td>
<td>19,240</td>
<td>21,985</td>
<td>5.6</td>
</tr>
<tr>
<td>13</td>
<td>11,425</td>
<td>9,525</td>
<td>3.7</td>
</tr>
<tr>
<td>14</td>
<td>15,655</td>
<td>12,745</td>
<td>3.5</td>
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<tr>
<td>15</td>
<td>11,880</td>
<td>11,765</td>
<td>4.5</td>
</tr>
<tr>
<td>16</td>
<td>24,490</td>
<td>23,780</td>
<td>4.4</td>
</tr>
<tr>
<td>17</td>
<td>12,895</td>
<td>12,030</td>
<td>4.9</td>
</tr>
<tr>
<td>18</td>
<td>24,310</td>
<td>22,545</td>
<td>4.4</td>
</tr>
<tr>
<td>19</td>
<td>9,350</td>
<td>11,035</td>
<td>3.7</td>
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<tr>
<td>20</td>
<td>11,920</td>
<td>10,150</td>
<td>4.5</td>
</tr>
<tr>
<td>21</td>
<td>18,840</td>
<td>18,545</td>
<td>3.8</td>
</tr>
<tr>
<td>22</td>
<td>26,385</td>
<td>23,295</td>
<td>4.7</td>
</tr>
<tr>
<td>23</td>
<td>23,135</td>
<td>22,385</td>
<td>4.5</td>
</tr>
</tbody>
</table>

For the events concerning the 1997 crisis, the significant duration tends to be greater when considering N-S duration, as shown in Figure 6.9.
In fact, for most of these events, the N-S component contained greater energy (and, subsequently, larger Arias Intensity). For the 1998 crisis, the latter observation regarding the horizontal components is also valid, with a great increase of energy during the significant duration. This observation is important, as the epicentral coordinates for the 1997 and 1998 crises are very different (1997 – epicenters at D. João de Castro Bank, near Terceira, towards SE; 1998 – epicenters near Faial island, ca. 100km towards W), which leads one to say that the N-S effect, as far as Escola site is concerned, is not affected by directivity. Figure 6.10 contains a Husid plot for a 1997 event; Figure 6.11 contains the same type of plot for a 1998 event.
Considering the PGA from the filtered acceleration time series, there is no trend regarding directivity, as the maximum PGA occurs sometimes for the N-S component, and others for the E-W component, as shown in Table 6.4:

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S PGA (m/s²)</th>
<th>E-W PGA (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0604</td>
<td>0.0650</td>
</tr>
<tr>
<td>2</td>
<td>0.0622</td>
<td>0.0560</td>
</tr>
<tr>
<td>3</td>
<td>0.0334</td>
<td>0.0401</td>
</tr>
<tr>
<td>4</td>
<td>0.0569</td>
<td>0.0775</td>
</tr>
<tr>
<td>5</td>
<td>0.2291</td>
<td>0.1537</td>
</tr>
<tr>
<td>6</td>
<td>0.0405</td>
<td>0.0406</td>
</tr>
<tr>
<td>7</td>
<td>0.0077</td>
<td>0.0053</td>
</tr>
<tr>
<td>8</td>
<td>0.0121</td>
<td>0.0053</td>
</tr>
<tr>
<td>9</td>
<td>0.0086</td>
<td>0.0069</td>
</tr>
<tr>
<td>10</td>
<td>0.0409</td>
<td>0.0461</td>
</tr>
<tr>
<td>11</td>
<td>0.0328</td>
<td>0.0237</td>
</tr>
<tr>
<td>12</td>
<td>0.4985</td>
<td>0.3856</td>
</tr>
<tr>
<td>13</td>
<td>0.0946</td>
<td>0.0925</td>
</tr>
<tr>
<td>14</td>
<td>0.2750</td>
<td>0.2273</td>
</tr>
<tr>
<td>15</td>
<td>0.0403</td>
<td>0.0446</td>
</tr>
<tr>
<td>16</td>
<td>0.0265</td>
<td>0.0227</td>
</tr>
<tr>
<td>17</td>
<td>0.0387</td>
<td>0.0426</td>
</tr>
<tr>
<td>18</td>
<td>0.0461</td>
<td>0.0611</td>
</tr>
<tr>
<td>19</td>
<td>0.1307</td>
<td>0.0936</td>
</tr>
<tr>
<td>20</td>
<td>0.1381</td>
<td>0.1885</td>
</tr>
<tr>
<td>21</td>
<td>0.0257</td>
<td>0.0257</td>
</tr>
<tr>
<td>22</td>
<td>0.0210</td>
<td>0.0233</td>
</tr>
<tr>
<td>23</td>
<td>0.0286</td>
<td>0.0457</td>
</tr>
</tbody>
</table>

Figure 6.12 allows a graphical comparison of the filtered PGA concerning the horizontal direction.
As far as the Fourier amplitude spectrum is concerned, there is clearly an interval, for which there is a major peak, between 2.0Hz and 2.5Hz. One may also find local peaks in terms of amplitude for the following frequency bandwidths:

- 1.0Hz and 1.5Hz
- 4.0Hz and 5.5Hz
- 7.0Hz and 8.0Hz

Amplification at 7.0Hz-8.0Hz bandwidth may correspond to modal amplification of the building where the Escola station was placed, as the building is a school with two floors. Figure 6.13 contains the amplitude spectra, concerning the horizontal components, for event #12, at the Escola site. One may clearly identify significant amplitude values ca. 1.1Hz and ca. 2.2Hz, corresponding to the first two of the mentioned amplification intervals. These peaks tend to be more important for events of higher magnitude, namely, events #2, and #5, as one may see in Figure 6.14 and Figure 6.15.
Figure 6.13 – Amplitude spectra of horizontal components for event #12, recorded at Escola

Figure 6.14 – Amplitude spectra of horizontal components for event #2, recorded at Escola

Figure 6.15 – Amplitude spectra of horizontal components for event #5, recorded at Escola
The power spectra identify the interval 2.0Hz-2.5Hz as the major bandwidth of amplification, especially for the N-S component, as shown clearly on Figure 6.16, Figure 6.17 and Figure 6.18. The other relevant intervals in terms of power spectral density are 1.0Hz-1.5Hz and 4.0-5.5Hz.

![Figure 6.16 – Power spectra of horizontal components for event #2, recorded at Escola](image1)

![Figure 6.17 – Power spectra of horizontal components for event #5, recorded at Escola](image2)
The other two parameters adopted to characterize the data were, as mentioned before, the central frequency and the shape factor. Both the central frequency and the shape factor tend to assume lower values for increasing magnitude, *i.e.*, for greater events, the relevant bandwidth diminishes, and the latter centers itself in the amplification intervals mentioned before, as shown in Table 6.5.

---

Figure 6.18 – Power spectra of horizontal components for event #12, recorded at Escola
### Table 6.5 – Central frequency and shape factor of horizontal components for events at Escola

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S Central Frequency (Hz)</th>
<th>E-W Central Frequency (Hz)</th>
<th>N-S Shape Factor</th>
<th>E-W Shape Factor</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.581</td>
<td>4.597</td>
<td>0.6148</td>
<td>0.6334</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>4.079</td>
<td>4.511</td>
<td>0.6859</td>
<td>0.7073</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>6.108</td>
<td>6.900</td>
<td>0.7715</td>
<td>0.7381</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>5.633</td>
<td>6.920</td>
<td>0.5850</td>
<td>0.6041</td>
<td>3.7</td>
</tr>
<tr>
<td>5</td>
<td>2.961</td>
<td>3.306</td>
<td>0.5754</td>
<td>0.5997</td>
<td>5.7</td>
</tr>
<tr>
<td>6</td>
<td>5.017</td>
<td>5.845</td>
<td>0.7355</td>
<td>0.7570</td>
<td>4.1</td>
</tr>
<tr>
<td>7</td>
<td>7.416</td>
<td>8.387</td>
<td>0.7281</td>
<td>0.7801</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>7.409</td>
<td>8.984</td>
<td>0.6488</td>
<td>0.7404</td>
<td>--</td>
</tr>
<tr>
<td>9</td>
<td>5.513</td>
<td>6.221</td>
<td>0.8131</td>
<td>0.7959</td>
<td>4.3</td>
</tr>
<tr>
<td>10</td>
<td>5.077</td>
<td>4.715</td>
<td>0.5302</td>
<td>0.4622</td>
<td>3.3</td>
</tr>
<tr>
<td>11</td>
<td>4.990</td>
<td>5.429</td>
<td>0.5938</td>
<td>0.6387</td>
<td>3.6</td>
</tr>
<tr>
<td>12</td>
<td>3.376</td>
<td>3.586</td>
<td>0.4910</td>
<td>0.5568</td>
<td>5.6</td>
</tr>
<tr>
<td>13</td>
<td>4.470</td>
<td>5.435</td>
<td>0.5346</td>
<td>0.4943</td>
<td>3.7</td>
</tr>
<tr>
<td>14</td>
<td>3.657</td>
<td>3.785</td>
<td>0.5163</td>
<td>0.5456</td>
<td>3.5</td>
</tr>
<tr>
<td>15</td>
<td>4.816</td>
<td>5.626</td>
<td>0.6985</td>
<td>0.7174</td>
<td>4.5</td>
</tr>
<tr>
<td>16</td>
<td>6.987</td>
<td>7.815</td>
<td>0.7473</td>
<td>0.7070</td>
<td>4.4</td>
</tr>
<tr>
<td>17</td>
<td>6.203</td>
<td>7.176</td>
<td>0.7357</td>
<td>0.7048</td>
<td>4.9</td>
</tr>
<tr>
<td>18</td>
<td>5.840</td>
<td>6.592</td>
<td>0.6508</td>
<td>0.6939</td>
<td>4.4</td>
</tr>
<tr>
<td>19</td>
<td>5.640</td>
<td>6.940</td>
<td>0.5178</td>
<td>0.5527</td>
<td>3.7</td>
</tr>
<tr>
<td>20</td>
<td>5.009</td>
<td>5.997</td>
<td>0.4841</td>
<td>0.4401</td>
<td>4.5</td>
</tr>
<tr>
<td>21</td>
<td>7.571</td>
<td>9.480</td>
<td>0.7360</td>
<td>0.7222</td>
<td>3.8</td>
</tr>
<tr>
<td>22</td>
<td>7.599</td>
<td>7.259</td>
<td>0.8201</td>
<td>0.8081</td>
<td>4.7</td>
</tr>
<tr>
<td>23</td>
<td>5.175</td>
<td>4.890</td>
<td>0.7360</td>
<td>0.6921</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Comparing the horizontal components for the same event, the N-S component tends to have lower central frequency, which falls in line with greater amount of energy (and power) in the N-S direction for the frequencies of engineering interest (Figure 6.19). For the shape factor, there is no clear trend (Figure 6.20).
6.4.2.2. Characterization of records at Junta

Following the same methodology used for the Escola station records, the significant duration was determined for the horizontal components at Junta. The results are displayed in Table 6.6.

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S duration (s)</th>
<th>E-W duration (s)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24,955</td>
<td>26,945</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>33,200</td>
<td>35,705</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>27,730</td>
<td>27,835</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>25,465</td>
<td>22,600</td>
<td>3.7</td>
</tr>
<tr>
<td>6</td>
<td>29,155</td>
<td>25,600</td>
<td>4.1</td>
</tr>
<tr>
<td>24</td>
<td>20,825</td>
<td>17,590</td>
<td>--</td>
</tr>
<tr>
<td>25</td>
<td>27,085</td>
<td>22,560</td>
<td>3.7</td>
</tr>
<tr>
<td>26</td>
<td>22,790</td>
<td>23,190</td>
<td>4.4</td>
</tr>
</tbody>
</table>

There is a trend for the N-S duration to be slightly longer than the E-W duration, but this tendency is not as much evident as in the case of the records at Escola, as one may see in Figure 6.21.
In fact, comparing the Arias intensity for the Junta records, both N-S and E-W have much more similar energy evolution than the ones recorded at Escola (Figure 6.22).

PGA concerning the filtered acceleration time series, considering the Junta records, supports the fact that N-S component doesn’t have prevalence as significant as in the Escola station. Table 6.7 contains the filtered PGA value. In fact, the filtered PGA tends to assume a greater value for E-W component, as one may see in Figure 6.23.
Table 6.7 – Filtered PGA for the horizontal components at Junta

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S PGA (m/s²)</th>
<th>E-W PGA (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0265</td>
<td>0.0296</td>
</tr>
<tr>
<td>2</td>
<td>0.0283</td>
<td>0.0288</td>
</tr>
<tr>
<td>3</td>
<td>0.0138</td>
<td>0.0148</td>
</tr>
<tr>
<td>4</td>
<td>0.0235</td>
<td>0.0267</td>
</tr>
<tr>
<td>6</td>
<td>0.0179</td>
<td>0.0155</td>
</tr>
<tr>
<td>24</td>
<td>0.0139</td>
<td>0.0206</td>
</tr>
<tr>
<td>25</td>
<td>0.0131</td>
<td>0.0228</td>
</tr>
<tr>
<td>26</td>
<td>0.0123</td>
<td>0.0166</td>
</tr>
</tbody>
</table>

Figure 6.23 – Comparison between filtered PGA of horizontal components at Junta

The analysis in the frequency domain corroborates the similitude between the horizontal components. The bandwidths for which there is significant amplitude are:

- 1.0Hz and 1.5Hz
- 2.5Hz and 3.0Hz
- 3.5Hz and 4.5Hz

In spite of similar shapes between both horizontal components, for some of the records at Junta, the relative extreme at 2.5Hz-3.0Hz tends to have somewhat greater value for the N-S component than for the E-W one, e.g. event #3 (Figure 6.24). Most times, the maximum of the amplitude spectrum occurs for the first of these bandwidths. As an example, Figure 6.25 contains the amplitude spectra of the horizontal components for event #2 at Junta:
One may also identify these peaks at the power spectra. The 1.0Hz-1.5Hz bandwidth is significant for both horizontal components, the same happening for the 2.5Hz-3.0Hz bandwidth. The first of the mentioned bandwidths tends to contain the absolute maximum of the amplitude spectrum; the second gains importance for weaker events. In terms of N-S and E-W components, the amplitude value of the N-S component tends to be greater for the 1.0Hz-1.5Hz and 2.5Hz-3.0Hz bandwidth, e.g. event #3 (Figure 6.26). The E-W component has, for some events, higher content in higher frequencies, e.g. event #25 (Figure 6.27).
From the previous data, one should expect slightly lower central frequency for the N-S component, for the Junta records, and similar shape factors for both components (concentration of energy at low frequencies for N-S component, similar process nature for both components). Table 6.8 contains the central frequency and the shape factor of the events recorded at Junta.
Table 6.8 – Central frequency and shape factor of horizontal components for events at Junta

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S Central Frequency (Hz)</th>
<th>E-W Central Frequency (Hz)</th>
<th>N-S Shape Factor</th>
<th>E-W Shape Factor</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.069</td>
<td>5.464</td>
<td>0.6584</td>
<td>0.6469</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>3.846</td>
<td>4.773</td>
<td>0.7458</td>
<td>0.7341</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>7.555</td>
<td>7.994</td>
<td>0.7027</td>
<td>0.6540</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>6.431</td>
<td>6.886</td>
<td>0.6305</td>
<td>0.6194</td>
<td>3.7</td>
</tr>
<tr>
<td>6</td>
<td>6.231</td>
<td>6.236</td>
<td>0.7527</td>
<td>0.7242</td>
<td>4.1</td>
</tr>
<tr>
<td>24</td>
<td>7.184</td>
<td>7.295</td>
<td>0.6564</td>
<td>0.6406</td>
<td>--</td>
</tr>
<tr>
<td>25</td>
<td>9.862</td>
<td>9.811</td>
<td>0.7062</td>
<td>0.6895</td>
<td>3.7</td>
</tr>
<tr>
<td>26</td>
<td>6.490</td>
<td>7.121</td>
<td>0.7387</td>
<td>0.6971</td>
<td>4.4</td>
</tr>
</tbody>
</table>

In fact, the central frequency of N-S component is generally lower than the one for the E-W component (Figure 6.28). Considering the shape factor, one has, for all records, broader band for the N-S component (Figure 6.29).

Figure 6.28 – Comparison between the central frequency of horizontal components at Junta
6.4.2.3. Characterization of records at Misericórdia

The records at Misericórdia have quite similar behavior to the ones recorded at Junta, as far as significant duration is concerned. For the events at Misericórdia, Table 6.9 displays the significant duration, where one may see that the significant duration of N-S component is slightly longer than the one of the E-W component.

Table 6.9 – Trifunac & Brady duration of horizontal components at Misericórdia

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S duration (s)</th>
<th>E-W duration (s)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20,730</td>
<td>19,950</td>
<td>4,5</td>
</tr>
<tr>
<td>2</td>
<td>35,480</td>
<td>34,175</td>
<td>4,9</td>
</tr>
<tr>
<td>3</td>
<td>27,900</td>
<td>30,125</td>
<td>4,5</td>
</tr>
<tr>
<td>4</td>
<td>22,835</td>
<td>21,490</td>
<td>3,7</td>
</tr>
<tr>
<td>24</td>
<td>12,855</td>
<td>12,850</td>
<td>--</td>
</tr>
<tr>
<td>27</td>
<td>23,905</td>
<td>20,215</td>
<td>--</td>
</tr>
</tbody>
</table>

In terms of Arias intensity (Figure 6.30), one may see that the N-S, for almost all the records, carries more energy along the significant duration:
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Figure 6.30 – Comparison between Arias intensity for the horizontal components at Misericórdia

Considering the filtered PGA, the records at Misericórdia have similar values for the horizontal components, with greater values, usually, for the E-W PGA (much as happened for the records at Junta). Table 6.10 contains the filtered PGA:

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S PGA (m/s²)</th>
<th>E-W PGA (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0310</td>
<td>0.0316</td>
</tr>
<tr>
<td>2</td>
<td>0.0287</td>
<td>0.0342</td>
</tr>
<tr>
<td>3</td>
<td>0.0153</td>
<td>0.0107</td>
</tr>
<tr>
<td>4</td>
<td>0.0261</td>
<td>0.0261</td>
</tr>
<tr>
<td>24</td>
<td>0.0160</td>
<td>0.0213</td>
</tr>
<tr>
<td>27</td>
<td>0.0110</td>
<td>0.0121</td>
</tr>
</tbody>
</table>

In the frequency domain, the Fourier amplitude spectrum has significant values for the following bandwidths:

- 1.0Hz-1.5Hz
- 2.5Hz-3.0Hz
- 4.0Hz-5.0Hz

Normally, the amplitude values for these bandwidths are similar, being the absolute maximum for 2.5Hz-3.0Hz bandwidth. For increasing magnitude, the 1.0-1.5Hz bandwidth assumes greater importance, and the inverse happens for the 4.0-5.0Hz bandwidth. Figure 6.31 contains the amplitude spectra of the record of higher magnitude obtained at Misericórdia. At least for event #1, there seems to exist a local source, as there are two narrow peaks (Figure 6.32).
The referred bandwidths are clearly identifiable in the power spectra, shown in Figure 6.33, concerning event #2.
Figure 6.33 – Power spectra of horizontal components for event #2, recorded at Misericórdia

The central frequency for Misericórdia has no fixed trend, as there aren’t enough events to detect it. There are more records with higher central frequency for the N-S component and for the E-W component. The shape factors concerning the horizontal components are similar, but the E-W shape factor assumes a greater value for five of the six events recorded at Misericórdia. Table 6.11 contains the data for the Misericórdia records. Figure 6.34 and Figure 6.35 assess the results graphically.

Table 6.11 – Central frequency and shape factor of horizontal components for events at Misericórdia

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S Central Frequency (Hz)</th>
<th>E-W Central Frequency (Hz)</th>
<th>N-S Shape Factor</th>
<th>E-W Shape Factor</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.203</td>
<td>5.998</td>
<td>0.5623</td>
<td>0.5756</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>5.112</td>
<td>4.470</td>
<td>0.6716</td>
<td>0.6810</td>
<td>4.9</td>
</tr>
<tr>
<td>3</td>
<td>7.245</td>
<td>7.693</td>
<td>0.6453</td>
<td>0.6707</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>6.608</td>
<td>6.003</td>
<td>0.5318</td>
<td>0.5102</td>
<td>3.7</td>
</tr>
<tr>
<td>24</td>
<td>7.120</td>
<td>6.560</td>
<td>0.5110</td>
<td>0.5260</td>
<td>--</td>
</tr>
<tr>
<td>27</td>
<td>6.894</td>
<td>6.000</td>
<td>0.6283</td>
<td>0.6462</td>
<td>--</td>
</tr>
</tbody>
</table>
6.4.2.4. Comparison between characteristics of records for the same event at different stations

As mentioned before, there were six events for which, at least, there were two station acquiring time series. A direct comparison was made using the parameters chosen to characterize the seismic motion.

In terms of significant duration, the records at Escola present almost for every event a shorter duration than the records at Misericórdia and at Junta, which may be explained by the fact that signal-to-noise ratio is significantly higher at Escola than at the other station (significant duration is energy-related). The duration at Junta and Misericórdia tends to be more similar, but with lower values for Misericórdia. The difference of behavior in terms of significant duration is especially relevant considering the E-W component. Table 6.12 and Table 6.13 contain the data referent to significant duration.
Despite the shorter duration of the records at Escola, the latter have a much greater increase of energy along the former, as one may clearly see on the Husid plot shown in Figure 6.36. The Junta and Misericórdia records have much more similar Arias Intensity considering the same event.

**Figure 6.36 – Husid plot for N-S component concerning event #3 at different stations**

The differences regarding PGA inside the crater were first mentioned by Oliveira *et al.* (1997), and later appointed by Montesinos *et al.* (2003). The filtered PGA in Escola is almost twice the ones obtained at Junta and Misericórdia. This observation is true for both horizontal components, as well as for the vertical component. For the N-S component, the PGA at Misericórdia has a higher value than
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the one at Junta; for the E-W there is no fixed tendency (Table 6.14 and Table 6.15). Figure 6.37 allows the comparison between the filtered PGA at the different stations.

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S PGA Escola (m/s²)</th>
<th>N-S PGA Junta (m/s²)</th>
<th>N-S PGA Misericórdia (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0604</td>
<td>0.0265</td>
<td>0.0310</td>
</tr>
<tr>
<td>2</td>
<td>0.0622</td>
<td>0.0283</td>
<td>0.0287</td>
</tr>
<tr>
<td>3</td>
<td>0.0334</td>
<td>0.0138</td>
<td>0.0153</td>
</tr>
<tr>
<td>4</td>
<td>0.0569</td>
<td>0.0235</td>
<td>0.0261</td>
</tr>
<tr>
<td>6</td>
<td>0.0405</td>
<td>0.0179</td>
<td>--</td>
</tr>
<tr>
<td>24</td>
<td>--</td>
<td>0.0139</td>
<td>0.0160</td>
</tr>
</tbody>
</table>

Table 6.15 – Filtered PGA for E-W component, concerning the same event at different stations

<table>
<thead>
<tr>
<th>Event Number</th>
<th>E-W PGA Escola (m/s²)</th>
<th>E-W PGA Junta (m/s²)</th>
<th>E-W PGA Misericórdia (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0650</td>
<td>0.0296</td>
<td>0.0316</td>
</tr>
<tr>
<td>2</td>
<td>0.0560</td>
<td>0.0288</td>
<td>0.0342</td>
</tr>
<tr>
<td>3</td>
<td>0.0401</td>
<td>0.0148</td>
<td>0.0107</td>
</tr>
<tr>
<td>4</td>
<td>0.0775</td>
<td>0.0267</td>
<td>0.0261</td>
</tr>
<tr>
<td>6</td>
<td>0.0406</td>
<td>0.0155</td>
<td>--</td>
</tr>
<tr>
<td>24</td>
<td>--</td>
<td>0.0206</td>
<td>0.0213</td>
</tr>
</tbody>
</table>

Figure 6.37 – Comparison between filtered PGA for the N-S component, concerning the same event at different stations

The amplitude spectra concerning the three stations for the same event present a clear distinction between the Escola site and the other two. The 2.0Hz-2.5Hz amplitude peak that exists for the Escola station has much lesser significance for both the Misericórdia and the Junta records. This statement is
reinforced by the data concerning the power spectral density. Figure 6.38 and Figure 6.39 contain the amplitude and the power spectra for event #3.

Figure 6.38 – Comparison between the amplitude spectra for the N-S component, concerning the event #3 at different stations

Figure 6.39 – Comparison between the power spectra for the N-S component, concerning the event #3 at different stations

The central frequency at Escola tends to be lower than the ones obtained for the other stations. However, in terms of shape factor, the Misericórdia records tend to have the lowest value. The Junta records have the highest value of both central frequency and shape factor. The first remark may be explained by the much greater power spectral density of Escola records for frequencies lower to 10Hz in relation to all the spectrum’s frequencies. The second one implies that the Misericórdia records are not as broadband as those relative to Escola and Junta. Table 6.16 and Table 6.17 contain the data
relative to the central frequency and to the shape factor; Figure 6.40 and Figure 6.41 show graphical comparison of central frequency for the different stations.

Table 6.16 – Central frequency and shape factor for N-S component, concerning the same event at different stations

<table>
<thead>
<tr>
<th>Event Number</th>
<th>N-S Central Frequency Escola (Hz)</th>
<th>N-S Central Frequency Junta (Hz)</th>
<th>N-S Central Frequency Miser. (Hz)</th>
<th>N-S Shape Factor Escola</th>
<th>N-S Shape Factor Junta</th>
<th>N-S Shape Factor Miser.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.581</td>
<td>5.069</td>
<td>5.203</td>
<td>0.6148</td>
<td>0.6584</td>
<td>0.5623</td>
</tr>
<tr>
<td>2</td>
<td>4.079</td>
<td>3.846</td>
<td>5.112</td>
<td>0.6859</td>
<td>0.7458</td>
<td>0.6716</td>
</tr>
<tr>
<td>3</td>
<td>6.108</td>
<td>7.555</td>
<td>7.245</td>
<td>0.7715</td>
<td>0.7027</td>
<td>0.6453</td>
</tr>
<tr>
<td>4</td>
<td>5.633</td>
<td>6.431</td>
<td>6.608</td>
<td>0.5850</td>
<td>0.6305</td>
<td>0.5318</td>
</tr>
<tr>
<td>6</td>
<td>5.017</td>
<td>6.231</td>
<td>--</td>
<td>0.7355</td>
<td>0.7527</td>
<td>0.5110</td>
</tr>
<tr>
<td>24</td>
<td>--</td>
<td>7.184</td>
<td>7.120</td>
<td>--</td>
<td>0.6564</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 6.17 – Central frequency and shape factor for E-W component, concerning the same event at different stations

<table>
<thead>
<tr>
<th>Event Number</th>
<th>E-W Central Frequency Escola (Hz)</th>
<th>E-W Central Frequency Junta (Hz)</th>
<th>E-W Central Frequency Miser. (Hz)</th>
<th>E-W Shape Factor Escola</th>
<th>E-W Shape Factor Junta</th>
<th>E-W Shape Factor Miser.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.597</td>
<td>5.464</td>
<td>5.998</td>
<td>0.6334</td>
<td>0.6469</td>
<td>0.5756</td>
</tr>
<tr>
<td>2</td>
<td>4.511</td>
<td>4.773</td>
<td>4.470</td>
<td>0.7073</td>
<td>0.7341</td>
<td>0.6810</td>
</tr>
<tr>
<td>3</td>
<td>6.900</td>
<td>7.994</td>
<td>7.693</td>
<td>0.7381</td>
<td>0.6540</td>
<td>0.6707</td>
</tr>
<tr>
<td>4</td>
<td>6.920</td>
<td>6.886</td>
<td>6.003</td>
<td>0.6041</td>
<td>0.6194</td>
<td>0.5102</td>
</tr>
<tr>
<td>6</td>
<td>5.845</td>
<td>6.236</td>
<td>--</td>
<td>0.7570</td>
<td>0.7242</td>
<td>--</td>
</tr>
<tr>
<td>24</td>
<td>--</td>
<td>7.295</td>
<td>6.560</td>
<td>--</td>
<td>0.6406</td>
<td>0.5260</td>
</tr>
</tbody>
</table>

Comparison between central frequencies at different stations for the same event - N-S

Figure 6.40 – Comparison between central frequencies for the N-S component, concerning the same event at different stations
6.4.2.5. Remarks

From what has been exposed, the following remarks contain the most relevant aspect that one must retain, in respect to site-effect assessment:

- For the most part of the records, the N-S component tends to carry more energy and have greater amplitude and power spectral density than the E-W component. This is especially true for the records at Escola (Figure 6.10, Figure 6.11 and Figure 6.16).

- There is a much greater increase of the measured Arias intensity along the significant duration for the Escola site (Figure 6.36). This fact may explain why one has shorter significant duration at Escola than the records at other stations and for records of the same magnitude and epicentral distance, as one has greater signal-to-noise ratio at Escola.

- The filtered PGA obtained for records at Escola is, for almost all the signals recorded simultaneously by other stations, about twice the value obtained at those stations (Figure 6.37).

- The Escola records present an absolute maximum of amplitude and power spectral density at around 2.5Hz (for all events) that is almost non-existent for the records at Junta and Misericórdia (Figure 6.38 and Figure 6.39).
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- There are relative extremes in terms of amplitude and power spectral density that are found at almost all the considered records, such as the peaks for 1,0Hz-1,5Hz and for 4,0-5,0Hz.

- Escola records tend to have lower central frequency, as there is much greater amplification for low frequencies (Figure 6.40 and Figure 6.41).

From the previous remarks, one may conclude that there is a local site effect at Escola. These conclusions are in line with the damage map for the 1980 January 1st earthquake, as the concentration of ruined buildings near the Escola site is greater than those for Junta and Misericórdia, as already shown in Figure 6.4.

6.4.3. One-dimensional modeling

After the characterization of the records for the stations inside the crater, the next goal was to get the soil “columns”, i.e., the best \( V_S \) profiles that could explain the difference between the records at the different stations. The velocity profiles were based on the already mentioned work by Lopes (2005).

In order to obtain the best fit between the soil profile and the obtained records, a parametric study was made. The adopted soil profiles were based on two \( V_S \) profiles obtained by Lopes, #11 and #12, respectively. As one may see in Figure 6.8, these profiles were the closest to the Junta and the Misericórdia Stations, and the Escola Station, respectively. Profiles #11 and #12 are presented in Figure 6.42. As one may see, the expected thickness of low-\( V_S \) layers for the Escola site is much greater than the one for both junta and Misericórdia. For profile #11, at a depth of 11m, there is a sudden increase of \( V_S \), which could correspond to the lava tongue.

![Figure 6.42 – \( V_S \) Profiles #11 and #12 (adapted from Lopes, 2005)](image)

For each soil column, the following parameters were varied:

- Depth of the bedrock.
• Thickness of each layer.

• $V_s$ of each layer.

• Hysteretic damping of each layer.

• Stiffness reduction curves.

In the case of the soil column considered for the Junta station, the thickness of the “lava flow” was also a parameter to vary.

The depth of the bedrock was the main issue in the modeling, as there are clear limitations of the SWM method for depths greater than 25m. As mentioned in 6.3, the results of the deep borehole in Lloyd and Collis (1981) pointed the depth of the bedrock, at the edge of the crater, at about 34m. This fact led to increasing bedrock depths profiles.

In close association with the bedrock depth, the thickness and the $V_s$ of each layer were also varied. This resulted in several changes in the transfer functions. One must make an important note concerning the thickness and $V_s$ of layers and the variation of $V_s$ in depth. As there were no reliable results for depths over 20m, one assumed increasing $V_s$ in depth, for layers of approximately 10m.

Concerning the stiffness reduction curves and the hysteretic damping of the different layers, at a first step, one considered, for the low-$V_s$ layers, the equations given by Vucetic & Dobry (1991). These equations were considered due to the fact that, usually, through progressive weathering, slope deposits and fills convert themselves into fine soils. As there were no Atterberg limits determined for these layers, one considered three equations, for different Plasticity Indexes (Pl=15, Pl=30 and Pl=50). These curves are shown in Figure 6.43.
However, as the records obtained in more than one station did not produce strains of over $10^{-5}$, one could consider that there was no degradation of stiffness and that the hysteretic damping ratio could remain constant. For the hysteretic damping, under the conditions of no degradation, one considered 2% or 5%, for the low-$V_S$, layers, and 1% or 2%, for the bedrock.

For each and all the tested soil profiles, the respective transfer function was determined, using the commercial software SHAKE2000. Considering the fact that the records obtained at the Junta and the Misericórdia sites showed similar features, the lower number of relevant records at Misericórdia and also the proximity of both stations to the $V_S$ profile #11 in Figure 6.8, the modeled soil columns corresponded only to the Escola and the Junta sites.

### 6.4.3.1. One-dimensional modeling of the soil profile for the Escola station

Figure 6.44 shows the $V_S$ profiles used to model the Escola response. Table 6.18 contains all the relevant data in terms of modeling. A common feature concerning all the models is the modeling of seismic bedrock, which was considered a semi-infinite medium.
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Profiles tested at Escola

Figure 6.44 – \( V_s \) profiles used in 1-D modeling of Escola site

Table 6.18 – Relevant data in terms of 1-D modeling – Escola

<table>
<thead>
<tr>
<th>Profile Number</th>
<th>G/Gmax</th>
<th>( \gamma_{\text{slope deposits}} ) (kN/m(^3))</th>
<th>( \gamma_{\text{basalt}} ) (kN/m(^3))</th>
<th>( \xi_{\text{slope deposits}} ) (%)</th>
<th>( \xi_{\text{basalt}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP’s #1, #6, and from #10 to #16</td>
<td>1</td>
<td>18</td>
<td>27</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>EP’s #2 and #7</td>
<td>Vucetic &amp; Dobry PI=15</td>
<td>18</td>
<td>27</td>
<td>Vucetic &amp; Dobry PI=15</td>
<td>Schnabel for Rock (1973)</td>
</tr>
<tr>
<td>EP #4 and EP #9</td>
<td>Vucetic &amp; Dobry PI=50</td>
<td>18</td>
<td>27</td>
<td>Vucetic &amp; Dobry PI=50</td>
<td>Schnabel for Rock (1973)</td>
</tr>
<tr>
<td>EP #5</td>
<td>1</td>
<td>18</td>
<td>27</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

From these profiles, the ones modeled with the degradation equations of Vucetic & Dobry led to unrealistic transfer functions, with sharp peaks of amplification of extreme value, as the hysteretic damping coefficient was extremely low (ca. 2%). As an example, Figure 6.45 presents the transfer function for the Escola profile #7.
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Figure 6.45 – Transfer function for Escola profile #7

The deconvolution of the times series recorded at Escola, using the soil columns modeled considering the Vucetic and Dobry equations resulted in an almost complete cut of the amplitude for the amplification frequencies at the admitted bedrock.

These facts excluded the soil profiles mentioned in the last two paragraphs. Hence, the soil profiles that were effectively considered in the 1-D modeling were profiles #1, #6, #11, #12, #13, #14, #15 and #16. Figure 6.46 and Figure 6.47 contain the transfer functions calculated for these profiles:

Figure 6.46 – Transfer functions considered at Escola: Profiles #1, #6, #11 and #12
Comparison was made between the amplitude spectra for the records at the surface and the obtained transfer functions. Only the events for which there were records both at Escola and Junta were used for this analysis, i.e., the considered amplitude spectra concerned events #1, #2, #3, #4 and #6. For all of the events, the transfer functions that, in a general manner, fitted better with the amplitude peaks at the surface were the ones corresponding to profiles #14, #15 and #16, as these transfer functions clearly identify the two lowest relative extremes of the Fourier amplitude spectrum (and, for #15 and #16, the third mode). An important remark that must be done at this point is the fact that these transfer functions, if valid, would imply that the strong amplification for bandwidth 2.0Hz-2.5Hz is due to the second mode of the soil column, and not to the first one. Figure 6.48, Figure 6.49, Figure 6.50 and Figure 6.51 show the comparison between the respective transfer functions and the N-S and E-W amplitude spectra for events #1 and #2, as evidence of what was mentioned.

Figure 6.48 – Comparison between the amplitude spectra and the transfer functions, for event #1 and N-S component
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Comparison between the Fourier amplitude spectrum and transfer functions - Event #1 - E-W

Amplification ratio

Figure 6.49 – Comparison between the amplitude spectra and the transfer functions, for event #1 and E-W component

Comparison between the Fourier amplitude spectrum and transfer functions - Event #2 - N-S

Amplification ratio

Figure 6.50 – Comparison between the amplitude spectra and the transfer functions, for event #2 and N-S component
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Comparison between the Fourier amplitude spectrum and transfer functions - Event #2 - E-W

![Graph showing Fourier amplitude spectrum and transfer functions](image)

Figure 6.51 – Comparison between the amplitude spectra and the transfer functions, for event #2 and E-W component

6.4.3.2. One-dimensional modeling of the soil profile for the Junta station

As mentioned before, for the soil column used for the analysis of the Junta station, according to the interpretation of the geological setting and the results of SWM, the $V_S$ profile must contemplate the basaltic intrusion and correspond, generally, to higher velocities along its depth. The latter fact, alongside the variation of stiffness reduction and hysteretic damping, caused a significant increase on the amount of tested $V_S$ profiles. Figure 6.52 contains the considered profiles. The relevant data in terms of modeling are shown in Table 6.19. Once again, seismic bedrock was modeled as a semi-infinite medium.
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Figure 6.52 – $V_s$ profiles used in 1-D modeling of Junta site

Table 6.19 – Relevant data in terms of 1-D modeling – Junta

<table>
<thead>
<tr>
<th>Profile Number</th>
<th>G/Gmax</th>
<th>$\gamma_{\text{slope deposits}}$ (kN/m$^3$)</th>
<th>$\gamma_{\text{basalt}}$ (kN/m$^3$) (tongue and bedrock)</th>
<th>$\xi_{\text{slope deposits}}$ (%)</th>
<th>$\xi_{\text{basalt}}$ (%) (tongue and bedrock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JP’s #1, #2, #6 and from #10 to #26</td>
<td>1</td>
<td>18</td>
<td>27</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>JP’s #3 and #7</td>
<td>Vucetic &amp; Dobry PI=15</td>
<td>18</td>
<td>27</td>
<td>Vucetic &amp; Dobry PI=15</td>
<td>Schnabel for Rock (1973)</td>
</tr>
<tr>
<td>EP #5 and EP #9</td>
<td>Vucetic &amp; Dobry PI=50</td>
<td>18</td>
<td>27</td>
<td>Vucetic &amp; Dobry PI=50</td>
<td>Schnabel for Rock (1973)</td>
</tr>
</tbody>
</table>

Just as happened for the Escola analysis, the profiles in which one considered the equations given by Vucetic & Dobry have given unrealistic results. The most important profiles in terms of the pretended analysis were profiles #14, #15, #16, #17, #18, #19, #20, #21, #22, #23, #24, #25 and #26. The respective transfer functions are shown in Figure 6.53, Figure 6.54 and Figure 6.55.
Figure 6.53 – Transfer functions considered at Junta: Profiles #14, #15, #16 and #17

Figure 6.54 – Transfer functions considered at Junta: Profiles #18, #19, #20 and #21
Aiming to clarify the influence that a basaltic layer would have in the response at the surface, comparison between the Fourier amplitude spectra for each event (both horizontal components) and the transfer functions was made. As one may see in Figure 6.56, concerning the transfer functions for bedrock depth equal to 55m, the basaltic layer modifies the third and fourth modes.

The best fit between the Fourier amplitude spectra and the transfer function is achieved for the profiles corresponding to 50m and 55m of bedrock depth (Profiles #21, #22, #23, #24, #25 and #26). These results are in line with those obtained at Escola, which appointed to a 50m to 60m bedrock depth at that site. Considering the altimetric difference between Junta and Escola (the Junta station is 5m higher than the Escola station), one may conclude that the one-dimensional modeling bedrock may not be horizontal, with a maximum 10m altimetric difference between Escola and Junta (deeper
at Escola). Figure 6.57 and Figure 6.58 present the results for events #1 (N-S component) and #2 (E-W component), and for the profiles considering 2m and 4m of basaltic intrusion. One fact that, in a global manner, is valid for all events, is that, in spite of the existence of some differences in terms of frequency content between N-S and E-W components, the transfer functions present a similar adjustment to both components.

Figure 6.57 – Comparison between the amplitude spectra and the transfer functions, for event #1 and N-S component (intrusion thickness equal to 2m)

Figure 6.58 – Comparison between the amplitude spectra and the transfer functions, for event #1 and N-S component (intrusion thickness equal to 4m)
6.4.3.3. Deconvolution and comparison of spectral content at the bedrock

In order to confirm the best fit in terms of one-dimensional modeling of the soil profiles, one obtained the acceleration time series for the latter through deconvolution at both Escola and Junta. The commercial program SHAKE2000 was used, and the acceleration time series at the admitted bedrock were obtained for the profiles for which the comparison between the Fourier amplitude spectra and the transfer functions presented the best fit, i.e., for the Escola station, profiles #14, #15 and #16; for the Junta station, profiles #21, #22, #23, #24, #25 and #26.
Two approaches were used to assess the best fit. In one hand, if the soil columns are to correctly model the soil profile, the amplitude spectra for the acceleration at the admitted bedrock obtained through deconvolution at both Escola and Junta should be, at least, similar (theoretically, should be the same), leading to similar central frequencies and shape factors. On the other hand, the spectral ratio between the amplitude spectrum at the surface for one station and the amplitude spectrum at the bedrock for the other station should correspond to the transfer function associated to the former station. The latter approach was named *cross spectral ratio*.

For the first approach, the adopted procedure was to compare the amplitude spectra for the acceleration time series obtained at both Junta and Escola. If the deconvolution is to correctly model the amplification, the amplitude values should be the same for the deconvolved signals for the whole frequency domain. If one of the compared spectra is a “scaled version” of the other, the value adopted for the hysteretic damping or for the impedance ratio is not correct, at least for one of the soil profiles. The central frequencies and shape factors were determined in order to minimize the event effect of a erroneous damping, as this parameters are not as much influenced the amplitude value but by the shape of the spectrum.

Comparing the amplitude spectra, one quickly understands that, for the adopted profiles, either the damping or the impedance ratio is not perfectly modeled, as the amplitude resultant from the Escola deconvolution contains higher values than the one resultant from the Junta deconvolution. Visually, the best fit occurs for the Escola profile #16 and the Junta profiles #23 and #26. An important fact that must be referred to is that, in certain events, there is a major difference for frequencies between 3.0Hz and 4.0Hz, as the records at Escola contain high amplitude content and, sometimes, sharp peaks. The Junta profiles #23 and #26 are the best simulation for the 4.0Hz amplification. Another fact that one must retain is that the misfit is more significant for the N-S component, especially for the mentioned bandwidth where amplitude peaks. Figure 6.61, Figure 6.62, Figure 6.63 and Figure 6.64 contain the comparison between the spectra for the events #2 and #3, concerning the profiles with the best fit (N-S component only). In spite of the difficulty determining the amplification ratio, one may say that the first two vibration modes, assumed according to the transfer function, have been clearly identified. This statement is especially valid for the first vibration mode, which occurs for a frequency between 1.1Hz and 1.5Hz:
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Figure 6.61 – Comparison between the frequency content of time series at the assumed bedrock (EP’s #14, #15 and #16 and JP #23), for event #2, and N-S component

Figure 6.62 – Comparison between the frequency content of time series at the assumed bedrock (EP’s #14, #15 and #16 and JP #26), for event #2, and N-S component
Figure 6.63 – Comparison between the frequency content of time series at the assumed bedrock (EP’s #14, #15 and #16 and JP #23), for event #3, and N-S component

Figure 6.64 – Comparison between the frequency content of time series at the assumed bedrock (EP’s #14, #15 and #16 and JP #26), for event #3, and N-S component

For the second approach, as mentioned before, one compared the spectral ratio surface/bedrock with the transfer function. Figure 6.65 and Figure 6.66 present a scheme of what is the meaning of spectral ratio.
The cross spectral ratio for Junta presents a better fit in terms of the amplification ratio predicted by the assumed transfer functions, i.e., there is a better simulation of either the hysteretic damping ration and/or the impedance ratio. For some events there are local extremes in terms of amplitude (normally, sharp peaks) for frequencies around 2.5Hz that have no match with the respective transfer function. This feature appears in both N-S and E-W components, but more frequently in the former. Figure 6.67, Figure 6.68, Figure 6.69 and Figure 6.70 contain the data concerning events #2 and #3, for the N-S component and for the Profiles #23 and #26.
Comparison between the spectral ratio and the transfer functions - Junta - Event #2 - N-S

Figure 6.67 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of EP’s #14, #15 and #16 and the transfer function of JP #23, for event #2 and N-S component

Comparison between the spectral ratio and the transfer functions - Junta - Event #2 - N-S

Figure 6.68 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of EP’s #14, #15 and #16 and the transfer function of JP #26, for event #2 and N-S component
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Figure 6.69 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of EP’s #14, #15 and #16 and the transfer function of JP #23, for event #3 and N-S component

Figure 6.70 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of EP’s #14, #15 and #16 and the transfer function of JP #26, for event #3 and N-S component

The amplification for the Junta deconvolution is much greater than the ones predicted by the assumed transfer functions. This may be the result of an underestimation of the amplitude resulting from the real acceleration time series at the bedrock, due to an underestimation of the hysteretic damping or the impedance ratio. Hence, the soil profiles adopted to model Escola could correspond to better simulations. A feature that was already mentioned when visual comparison was made is that there is a 3.0Hz-4.0Hz interval where there is an amplification that was best modeled considering the transfer function of Escola Profile #16. The results for events #2 and #3, for the N-S component, considering
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Escola profile #16 and Junta profiles #23 and #26 are shown in Figure 6.71, Figure 6.72, Figure 6.73 and Figure 6.74.

Figure 6.71 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of JP’s #21, #22 and #23 and the transfer function of EP #16, for event #2 and N-S component

Figure 6.72 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of JP’s #24, #25 and #26 and the transfer function of EP #16, for event #2 and N-S component
Figure 6.73 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of JP’s #21, #22 and #23 and the transfer function of EP #16, for event #3 and N-S component

Figure 6.74 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of JP’s #24, #25 and #26 and the transfer function of EP #16, for event #3 and N-S component

It is possible to say that, in spite of the erroneous estimation of either the hysteretic damping or the impedance ratio, the first and second vibration modes are well identified for both stations. For the first amplification peak, for the Escola profiles, profile #14 fits the spectral ratio the best (Figure 6.75). The second and third amplification peaks presented the best fit for Escola Profiles #15 and #16, and for Junta profile #26, which is in agreement with the comparison previously made in 6.4.3.2. The latter profile is the best to model the high amplification for frequencies around 4.0Hz, albeit the results of the deconvolution for this frequency are not fully satisfactory.
Comparison between spectral ratio and transfer function -
Escola - Event #3 - N-S

Figure 6.75 – Comparison between the cross spectral ratio considering the signal at the bedrock resulting from deconvolution of JP’s #24, #25 and #26 and the transfer function of EP #14, for event #3 and N-S component

This feature of the Junta profiles that have a 4m intrusion, i.e., profiles #23 and #26, leads to higher central frequency and higher shape factor. Concerning the central frequency, the pair EP#14/JP#23 present the best fit; in terms of shape factor, for most cases, JP#25 is the junta profile with values closer to the ones verified at Escola.

Table 6.20 – Central frequency and shape factor of the deconvolution signals, for event #2

<table>
<thead>
<tr>
<th>Profile</th>
<th>N-S Central Frequency (Hz)</th>
<th>E-W Central Frequency (Hz)</th>
<th>N-S Shape Factor</th>
<th>E-W Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP #14</td>
<td>6.3259</td>
<td>7.7335</td>
<td>0.6978</td>
<td>0.6906</td>
</tr>
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<td>EP #15</td>
<td>7.0932</td>
<td>8.6213</td>
<td>0.6975</td>
<td>0.6705</td>
</tr>
<tr>
<td>EP #16</td>
<td>7.5578</td>
<td>9.2289</td>
<td>0.6892</td>
<td>0.6508</td>
</tr>
<tr>
<td>JP #21</td>
<td>4.8251</td>
<td>6.2499</td>
<td>0.7387</td>
<td>0.7295</td>
</tr>
<tr>
<td>JP #22</td>
<td>5.3747</td>
<td>5.6670</td>
<td>0.6198</td>
<td>0.7243</td>
</tr>
<tr>
<td>JP #23</td>
<td>5.7455</td>
<td>7.6071</td>
<td>0.7317</td>
<td>0.7070</td>
</tr>
<tr>
<td>JP #24</td>
<td>4.6188</td>
<td>6.1311</td>
<td>0.7294</td>
<td>0.7218</td>
</tr>
<tr>
<td>JP #25</td>
<td>4.4283</td>
<td>5.6148</td>
<td>0.7198</td>
<td>0.7086</td>
</tr>
<tr>
<td>JP #26</td>
<td>5.4923</td>
<td>7.3679</td>
<td>0.7323</td>
<td>0.7103</td>
</tr>
</tbody>
</table>
Table 6.21 – Central frequency and shape factor of the deconvolution signals, for event #3

<table>
<thead>
<tr>
<th>Profile</th>
<th>N-S Central Frequency (Hz)</th>
<th>E-W Central Frequency (Hz)</th>
<th>N-S Shape Factor</th>
<th>E-W Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>EP #14</td>
<td>8.7351</td>
<td>9.6973</td>
<td>0.6632</td>
<td>0.6115</td>
</tr>
<tr>
<td>EP #15</td>
<td>9.7439</td>
<td>10.5887</td>
<td>0.6417</td>
<td>0.6062</td>
</tr>
<tr>
<td>EP #16</td>
<td>10.2018</td>
<td>10.7933</td>
<td>0.6376</td>
<td>0.5834</td>
</tr>
<tr>
<td>JP #21</td>
<td>7.0391</td>
<td>8.4911</td>
<td>0.6483</td>
<td>0.5687</td>
</tr>
<tr>
<td>JP #22</td>
<td>6.7254</td>
<td>7.9994</td>
<td>0.6462</td>
<td>0.5765</td>
</tr>
<tr>
<td>JP #23</td>
<td>8.2541</td>
<td>9.4150</td>
<td>0.6074</td>
<td>0.5127</td>
</tr>
<tr>
<td>JP #24</td>
<td>6.8850</td>
<td>8.3297</td>
<td>0.6563</td>
<td>0.6049</td>
</tr>
<tr>
<td>JP #25</td>
<td>6.6719</td>
<td>7.7531</td>
<td>0.6381</td>
<td>0.5985</td>
</tr>
<tr>
<td>JP #26</td>
<td>7.9654</td>
<td>9.5553</td>
<td>0.6536</td>
<td>0.5706</td>
</tr>
</tbody>
</table>

6.4.3.4. Interpretation and remarks

Concerning one-dimensional modeling of the soil columns underneath the Escola and Junta station, having as reference the tested profiles, the graphical comparison between the amplitude spectra at the surface and the comparison approaches referent to the results of deconvolution; the most important remarks are the following:

- There was a misfit in the amplification associated to transfer function assumed and the one obtained through the techniques used to analyze deconvolution results (Figure 6.71, Figure 6.72, Figure 6.73 and Figure 6.74).

- The first vibration mode has been identified by all techniques, for both stations. Major amplification occurs for a frequency of approximately 1.2Hz (Figures from 6.57 to 6.70, Figure 6.75).

- There is amplification that 1-D modeling doesn’t explain in a satisfactory way, such as the sharp peaks that appear around 2.0Hz and the 3.0-4.0 amplification (Figures from 6.57 to 6.70).

- The Junta profiles considering a 4m intrusion lead to the best results considering all the techniques. The results of the deconvolution using the Junta profile #23 present a good fit with Escola profile #14 in terms of central frequency (Table 6.20 and
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• Table 6.21).

• Comparing the spectral ratio through deconvolution and the transfer functions, for the Escola station, profile #14 presents the best fit for the first mode (Figure 6.75); the next two modes, both profiles #15 and #16 present a better fit (Figure 6.71 to Figure 6.74). For the Junta station, profile #26 presents the best fit, especially for the second and third modes (Figure 6.68).

• Considering all the techniques, at Junta, profile #23 is the best in explaining the amplification. At Escola, profile #14 leads to better results, globally speaking. However, Escola profiles #15 and #16 give more satisfactory results for higher frequencies.

• All the techniques point to a bedrock depth between 50m and 60m for Escola. For the Junta station results point out a bedrock depth between 50m and 55m. This may imply that the bedrock is not horizontal.

6.4.4. Reference-site spectral ratio (RSR)

For event #5, there was the possibility to obtain records at Escola and at a station at Praia da Vitória town (outcrop), outside the crater. For this event the records fall on the definition of similar epicentral distance (Oliveira et al., 1998). As the epicenter of this event, near the Faial island, was farther than 100km of both sites, there were no near-source effects.

The isoseismic map of Terceira for the January 1st 1980 earthquake, which also had epicenter near Faial, justifies the use of the Praia da Vitória station as a reference site. For the Praia da Vitória station, the Modified Mercalli Intensity values did not exceed V (in fact IV/V), as for São Sebastião it reached VIII at some places. Hence, even there was to be any amplification at Praia da Vitória, it was much less significant than on S. Sebastião, allowing one to detect the vibration modes.

The characterization of seismic motion at Escola and Praia da Vitória stations makes evidence of the site effect at Escola. The PGA has significantly higher values at the latter station comparing with the record at Praia da Vitória for both directions. In terms of Fourier amplitude spectra, Figure 6.76 and Figure 6.77 allow the respective comparison for the horizontal component, being clear that at Escola the amplitude values are much higher than the ones obtained at Praia da Vitória.
Figure 6.76 – Comparison between amplitude spectra at Escola and at the reference site, for event #5, for N-S component

Figure 6.77 – Comparison between amplitude spectra at Escola and at the reference site, for event #5, for E-W component

In terms of spectral ratio, the first vibration mode is clearly identified for the previously mentioned frequency of 1.2Hz. For the N-S component, the obtained amplification for the second mode, for previously mentioned bandwidth of 2.0Hz-2.5Hz, is clearly higher than the one predicted by the transfer functions. There is also a great amplification for frequencies around 5.0Hz, present in both components. Figure 6.78 and Figure 6.79 compare the spectral ratio with the transfer functions assumed for the Escola station.
Figure 6.78 – Comparison between reference-site spectral and the assumed transfer functions for Escola - N-S component

Figure 6.79 – Comparison between reference-site spectral and the assumed transfer functions for Escola - E-W component

This comparison confirms the remarks made in 6.4.3.3: Escola profile #14 gives the best results in terms of modeling the first vibration mode, and Escola profile #16 is, by far, the best in modeling the higher vibration modes. One important fact that must be referred to is that, for this specific event, the amplification for the fundamental frequency is well predicted by the adopted transfer functions. Despite the lack of more numerous comparisons, this technique is in general agreement with the previously obtained results. It confirms the presence of a special feature for the second vibration mode for the N-S component that is not completely explained using one-dimensional modeling.
6.4.5. **Horizontal-to-vertical spectral ratio**

For all the characterized events inside the crater of São Sebastião, one calculated the spectral ratio between the horizontal amplitudes and the vertical amplitude. This technique was made for 20 second S-wave time windows. There are several reasons to adopt these windows. The main underlying reason to choose them is the greater energy that the signal carries for this part of the waveform (for 20-second windows, there will be surface waves and noise in the waveform, but they should give the same result, at least for the fundamental frequency). Two other advantages, inherent to the use of body waves when applying this technique, are the identification of other vibration modes (even the amplification ratio is not correct) and the estimated amplification ratio should be closer to real amplification than the one given by noise windows. The use of P-waves windows presents as disadvantage the fact that the duration of the latter is not long enough to provide reliable results. The use of noise windows in accelerometric data is not advisable, as the accelerometers aren’t sensitive enough. Also associated to the use of accelerometers in data acquisition is the fact that there are no reliable results for frequencies under 1.0Hz.

The SESAME project recommends the following criteria to check the reliability of the H/V curve:

- The adopted window length should be greater than ten times the determined fundamental frequency.

- The number of cycles, considering all the time windows, should exceed 200.

These criteria were completely fulfilled for the Escola station, as there were 23 records. However, for the Junta and the Misericórdia, that was not true, as the number of windows was not enough to fulfill the criteria. For the Junta station, if the fundamental frequency were to be 1.2Hz, as given by the adopted profile, the number of cycles contained would be 192, a value pretty close to the criterion. Hence, despite not fulfilling the number of cycles criterion, the H/V curve was calculated for the Junta station.

The H/V curve was obtained through the following procedure:

1. A 5% co-sine tapering window was applied to the S-wave part of the acceleration time series (for the three components).

2. The FFT algorithm was applied to the three components of each selected time window.

3. The spectral ratios were calculated for the two horizontal components for each window.
4. Considering all the windows, the geometric mean for the two horizontal components was determined: 
\[ H/V = 10^{\frac{\sum \log(H/V)_i}{N}} \]

5. After checking if there were differences in the two components, these were combined using a quadratic mean: 
\[ (H/V)_{comb} = \sqrt{\frac{(H/V)_{NS}^2 + (H/V)_{EW}^2}{2}} \]

Dispersion was accounted considering the standard deviation of the logarithmic curve, for each component. After applying these procedures, the results at Escola are shown in Figure 6.80, Figure 6.81 and Figure 6.82.
The obtained H/V curve identifies clearly two amplification peaks, one for 1.0Hz and other for 2.3Hz. Both components exhibit these peaks. The peaks are sharper and with greater value for the N-S component.

There are several considerations that must be made upon the obtained curve. First, in spite of the fact that the first identified peak is over the reliable-result threshold, the predicted values by the assumed transfer functions at 6.4.3 for the fundamental frequency are close to 1.0Hz. Another fact is that the second peak happens clearly for the second vibration mode predicted by the Escola profile #16, but with a value that is similar to the one obtained for the first peak. One of the possible causes to this fact may be a complex subsurface topography, or the presence of two large impedance contrasts (SESAME project, 2004). Still concerning the second peak, the use of the geometric mean reduces the influence of high amplification values. In fact, if one used an arithmetic averaging procedure, the amplification would the slightly higher.

Graphical comparison between the obtained H/V curve and the adopted transfer functions is given by Figure 6.83.
The first two modes were clearly identified by the assumed transfer functions. The predicted amplification is higher than the obtained via H/V technique, especially for the first amplification peak.

The results for the Junta station present a single peak for a frequency of approximately 1.0Hz, just as happens for the Escola site. However, the second peak doesn’t appear, as seen in Figure 6.84.
These results must be seen with caution. On one hand, the reliability criterion was not fulfilled; on other one, the peak for 1.0 Hz, at least for the N-S component, is not clear. Nevertheless, one may say that the fundamental frequency determined for Junta is similar to the one obtained at Escola but higher modes cannot be identified. This feature falls in line with the results of the reference-site spectral ratio and one-dimensional modeling. Once again, this effect is more visible for the N-S component.

6.5. Borehole campaign and resonant column test

Considering all the results in 6.3 and in 6.4, and having the financial support to do more tests in São Sebastião, a borehole campaign was performed during the summer of 2007. The location of the boreholes was decided based the following factors:
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- The boreholes should corroborate the presence of a lava tongue at the western part of the crater, as proposed by Santos et al. (2007);

- The boreholes should be made as near as possible to the accelerometric stations’ locations.

Figure 6.87 contains the location of all the tests performed at São Sebastião, including the mentioned boreholes.

Figure 6.87 – Location of the main tests performed in São Sebastião volcanic crater (Basis map: IGEOE, 2002) (in: Lopes et al., 2008)

The 2007 borehole campaign consisted on four boreholes with SPT tests. SPT samples and undisturbed samples were collected for laboratory tests which are currently ongoing (physical and mineralogical identification, triaxial and dynamic tests).

The first important result obtained from the boreholes was the confirmation of the presence of a lava tongue, as two of the boreholes (the ones placed near to the south part of the crater) crossed a shallow layer of basaltic lavas, consistent with the premise stated before regarding the presence of a lava tongue. The lava tongue is set in the NW-SE direction.

Another important fact concerning the geological and geotechnical setting is that none of the boreholes reached what could be called as “bedrock”. The deepest borehole, with Escola nearby, reached 40m, and no basaltic layer was found.
In Figure 6.87, a cross-section line is drawn in the SW-NE direction. The adopted direction aimed to have a geological interpretation right underneath the accelerometric stations at Escola and at Junta. Figure 6.88 contains the geological interpretation, based on all the characterization tests made at São Sebastião.

The lava tongue appears with a “V” shape, which can be an indication of a valley in the paleomorphology. This hypothesis is also confirmed by the presence of an organic soil deposit detected by the boreholes.

The ongoing laboratory tests indicate that the “Fluvial and slope deposits” mentioned in 6.3 are mainly composed by sandy soils with a fine content lesser than 20%. The layers showing a higher percentage of fines are essentially non-plastic materials.

The SPT test results show that until more or less 15 m of depth the $N_{SPT}$ is lower than 20 but bellow that depth the $N_{SPT}$ starts increasing. Comparing the obtained values of $N_{SPT}$ with the velocity profiles assumed in 6.4.3.1 and 6.4.3.2, one may say that the latter were realistic.

In important remark that must be made concerning the campaign is that the boreholes didn’t allow a final conclusion on the genetic process that led to the formation of São Sebastião.

As previously mentioned, undisturbed samples were collected for laboratory tests. One of these samples was tested at the Resonant Column apparatus available at Instituto Superior Técnico, in order to determine the stiffness reduction/damping increase curves. The undisturbed sample was collected at
a depth of 13.5m, for the borehole near Escola. The sampled soil could be classified as the already mentioned “Fluvial and slope deposits”, of sandy nature.

The results of this test are shown in Figure 6.89, Figure 6.90 and Figure 6.91.

![Figure 6.89 – Results from the Resonant Column test made on the undisturbed sample collected at São Sebastião, for different consolidation stresses: Shear strain vs. Shear modulus](image)

![Figure 6.90 – Results from the Resonant Column test made on the undisturbed sample collected at São Sebastião, for different consolidation stresses: Shear strain vs. Normalized shear modulus](image)
The results obtained by the Resonant Column test were used in the two-dimensional modeling of São Sebastião volcanic crater, along with the geological interpretation as shown in Figure 6.88.

### 6.6. Two-dimensional modeling of São Sebastião volcanic crater.

#### 6.6.1. Modeling issues

Two-dimensional models were built in the finite-element program PLAXIS, in order to study site effects at São Sebastião. The main issues concerning the models will be presented from now on.

Using the constitutive relation developed at Chapter 5 as the adopted soil model, the first step in terms of modeling the crater was to build a finite-element mesh that would translate the different geological/geotechnical characteristics of the crater that could be distinguished as different materials in PLAXIS.

As mentioned in 6.5, a lava tongue was detected underneath Junta. The lava tongue was modeled using the porous linear elastic model, specifying a material with a high value of shear wave velocity (800m/s) and density (2500kg/m$^3$).

Considering the one-dimensional models and the results of SPT tests, as one has discussed in 6.4.3.1, 6.4.3.2 and 6.5, in order for one to model the increase of shear wave velocity along the depth, one adopted four layers (clusters in PLAXIS) with different initial shear wave velocity, respectively,
125m/s, 200m/s, 300m/s and 350m/s. These layers obeyed to the constitutive relation as defined in Chapter 5. Table 6.22 contains the colors associated to each cluster:

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vs-125</td>
<td></td>
</tr>
<tr>
<td>Vs-200</td>
<td></td>
</tr>
<tr>
<td>Vs-300</td>
<td></td>
</tr>
<tr>
<td>Vs-350</td>
<td></td>
</tr>
<tr>
<td>bedrock</td>
<td></td>
</tr>
<tr>
<td>Lava tongue</td>
<td></td>
</tr>
</tbody>
</table>

Another important issue concerning the models’ definition was the shape of the crater, as far as bedrock was concerned. As discussed in 6.2 and in 6.5, none of the tests performed at São Sebastião allowed the lateral boundaries of the crater to be defined. Therefore, two geometric definitions were implemented in PLAXIS, one with steep vertical discontinuities, and another one with bowl-shaped bedrock. In either case, bedrock was modeled as a linear elastic material, with a shear wave velocity of 1000m/s. Concerning finite-element mesh, one adopted a smaller element size factor for shallower clusters and for lateral discontinuities. The adopted models are shown in Figure 6.92 and in Figure 6.93, where one may identify the lava tongue as the cluster with a “V” shape that is crossed by the water table. For both models, seismic bedrock depth was equal to 50m. This depth was chosen as it was associated to the best one-dimensional profiles, either at Escola (profile #14) and Junta (profile #23). An important issue that must be mentioned at this point is the computational effort required during the calculations of these models. The adopted meshes were such that results would be valid for frequencies under 10Hz.

![Figure 6.92 – Model adopted in PLAXIS, with steep lateral discontinuities (scale elevation factor 2x)](image-url)
A 4m-deep water table was defined for both models, according the average water-table depth detected by the boreholes. It may be seen in Figure 6.92 and in Figure 6.93 as a thick blue line.

A prescribed displacement boundary condition was applied to the base of the model. Lateral absorbent boundary conditions were adopted in the models.

The next step consisted in adopting Ramberg-Osgood parameters for each PLAXIS material that would fit the stiffness reduction law obtained using the Resonant Column test. For this purpose, one adopted the same $\alpha$ and $r$ to all the clusters that obeyed the Ramberg-Osgood model. Two curves were fitted to the normalized stiffness reduction law:

- $\alpha=1$ and $r=2$;
- $\alpha=0.5$ and $r=1.7$.

The curve fitting is shown at Figure 6.94 and at Figure 6.95, concerning the stiffness reduction and the hysteretic damping increase.
Figure 6.94 – Curve fitting concerning the Ramberg-Osgood parameters used as input values in PLAXIS.
Fitting to stiffness reduction curve

Figure 6.95 – Curve fitting concerning the Ramberg-Osgood parameters used as input values in PLAXIS.
Fitting to damping ratio

As one may see from Figure 6.94 and Figure 6.95, there is a misfit in terms of hysteretic damping ratio for normalized shear strain values over $10^{-1}$. One considered that the stiffness reduction measurement was more reliable, so, preference was given to curve-fitting the stiffness reduction.

The adoption of two separate fitting curves meant that four calculation processes were needed, two for each geometric configuration. The calculations were labeled as:
• Steep vertical discontinuities, \( \alpha = 1 \) and \( r = 2 \) – Setting #1a
• Steep vertical discontinuities, \( \alpha = 0.5 \) and \( r = 1.7 \) – Setting #1b
• Bowl-shaped bedrock, \( \alpha = 1 \) and \( r = 2 \) – Setting #2a
• Bowl-shaped bedrock, \( \alpha = 0.5 \) and \( r = 1.7 \) – Setting #2b

After the curve fitting procedure, all the input data concerning the models had to be defined. Table 6.23 and Table 6.24 contain the input data for all the clusters for the calculation procedure.

### Table 6.23 – Input data for the calculation process in PLAXIS: cluster according to Ramberg-Osgood model

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( \gamma_{sat} ) (kN/m(^3))</th>
<th>G (kPa)</th>
<th>K (kPa)</th>
<th>( \gamma_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vs-125</td>
<td>19.0</td>
<td>2,969x10(^4)</td>
<td>4,947x10(^4)</td>
<td>3\times10(^{-4})</td>
</tr>
<tr>
<td>Vs-200</td>
<td>20.0</td>
<td>8,000x10(^4)</td>
<td>1,733x10(^5)</td>
<td>3\times10(^{-4})</td>
</tr>
<tr>
<td>Vs-300</td>
<td>21.0</td>
<td>1,890x10(^5)</td>
<td>3,150x10(^5)</td>
<td>3\times10(^{-4})</td>
</tr>
<tr>
<td>Vs-350</td>
<td>21.0</td>
<td>2,573x10(^5)</td>
<td>4,288x10(^5)</td>
<td>3\times10(^{-4})</td>
</tr>
</tbody>
</table>

### Table 6.24 – Input data for the calculation process in PLAXIS: cluster according to linear elastic model

<table>
<thead>
<tr>
<th>Cluster</th>
<th>( \gamma_{sat} ) (kN/m(^3))</th>
<th>( V_s ) (m/s)</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lava tongue</td>
<td>25,0</td>
<td>600</td>
<td>0.20</td>
</tr>
<tr>
<td>Bedrock</td>
<td>27.0</td>
<td>1000</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Initial stress-state generation was made using the gravity loading procedure, with alias materials of similar deformation characteristics. This was done because there are several lateral discontinuities within the model, and a non-horizontal surface. After the generation of the stress-state, displacements were reset to zero, and the dynamic calculation was made.

An essential issue concerning dynamic models in PLAXIS must be made at this point. As mentioned in 5.5.5, the prescribed-displacement boundary condition in PLAXIS imposes vertical displacement equal to 0 at the base. This means that no radiation damping occurs through the base, and the effects of absorbent boundaries near the base are minimized, which means that PLAXIS greatly underestimates the overall damping. In order to minimize this effect, one adopted a formulation done by Roesset and Whitmann (1969), mentioned by Mineiro (1988). This formulation results from the comparison between one-dimensional transfer functions’ amplitude for two situations: for rigid bedrock and material damping (what will be considered in the model), and elastic bedrock without material damping (what one wants to model). The transfer functions’ amplitude is given, respectively for each situation, by Equation 6.1 and Equation 6.2:

\[
\frac{1}{\sqrt{\left(\cos(k \cdot H\right)^2 + \left(\sinh(\xi \cdot k \cdot H\right)^2}}
\]  

(6.1)
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\[
\frac{1}{\sqrt{\left(\cos(k \cdot H)\right)^2 + \left(\alpha \cdot \sin(k \cdot H)\right)^2}} \tag{6.2}
\]

where $\alpha$ stands for the impedance ratio, $H$ stands for the layer’s thickness and $k$ is the wavenumber as defined in 2.3. In order for both amplitudes to be exactly the same, one must have:

\[
\left(\sinh(\xi \cdot k \cdot H)\right)^2 = \left(\alpha \cdot \sin(\xi \cdot k \cdot H)\right)^2 \tag{6.3}
\]

For small values of hysteretic damping (as it usually happens), and considering that, for small values of a given value $y$, the hyperbolic sine of $y$ is approximately equal to $y$, one may write:

\[
\left(\xi \cdot k \cdot H\right)^2 = \left(\alpha \cdot \sin(k \cdot H)\right)^2 \Rightarrow \xi \cdot k \cdot H = \left|\alpha \cdot \sin(k \cdot H)\right| \Rightarrow \xi = \frac{\left|\alpha \cdot \sin(k \cdot H)\right|}{k \cdot H} \tag{6.4}
\]

The equivalent radiation damping is given by a $\text{sinc}$ function, with argument equal to $k \cdot H$, amplitude equal to the impedance ratio $\alpha$.

PLAXIS only allows one to implement Rayleigh damping as data input. This means that a curve-fitting procedure had to be performed in order for one to account the equivalent radiation damping. The first step concerning the fitting was to determine the fundamental frequency of each model in PLAXIS. This was done considering the same clusters as seen in Figure 6.92 and Figure 6.93, but defined as linear elastic materials. These modes were subjected to pulse acceleration of low amplitude (0.01 m/s$^2$) at the base. This allowed one to evaluate the vibration modes of each model. These calculations were done considering only the numerical damping provided by the default PLAXIS Newmark-beta method, with $\alpha$ equal to 0.6 and $\beta$ equal to 0.3025.
Both models had a fundamental frequency around 1.5Hz, similar to the fundamental frequency obtained for Junta using one-dimensional modeling, but slightly higher than the amplification obtained at Escola using the same technique. It was also a bit higher than the experimental site-effect assessment techniques. Setting #2 led to a much more scattered transfer function, and significant amplification for higher frequencies.

Considering as fundamental frequency, \( f_0 \), for both models a value equal to 1.5Hz, one may define an “average” shear wave velocity as given by Equation 6.5:

\[
V_{s,\text{average}} = 4 \cdot H \cdot f_0 = 4 \cdot 50m \cdot 1.5Hz = 300m / s
\]  

(6.5)

With this average shear wave velocity, one may determine an average impedance ratio, considering the bedrock impedance as defined by the cluster’s properties:

\[
\alpha = \frac{2000kg / m^3 \cdot 300m / s}{2700kg / m^3 \cdot 1000m / s} = 0.222
\]  

(6.6)

Having an average impedance ratio, an average shear wave velocity and a thickness equal to 50m, one may determine an analytical solution for the equivalent radiation damping. Recalling Equation 6.4, one has:
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Rayleigh damping is frequency-dependent, and consists on a linear combination of a hyperbolic function (depending on the Rayleigh coefficient, \( a_0 \), and applied to the system’s mass) with a straight line (depending on the Rayleigh coefficient, \( a_1 \), and applied to the system’s stiffness). The Rayleigh damping is given by Equation 6.8:

\[
\xi = \frac{a_0}{2 \cdot \omega} + \frac{a_1 \cdot \omega}{2}
\]  

(6.8)

Knowing the damping at two determined values, the Rayleigh coefficients may be found. At 5.5.5, the Rayleigh coefficients were fitted to the first and third vibration modes, considering equal value of damping. For the two-dimensional modeling at São Sebastião, one opted to fit the coefficient to the damping values obtained by Equation 6.7 for two frequencies, namely, 1.0Hz and 5.0Hz. The obtained values were \( a_0 = 2.309 \) and \( a_1 = 1.237 \times 10^{-16} \). Figure 6.97 shows the analytical solution for the equivalent radiation damping and the adopted Rayleigh damping.

![Figure 6.97 – Curve fitting between Rayleigh damping and equivalent radiation damping](image)

One opted for these frequency fitting values for two main reasons. First, adopting vibration-mode frequencies would lead to a curve fitting at the crests of the analytical solution, which meant an overestimation of radiation damping for all the bandwidth. Second, having as lower frequency fitting
a value equal to 1,0Hz meant that over-damping would occur for low frequencies, where significant amplification wasn’t expected.

The Rayleigh damping coefficients were applied to all clusters in PLAXIS, i.e., one considered that radiation damping affected the model as a whole.

After the definition of the Rayleigh coefficients, the last input data to define was the acceleration time series at the base. In order to compare results with experimental site-effect assessment techniques, the acceleration time series obtained at Praia da Vitória for event #5 was used as bedrock motion, in E-W direction. Note that the same acceleration time series was used to determine reference-site spectral ratio. A 30-second window, with a sampling rate equal to 200Hz was applied to the base of all the models. Figure 6.98 contains the mentioned acceleration time series.

![Acceleration at the bedrock](image)

**Figure 6.98 – Acceleration time series applied to the base of PLAXIS models used to simulate the crater**

**6.6.2. Results from two-dimension modeling**

After the calculations were made, the obtained results were analyzed having as background the following remarks:

- The acceleration time series resulting from the model at Escola should lead to greater Peak Ground Acceleration than the time series concerning Junta;

- If the remarks made at 6.4.3.4, 6.4.4 and 6.4.5 were to be correct, the transfer function at Escola should translate into greater amplification ratio for a bandwidth between 2,0Hz and 2,5Hz, as well as for higher frequencies;
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- The Fourier spectrum resulting from the model at Escola should be similar to the one obtained using the accelerometric station;

- The transfer function at Escola obtained using the model should resemble the reference-site spectral ratio, as well as the combined H/V curve.

These remarks translated into the comparison between the times series obtained using the finite-element model at Escola and at Junta; into the comparison between the Fourier spectra obtained using the model at Escola and the one obtained using the registered time series for the event at the accelerometric station; and into the comparison the transfer function at Escola using the model and the reference-site and horizontal-to vertical spectral ratios.

Comparing the acceleration time series at Escola and at Junta, one obtains higher Peak ground Acceleration at Escola, and globally higher absolute values of horizontal acceleration for Escola. These results are as one would expect, from all the data registered by the accelerometric stations. This feature was found for all the calculated models, and the difference in terms of absolute value between Escola and Junta was similar. These facts lead one to say that the lava tongue, as well as some differences in terms of shear wave velocity profile between the soil underneath Escola and Junta (especially for shallower formations), are the main reasons that justify the differences between the acceleration time series registered at these stations.

Figure 6.99, Figure 6.100, Figure 6.101 and Figure 6.102 allow graphical comparison for all the calculated settings.

![Graphical Comparison](image.png)

**Figure 6.99 – Comparison between acceleration time series obtained at Escola and at Junta using the two-dimensional model, for setting #1a**
Figure 6.100 – Comparison between acceleration time series obtained at Escola and at Junta using the two-dimensional model, for setting #1b

Figure 6.101 – Comparison between acceleration time series obtained at Escola and at Junta using the two-dimensional model, for setting #2a

Figure 6.102 – Comparison between acceleration time series obtained at Escola and at Junta using the two-dimensional model, for setting #2b

A feature that may be seen analyzing Figure 6.99, Figure 6.100, Figure 6.101 and Figure 6.102 is the fact that there wasn’t significant difference in terms of amplification between the adopted Ramberg-Osgood parameters. This was valid for both settings. Thus, all the results presented from this point concern only the comparison between setting #1a and setting #2a.
Another feature that may be clearly seen comparing Figure 6.100 and Figure 6.101 is that setting #2 leads to much higher amplification than the setting #1. This was expected, as setting #2a is globally stiffer and it is much more prone to wave trapping, due to the bowl-shaped bedrock. Comparing the acceleration time series at Escola for setting #1a and setting #2a, one may say that setting #1a leads to a time series that looks more like the one registered for event #5 at Escola (Figure 6.103 and Figure 6.104). In either case, PLAXIS model led to higher amplification of ground motion.

Figure 6.103 – Comparison between acceleration time series obtained at Escola using the two-dimensional model and the time series registered by the accelerometric station, for setting #1a

Figure 6.104 – Comparison between acceleration time series obtained at Escola using the two-dimensional model and the time series registered by the accelerometric station, for setting #2a

In terms of frequency content, the acceleration time series at Escola is richer for higher frequencies than the record at Junta. There is also a much more significant second amplification peak at Escola for a frequency around 3.0Hz almost non-existent at Junta. Once again, this statement was true for both settings, as one may see in Figure 6.105 and Figure 6.106, concerning the transfer function obtained using PLAXIS. Setting #2a led to amplification for a wider band, and with higher amplification values.
Comparing the Fourier spectrum obtained at Escola using PLAXIS with the real one; once again, setting #1a led to better results. Both models led to an overestimation of the maximum Fourier amplitude (Figure 6.107 and Figure 6.108).
The transfer function obtained at Escola for setting #1a overestimates the fundamental frequency given by the experimental site-effect assessment techniques, especially when comparing with horizontal-to-vertical spectral ratio, as one obtained a fundamental frequency using PLAXIS around 1.5Hz. The second amplification frequency for both settings presents a good fit to the reference-site spectral ratio, but overestimates the second peak when comparing to horizontal-to-vertical spectral ratio. One may say that, globally, the transfer function present similarities with the curve obtained using horizontal-to-vertical spectral ratio, and presents good fitting with the reference-site spectral ratio (Figure 6.109). The overestimation of the fundamental frequency may indicate that bedrock may be deeper than the one assumed for the PLAXIS model. When considering setting #2a, amplification associated to the first vibration mode, as previously mentioned, occurred for a wider band. Setting #2a leads to the best fitting to the reference-site spectral ratio for frequencies over 4.0Hz (Figure 6.110).

An important remark concerns the obtained amplification ratios. The amplification ratios obtained using the two-dimensional model are much lower than the ones obtained using one-dimensional
modeling, and very similar to the horizontal-to-vertical spectral ratio curve. One may say that the overall estimation of damping was better for the two-dimensional model.

![Transfer functions, Setting 1a](image1)

**Figure 6.109** – Comparison between the transfer function for Escola and experimental site-effect assessment techniques, for setting #1a

![Transfer functions, Setting 2a](image2)

**Figure 6.110** – Comparison between the transfer function for Escola and experimental site-effect assessment techniques, for setting #2a

### 6.6.3. Remarks

From what has been exposed, two-dimensional modeling allowed one to do the following remarks:

- Two-dimensional modeling has shown that the differences between the ground-motion records at Escola and at Junta are due to the presence of a lava tongue underneath Junta, independently of what subsurface topography was considered (Figure 6.99, Figure 6.100, Figure 6.101 and Figure 6.102);

- Comparing time-domain characteristics, setting #1a gave the best estimation of the acceleration time series at Escola (Figure 6.103 and Figure 6.104);
• Escola presents higher amplification ratio for almost all the frequencies below 8.0Hz. This was true for both settings, and especially for frequencies below 3.0Hz (Figure 6.105 and Figure 6.106);

• Both models overestimated maximum amplification, with setting #1 giving the best estimation in terms Fourier amplitude spectrum (Figure 6.107 and Figure 6.108);

• The adopted models led to an overestimation the fundamental frequency, when comparing with the experimental site-effect assessment techniques (Figure 6.109 and Figure 6.110). Setting #1a gave the best estimation for frequencies lower than 4.0Hz. Setting 2a presented a good fitting to reference-site spectral ratio for frequencies over 4.0Hz (Figure 6.110);

• Setting #1a was the better in modeling site effects, even though the fundamental frequency was overestimated. This overestimation may be due to deeper bedrock or to a lower Vs profile.

6.7. Concluding Remarks

Considering all the work done over São Sebastião, one may say that evidence was made on the existence of site effects at São Sebastião, when comparing to other sites with the same epicentral distance. This statement is based on the results from event #5 at São Sebastião (Escola station) and at Praia da Vitória in terms of acceleration time series characterization, as shown in 6.4.2.4.

Another result obtained from direct characterization of acceleration time series concerns the distinct behavior of Escola station when compared to other stations at São Sebastião. This difference in terms of amplification was also corroborated by one- and two-dimensional modeling and by horizontal-to-vertical spectral ratio. The difference between stations seems to result mainly from the existence of a lava tongue underneath Junta station. This lava tongue, first considered by Santos et al. (2007), and detected by the boreholes made at São Sebastião during the summer of 2007, led to the best results in terms of one-dimensional modeling at Junta (profile #23). Considering the two-dimensional model with the lava tongue, and using as input acceleration time series the one obtained at Praia da Vitória for event #5, the acceleration time series at Junta and at Escola present features as the ones predicted by all the other adopted techniques, namely:

• Horizontal PGA at Escola is about twice the one recorded at Junta;
A second amplification peak appears at Escola, but not at Junta.

It seems that the lava tongue de-amplifies strong motion, diminishing ground motion at Junta.

Evaluation of the fundamental frequency for all stations at São Sebastião using acceleration time series characterization parameters gave an estimation for the fundamental frequency ranging from 1.0Hz to 1.5Hz. The value of the fundamental frequency at Escola and at Junta using experimental site-effect assessment techniques was approximately 1.0Hz. Modeling techniques led to higher values than the experimental site-effect assessment techniques, namely 1.2Hz for the one-dimensional model of Escola; and 1.5Hz for the one-dimensional model of Junta and both stations considering two-dimensional modeling. Concerning the second amplification peak at Escola, most techniques gave an amplification frequency ranging from 2.0Hz to 2.5Hz.

This slight misfit between experimental and modeling techniques may be justified mainly by the uncertainty in evaluating seismic bedrock depth. Geophysical and geotechnical tests at São Sebastião haven’t reached depths over 40m. All modeling techniques assumed a depth of over 50m. If one considered deeper seismic bedrock, one would have obtained lower fundamental frequency. The misfit in terms of estimated amplification frequency also affected the second peak at Escola for the two-dimensional model of the crater.

There weren’t decisive results concerning the genetic process of the crater. The differences between the two genetic processes in terms of ground motion weren’t as much important as the presence of the lava tongue. However, one may say the the model with steeper lateral discontinuities, according to a pit-fall crater, gave better results in terms of two-dimensional modeling, especially for frequencies lower than 4.0Hz.

Considering overall results, one may clearly say that all the techniques gave their respective and important contribution in building a model that explained the anomalous behavior in terms of ground motion at São Sebastião. These techniques pointed out in the same direction, giving coherent results. Geological, geophysical and geotechnical considerations were necessary to understand the amplification features of the different stations.
7. Conclusions

7.1. Final considerations

At this point, one must do a balance of all contents discussed along the thesis. This balance must account not only the considerations and the activities described from Chapter 2 to Chapter 6, but also the author’s view on the subject of site effects.

The author believes that the present work proves the importance of all scientific areas that have site effects as a subject in order to assess them. Methods associated to Seismology, Geology and Geotechnical Earthquake Engineering were successfully applied to describe site effects at São Sebastião, and each one of them was extremely useful in the building of a global model that allowed one to explain the difference between amplification not only comparing São Sebastião to its surroundings, but also, and mainly, the difference between amplification at different places within the crater. The author also believes that proof was made on the interest of using acceleration time series in site-effect assessment, which is of extreme interest to seismic risk management and assessment, a fundamental goal of Geotechnical Earthquake Engineering.

Characterization of ground motion, as described in Chapter 2, was essential in making evidence that São Sebastião presents ground-motion amplification only explained by local factors. Several parameters and analysis techniques presented at the mentioned Chapter, namely horizontal PGA, Fourier amplitude and power spectra, central frequency and shape factor were applied to the data set of acceleration time series, and a pattern concerning different ground motion at Escola when comparing to Junta, Misericórdia and Praia da Vitória emerged.

The geophysical and geotechnical campaigns performed at São Sebastião were essential to bring information concerning the geological and geotechnical setting of the crater. The Surface Wave Method, the execution of boreholes and the use of the Resonant Column test, as presented at Chapter 3, concerning soil behavior under cyclic loading, allowed the gathering of more precise knowledge about the different formations within the crater.

The review of soil behavior under cyclic loading also gave the theoretical background for one to do the modeling of site effects at São Sebastião. A viscoelastic model and the Ramberg-Osgood model were directly used to model ground motion at São Sebastião. The linear equivalent model, based on the viscoelastic model presented at Chapter 4, was used in one-dimensional modeling of the soil
columns underneath the accelerometric stations. The Ramberg-Osgood model was the cornerstone of all the implementation described at Chapter 5, and was successfully applied at the two-dimensional modeling of the crater.

The experimental site-effect assessment techniques presented at Chapter 4 were used having as input signals acceleration time series. These techniques allowed comparing the results of soil modeling, leading to very satisfactory results.

One must also make further comment on the implementation of the Ramberg-Osgood in PLAXIS. A complete description of a constitutive model which allows one to account hysteretic damping in one of the most used tools in Geotechnical Engineering. Several tests in plane-strain conditions were made, with obtained results similar to those given by theoretical solutions. The author hopes that his contribution on this domain leads to wider implementation of user-defined soil models in powerful finite-element codes (not necessarily PLAXIS, but also codes such as FLAC, and others).

### 7.2. Future works

Despite all the work done having São Sebastião as subject, many issues concerning the volcanic crater remain unclear. Factors such as the crater limits, both the lateral boundaries and seismic bedrock are still to be determined. No decisive proof was made on the genetic process of the crater. Another issue to be tackled as far as São Sebastião concerns three-dimensional modeling of the crater. Considering the round shape of the crater, there is a strong possibility of the existence of three-dimensional site effects. These issues require further efforts in characterizing the crater, either geotechnical, geophysical or both.

As this work is being concluded, resonant column and torsional shear tests on undisturbed samples collected at São Sebastião are still running at Instituto Superior Técnico. Also, an accelerometric station is to be placed inside one of the boreholes performed during the summer of 2007, at an estimated depth of 30m. The comparison between records in depth and at the surface will help on the understanding of amplification at São Sebastião.

The use of acceleration time series to assess site effects at São Sebastião isn’t runout. A deep analysis of the vertical component recorded at different places at São Sebastião may help to determine the importance of basin-like effects, and therefore, help on determining the genetic process of the crater. Emerging techniques that were not presented in this work, such as spectral phase analysis, namely on the coda-wave part of acceleration time series, present a remarkable potential, not only in site-effect assessment, but in other areas linked to Seismology. The use of wavelets is also a major area in development as far as experimental site-effect assessment techniques are concerned.
Another area of this thesis that may present an area of development is the constitutive model implemented in Chapter 5. As the model was only applied in plane-strain calculations, its axissymmetrical and three-dimensional behaviors were not tested. Therefore, further validation tests are to be performed.

The formulation adopted in Chapter 5 presented features that, in the future, may be improved. It didn’t account non-linear volumetric stiffness, as happens in real soils. An interesting work to be done would be the implementation of coupled volumetric/distortional behavior. Nonetheless, one must always bear in mind that non-linear elastic models, such as the one presented in Chapter 5, are used in soil modeling for small to medium strains. Thus, the presented formulation should not be used for the analysis of dynamic problems where large strain intervenes, such as liquefaction and other failure situations.
References


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References


Schnabel, P.B. (1973). “Effects of local geology and distance from source on earthquake ground motions”, *PhD thesis*, University of California, Berkeley


SESAME project team (2004) “Guidelines for the implementation of the H/V spectral ratio technique on ambient vibrations. Measurements, processing and interpretation”.


References


Annex A

Source-code of subroutine “secante_RO”

```fortran
subroutine secante_RO(G,g_ref,alfa,exp_r,g_oct,G_sec)
    double precision, intent(in) :: G, g_ref, alfa, exp_r, g_oct
    double precision, intent(out) :: G_sec
    Real(8) :: erro, aux1, aux2, x, x_pos, f, f_linha
    erro = 1
    x = .999999
    x_menos1 = 1.0
    erro = 1.0
    do while (erro > 1.0e-6)
        aux1 = x_menos1-1/(1+alfa*(x_menos1*(g_oct/g_ref))**(exp_r-1))
        aux2 = x-1/(1+alfa*(x*(g_oct/g_ref))**(exp_r-1))
        x_pos = (aux2*x_menos1-aux1*x)/(aux2-aux1)
        erro = abs(x - x_pos)
        x_menos1 = x
        x = x_pos
    end do
    G_sec = 1.0*x_pos*G

    return
end subroutine
```

return

end subroutine
Source-code of subroutine “GrandezasD”

subroutine grandezasD (dEps, Eps, Eps_0, defdist, g_oct, *g_oct_0, d_g_oct)

  implicit none

  Real(8), Dimension(6), intent(in) :: dEps, Eps, Eps_0
  Real(8), Dimension(6), intent(out) :: defdist
  Real(8), intent(out) :: g_oct, g_oct_0, d_g_oct
  Real(8) :: comp1, comp2, comp3, comp4, comp5, comp6, parc1, parc2
  Real(8) :: teste, Eps_vol

  Eps_vol = Eps(1) + Eps(2) + Eps(3)

  Call MZeroR(defdist,6)
  defdist(1) = Eps(1) - Eps_vol/3.0
  defdist(2) = Eps(2) - Eps_vol/3.0
  defdist(3) = Eps(3) - Eps_vol/3.0
  defdist(4) = Eps(4)
  defdist(5) = Eps(5)
  defdist(6) = Eps(6)

  comp1 = 1.0*(Eps(1)-Eps(2))**2.0
  comp2 = 1.0*(Eps(2)-Eps(3))**2.0
  comp3 = 1.0*(Eps(3)-Eps(1))**2.0
  comp4 = 6.0*Eps(4)**2.0
  comp5 = 6.0*Eps(5)**2.0
  comp6 = 6.0*Eps(6)**2.0

  parc1 = 1.0*comp1+1.0*comp2+1.0*comp3
  parc2 = 1.0*comp4+1.0*comp5+1.0*comp6
  teste = 1.0*parc1+1.0*parc2
  teste = sqrt(teste)
g_oct = (2.0/3.0)*teste
comp1 = 1.0*(Eps_0(1)-Eps_0(2))**2.0
comp2 = 1.0*(Eps_0(2)-Eps_0(3))**2.0
comp3 = 1.0*(Eps_0(3)-Eps_0(1))**2.0
comp4 = 6.0*Eps_0(4)**2.0
comp5 = 6.0*Eps_0(5)**2.0
comp6 = 6.0*Eps_0(6)**2.0
parc1 = 1.0*comp1+1.0*comp2+1.0*comp3
parc2 = 1.0*comp4+1.0*comp5+1.0*comp6
teste = 1.0*parc1+1.0*parc2
teste = sqrt(teste)
g_oct_0 = (2.0/3.0)*teste
d_g_oct = 1.0*g_oct-1.0*g_oct_0
return
end subroutine

Source-code of subroutine “CalculoD”

subroutine CalculoD (K, G, alfa, exp_r, g_ref, g_oct, g_oct_0, *
                      d_g_oct, defdist, D_ne)

  implicit none
  Real(8), dimension(6), intent(in) :: defdist
  Real(8), intent(in) :: K, G, alfa, exp_r, g_ref, g_oct, g_oct_0
  Real(8), intent(in) :: d_g_oct
  Real(8), dimension(6,6), intent(out) :: D_ne
  Real(8), dimension(6,6) :: Z_MatA, z_MatB
  Real(8) :: factor_A1, factor_A2, G_sec, G_sec_0, G_tan, eta
integer(4) :: i,j

Call secante_RO(G,g_ref,alfa,exp_r,g_oct_0,G_sec_0)
Call secante_RO(G,g_ref,alfa,exp_r,g_oct,G_sec)

Call MZeroR(Z_MatA,36)
factor_A1 = 1.0*K+4.0/3.0*G_sec
factor_A2 = 1.0*K-2.0/3.0*G_sec

Do i=1,3
    Do j=1,3
        Z_MatA(i,j) = factor_A2
    End Do
    Z_MatA(i,i) = factor_A1
    Z_MatA(i+3,i+3) = G_sec
End Do

Call MZeroR(Z_MatB,36)

G_tan = G_sec + g_oct*((G_sec-G_sec_0)/(d_g_oct))

if (g_oct_0.EQ.0.0) then
    eta=0.0
else
    eta = 4.0/3.0*(G_tan-G_sec)/(g_oct*g_oct)
end if

do i=1,6
    do j=1,6
        Z_MatB(i,j)=2.0*eta*defdist(i)*defdist(j)
    end do
end do

Call MZeroR(D_ne,36)

Do i=1,6
    do j=1,6
D_ne(i,j) = Z_MatA(i,j) + Z_MatB(i,j)
end do
end do
return
end subroutine

Source-code of subroutine “usermod”

Subroutine User_Mod ( IDTask, iMod, IsUndr, iStep, iter, Iel, Int,
X, Y, Z,
Time0, dTime,
Props, Sig0, Swp0, StVar0,
dEps, D, BulkW,
Sig, Swp, StVar, ipl,
nStat, NonSym, iStrsDep, iTimeDep, iTang,
iPrjDir, iPrjLen, iAbort )

implicit none
Real(8), Dimension(6) :: Sig
Real(8), Dimension(6) :: dEps, Sig0
Real(8), Dimension(6) :: dSig
Real(8), Dimension(6,6) :: D(6,6), D_ne(6,6)
Real(8), Dimension(50) :: Props
Real(8), Dimension(232) :: StVar
Real(8), Dimension(232) :: StVar0
Real(8), Dimension(6) :: Eps, Eps0
Real(8), Dimension(6) :: defdist
integer(4) :: iMod, IDTask, IsUndr, iStep, iTer, iEl
integer(4) :: Int
integer(4) :: iAbort
integer(4) :: ipl, nStat, NonSym, iStrsDep, iTimeDep
integer(4) :: iTang, iPrjLen
integer(4) :: nStatV, iPrjDir
Real(8) :: X, Y, Z, Time0, dTime, Swp0, dEpsV
Real(8) :: BulkW, Swp, Fac, Nu_U
Real(8) :: dSwp = 0, G, Nu
integer(4) i, j, caso, ki
  Real(8) :: g_ref, g_oct, G_sec, G_sec_0, g_oct_0
Real(8) :: alfa, exp_r, K_sobre_G, K
Real(8) :: factor_A1, factor_A2
Real(8) :: d_g_oct, sinal_d_g_oct, aux
nStatV = 232
include 'impexp'
Select Case (iMod)
  Case (1)
    Select case (IDTask)
      Case (1)
      Case (2)
      If (IsUndr.Eq.1) Then
        dEpsV = dEps(1) + dEps(2) + dEps(3)
        dSwp = BulkW * dEpsV
        Swp = Swp0 + dSwp
      Else
        Swp = Swp0
      End If
    G = Props(1) ! G
\[ K = \text{Props}(2) \div K \]
\[ g_{\text{ref}} = \text{Props}(3) \]
\[ \alpha = \text{Props}(4) \]
\[ \exp_r = \text{Props}(5) \]
\[ ipl = 0 \]
\[ \text{do } i=1,6 \]
\[ \quad \text{aux} = \text{aux} + \text{abs}(dEps(i)) \]
\[ \text{end do} \]
\[ \text{if } (\text{aux}.\text{EQ}.0.0)\text{then} \]
\[ \quad \text{call CopyRVec(Sig0,Sig,6)} \]
\[ \quad \text{caso} = 1 \]
\[ \text{else} \]
\[ \quad ipl = 1 \]
\[ \quad \text{caso} = 1 \]
\[ \quad \text{Call MZeroR(Eps,36)} \]
\[ \quad \text{Call MZeroR(Eps0,36)} \]
\[ \quad \text{Call MZeroR(defdist,6)} \]
\[ \quad g_{\text{oct}} = 0.0 \]
\[ \quad g_{\text{oct}0} = 0.0 \]
\[ \quad d_{g_{\text{oct}}} = 0.0 \]
\[ \quad \text{if } (\text{caso}.\text{GT}.32) \text{then} \]
\[ \quad \quad \text{goto 101} \]
\[ \quad \text{end if} \]
\[ \text{end if} \]
\[ \text{do } i=1,3 \]
\[ \quad \text{Eps}(i) = 1.0*\text{StVar0}(i+6*(\text{caso}-1))+1.0*dEps(i) \]
\[ \quad \text{Eps}(3+i) = 1.0*\text{StVar0}(3+i+6*(\text{caso}-1))+1.0*dEps(3+i)/2.0 \]
Annex A

```fortran
end do

do i=1,6
    StVar(6*(caso-1)+i) = 1.0*Eps(i)
    Eps0(i) = StVar0(6*(caso-1)+i)
end do
doi
call grandezasD (dEps, Eps, Eps0, defdist, g_oct, g_oct_0,
                *     d_g_oct)
if (g_oct.GE.StVar0(200+caso)) then
    ki = 200+caso
    if (ki.LT.202) then ! Carregamento na curva virgem
        call CalculoD (K, G, alfa, exp_r, g_ref, g_oct, g_oct_0,
                        *     d_g_oct, defdist, D_ne)
    else
        call CalculoD (K, G, alfa, exp_r, 2.0*g_ref, g_oct, g_oct_0,
                        *     d_g_oct, defdist, D_ne)
    end if
call MatVec( D_ne, 6, dEps, 6, dSig)
call AddVec( Sig0, dSig, 1d0, 1d0, 6, Sig )
StVar(200+caso) = g_oct
do i=caso,32
    StVar(201+i) = 0.0
end do
doi
i=200
ki=6*(caso)
doi while (i .Gt. ki)
    StVar(i) = 0.0
```
i=i-1
end do
else

StVar(200+caso) = StVar0(200+caso)
caso = caso+1
Goto 100
end if
end if

101 continue

Case (3)
G = Props(1) ! G
K = Props(2) ! K
call MZeroR(D,36)
factor_A1 = 1.0*K+4.0/3.0*G
factor_A2 = 1.0*K-2.0/3.0*G
Do i=1,3
  Do j=1,3
    D(i,j) = factor_A2
  End Do
  D(i,i) = factor_A1
  D(i+3,i+3) = G
End Do

BulkW = 0
If (IsUndr.Eq.1) Then
  Nu = (3.0*K-2.0*G)/(2.0*(3.0*K+1.0*G))
  Nu_U = 0.495d0
  Fac=(1+Nu_U)/(1-2*Nu_U) - (1+Nu)/(1-2*Nu)
Fac=2D0*G/3D0  * Fac

BulkW = Fac

End If

Case (4)

nStat = nStatV

Case (5) ! matrix type

NonSym = 0 ! 1 for non-symmetric D-matrix

iStrsDep = 0 ! 1 for stress dependent D-matrix

iTang = 0 ! 1 for tangent D-matrix

iTimeDep = 0 ! 1 for time dependent D-matrix

Case (6)

G = Props(1) ! G

K = Props(2) ! K

call MZeroR(D,36)

factor_A1 = 1.0*K+4.0/3.0*G

factor_A2 = 1.0*K-2.0/3.0*G

Do i=1,3

    Do j=1,3

        D(i,j) = factor_A2

    End Do

    D(i,i) = factor_A1

    D(i+3,i+3) = G

End Do

BulkW = 0

If (IsUndr.Eq.1) Then

    Nu = (3.0*K-2.0*G)/(2.0*(3.0*K+1.0*G))

Nu_U = 0.495d0

Fac=(1+Nu_U)/(1-2*Nu_U) - (1+Nu)/(1-2*Nu)
Fac=2D0*G/3D0  * Fac

BulkW = Fac

End If

case default

   Write(1,*) 'Erro no IDTask', IDTask
   Write(1,*) 'IDTask: ',IDTask
   iAbort=1

End Select

Case Default

   Write(1,*) 'invalid model number in UsrMod', iMod
   Write(1,*) 'IDTask: ',IDTask
   Stop 'invalid model number in UsrMod'
   iAbort=1

   Return

End Select

   Return

End

include 'usrlib.for'
include 'secante_RO.for'
include 'grandezasD.for'
include 'CalculoD.for'