One Dimensional Mixed Hybrid Element for the Early Age Transient Thermal Response of Concrete

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1 Introduction

In this report a one dimensional hybrid-mixed temperature-heat finite element model is formulated for the transient thermal response of concrete, from an initially poured and compacted state, to fully hardened state. One dimensional heat models for the hydration process of concrete are applicable for the study of the early age behaviour of bridges and slabs. The objective of this work is to develop accurate and reliable and computationally efficient numerical models for the study of the early age behaviour of these structures with the view of better understanding and controlling the thermo-mechanical process of concrete hardening. The ultimate aim is to provide improved design guidance and improve construction procedures. Issues that may be studied include selection of the most appropriate cement to be used, curing requirements and the time after casting that falsework can be removed and formwork struck. In the case of prestressed concrete, the time at which prestress can be applied may also be studied. The principle motives for the development of numerical models and studies are to minimize the potential of early age heat induced cracks, which are visually unsightly and detrimental to the durability of concrete, and to reduce construction times.

The early age mechanical behaviour of concrete is characterised by the evolution of the hydration of concrete with strength and heat gains, and thermal stresses and strains. From a modelling point of view the evolution of the heat generated can be viewed as an internal heat source. The development of thermo-mechanical models to model the early age behaviour of concrete is a relatively new research field with scope for significant advances to be made in the state of the art. The study is also becoming more pertinent because of the increasing use of High Performance Cement (HPC), which possess heat of hydration potentials greater than that of Normal Strength Concrete (NSC).
The generation of heat during hydration of several cements used in Portugal and the accompanying development of mechanical properties have been obtained through an extensive experimental campaign (Azenha, 2009) to calibrate the numerical models. The evolutions of heat during hydration have been evaluated by use of an isothermal calorimeter which measures the heat expelled from a mortar sample maintained at a fixed temperature. The development of the mechanical properties of concrete has been measured by tests on mini structural columns.

The transient thermal behaviour of concrete during it early age is governed by the Fourier law of thermodynamics which is considered in incremental form to model the transient response. Nonlinear terms are used for the thermal conductivity of concrete, the heat generation rate which is governed by the Arrhenius law, and the thermal boundary conditions. The term for the conductivity is that given by Ruiz et al, (2001) and the term the thermal boundary condition is that given by Branco et al, (1992) and Silveira, (1996). These nonlinear terms are the same as those used by Faria et al, (2006), where the thermal behaviour of concrete during its early age was modelled using conventional finite elements and several studies were carried out. In this work Lagrange polynomials of order N are used to interpolate the temperature and heat evolution. The Crank-Nicholson time integration method is employed to interpolate the variation of temperature and heat with time.

It is generally accepted that hybrid finite elements, elements of the type developed here, are the most powerful variant of finite elements, with a higher performance than conventional finite elements (Pian, and Tong, (1969), Atluri et al, (1983), Brezzi, and Fortin, (1991) and Fellipa, (1996)). Hybrid finite elements are able to overcome many of the difficulties associated with specific problems, and can be used as a specialized tool for this effect. These problems include the problem of the early age behaviour of concrete. Hybrid finite elements have the drawback of being more complex to formulate and implement. They do however possess significant advantage in terms of accuracy and reliability of analysis, and computational expense; making their development and use justified for certain classes of problem, such as that considered here.
2 Governing Equations

The Fourier law of thermodynamic equilibrium which governs the transient thermal response of concrete during its early age, in the domain V, is given by the following equation,

\[ k \nabla^2 T + \dot{Q} = \rho c \frac{\partial T}{\partial t} \quad \text{in } V \quad (1) \]

where \( T \) is the temperature, \( \partial T / \partial t \) is the rate of change of temperature with respect to time \( t \), \( k \) is the thermal conductivity, \( \rho c \) is the volumetric specific heat, \( \dot{Q} \) is the heat generated and \( \nabla^2 \) is the Laplacian. For the one dimensional case considered here the Laplacian degenerates to \( \partial^2 / \partial x^2 \), where, \( x \) is the dimension across the depth of the structure.

During hydration the thermal conductivity of concrete \( (k) \) undergoes substantial change. It has been established by Ruiz et al. (2001) that the thermal conductivity of concrete can be expressed by the following equation,

\[ k = k_\infty (1.33 - 0.33\alpha) \quad (2) \]

where \( k_\infty \) is the value of the thermal conductivity of concrete in its hardened state. \( \alpha \) is the hydration degree, defined as the ratio of heat released up to time \( t \) and the total heat expected upon completion of cement hydration \( Q_\infty \), it follows that,

\[ \alpha = \frac{Q}{Q_\infty} \quad (3) \]

Experimental evidence shows that the rate of heat generated during cement hydration \( \dot{Q} \) can be represented by the Arrhenius law, which has the form,

\[ \dot{Q} = A_T f(\alpha) \exp \left( \frac{-E_a}{RT} \right) \quad (4) \]

\( E_a \) is the activation energy (J/mol), \( R \) is the universal gas constant (J/mol K\(^{-1}\)), \( A_T \) is the
maximum value of the heat production rate (J/s) and the function \( f(\alpha) \) defines the evolution of the normalised heat production rate as a function of the hydration degree \( \alpha \). The values of \( A_T \), \( f(\alpha) \) and \( E_a \) calibrated for various cements used in Portugal by experimental means, by Azenha (2009), are used in this work.

The thermal boundary condition that concrete structures are subjected to, are convection and radiation. These two phenomena can be treated jointly by Newton’s cooling law applying a single convection-radiation coefficient, Branco et al, (1992) and Silveira, (1996). With the outward normal denoted \( n \), the thermal boundary condition, on boundary \( \Gamma = \Gamma_q \) is as follows,

\[
\mathbf{n} \cdot \mathbf{k} \Delta T = h_{cr} (T_s - T_a) \quad \text{on } \Gamma_q
\]

where \( h_{cr} \) is the single heat flux convection-radiation coefficient, \( T_s \) and \( T_a \) are the surface and air temperature respectively, and \( \nabla \) is the space gradient. For the one dimensional case treated in this report the space gradient, \( \nabla \), degenerates to \( \partial_x \) where, \( x \) is the dimension across the depth of the structure. \( h_{cr} \) is given by,

\[
h_{cr} = h_c + h_r
\]

where the convection coefficient is given by, Jonasson, (1994),

\[
h_c = \begin{cases} 
5.6 + 3.95 \nu, & \nu \leq 5 \text{ m/s} \\
7.6 \nu^{0.7}, & \nu > 5 \text{ m/s}
\end{cases}
\]

and the radiation coefficient is given by Silveira, (1996) and Branco et al, (1992),

\[
h_r = \begin{cases} 
\varepsilon \left[ 4.8 + 0.075(T_a - 278.15) \right], & T_a \geq 278.15 \\
4.8 \varepsilon, & T_a < 278.15
\end{cases}
\]

\( \nu \) (m/s) is the wind speed at the surface of the concrete, and \( \varepsilon \) is emissivity of concrete, which is usually in the range of 0.85-0.95.

In the most general sense the boundary conditions that are applicable to transient thermal problems also include, imposed temperature and imposed velocity of heat flow.
The imposed temperature boundary condition is of no practical consequence and is therefore not considered here. However, the imposed velocity of heat flow is useful to simulate adiabatic conditions to calibrate cement properties established under adiabatic test conditions, where a sample is tested without any heat being expelled. To simulate this condition the imposed velocity of heat flow is set equal to zero.

The velocity of heat flow boundary condition, on boundary $\Gamma=\Gamma_{\sigma}$ is as follows,

$$n^T \sigma = \bar{T} \quad \text{on } \Gamma_{\sigma}$$

(9)

where $\sigma$ represents velocity of heat flow and $\bar{T}$ is the imposed velocity of heat flow.

The initial conditions are that at the start of time the temperature $T$, (denoted $T_0$) is the pouring temperature of the concrete and $\dot{T}$, (denoted $T_0\dot{T}$) is zero. It is thus assumed that hydration begins when the concrete has been poured and compacted.

For familiarity, it is convenient to cast Fourier’s law of thermodynamics in a form analogous to equations found in Solid Mechanics. From here on all equations are written in the one dimensional space $S$. The domain equations become,

$$-\partial_x \sigma + Q = \rho c \dot{T} \quad \text{in } S$$

(10)

$$\varepsilon = \partial_x T \quad \text{in } S$$

(11)

$$\sigma = -k \varepsilon \quad \text{in } S$$

(12)

The boundary condition for convection takes the form,

$$n^T \sigma = h_{\sigma} (T_s - T_a) \quad \text{on } \Gamma_q$$

(13)

and the boundary condition for imposed velocity of heat takes the form,

$$n^T \sigma = \bar{T} \quad \text{on } \Gamma_{\sigma}$$

(14)

In Solid Mechanics terminology, equation (10) is analogous to the domain equilibrium
equation, equation (11) to the compatibility condition and equation (12) to the constitutive relationship. It follows that $\sigma$ and $\varepsilon$ are analogues to stress and strain tensors respectively. $T$ stands for displacement, $\dot{T}$ for velocity, $Q$ for body force, $\rho c$ for damping coefficient, $k$ for stiffness of material, and $h_c$ for stiffness of boundary spring.

3 Formulation of Element

3.1 Domain and Boundaries

The element occupies the one dimensional space $S$ and allows for convection and imposed velocity of heat flow boundaries denoted $\Gamma_q$ and $\Gamma_\sigma$ respectively. Discretisation is shown in Figure. 1.

3.2 Interpolation Functions

Scaled Lagrange polynomials are used to interpolate temperature $T$ and heat $Q$ in each element. Accordingly $T$ and $Q$ are as follows,

$$T = T_V^T \xi_T$$  \hspace{1cm} (15)

and,

$$Q = Q_V^T \xi_T$$  \hspace{1cm} (16)
where,

\[
T_v = [P_0, P_1, \ldots, P_n]
\]  

(17)

and,

\[
Q_v = [P_0, P_1, \ldots, P_n]
\]  

(18)

\(\xi_T\) and \(\xi_Q\) are generalised weights, \(P_n\) is given by,

\[
P_n(\xi) = \sqrt{n + \frac{1}{2}} L_n(\xi)
\]  

(19)

where \(L_n\) is the \(n\)th degree Lagrange polynomial.

### 3.3 Zero state

At zero state, which represents the initially poured and compacted state, the domain equations degenerate to the following,

\[-\partial_\xi \sigma = 0 \quad \text{in } S\]

(20)

\[
\epsilon = \partial_\xi T \quad \text{in } S
\]

(21)

\[
\sigma = -k_\theta \epsilon \quad \text{in } S
\]

(22)

The boundary condition take the form,

\[
n^T \sigma = h_{0,\text{cr}} (T_c - T_a) \quad \text{on } \Gamma_q
\]

(23)

and,

\[
n^T \sigma = \bar{t}_0 \quad \text{on } \Gamma_\sigma
\]

(24)

Solution of the above equations yields that there is a linear temperature distribution in \(S\), across the depth of the structure.
The generalised weights for each of the elements, for the solution are as follows,

\[ \xi_{0T} = \frac{1}{\sqrt{2}} (T_{i+1} + T_i) \]  
\[ \xi_{1T} = \frac{1}{\sqrt{6}} (T_{i+1} - T_i) \]  
(25)  
(26)

all other weights being null.

### 3.4 Transient Phase

The transient thermal response of concrete, from an initially poured and compacted state, to fully hardened state is considered by considering increments of the domain and boundary equations.

The incremental domain equations become,

\[ -\partial_x \Delta \sigma + \Delta \dot{Q} = \rho c \Delta \dot{T} \text{ in } S \]  
(27)

\[ \Delta \epsilon = \partial_x \Delta T \text{ in } S \]  
(28)

\[ \Delta \sigma = -k \Delta \epsilon - R_k \text{ in } S \]  
(29)

The incremental boundary conditions become,

\[ n^T \Delta \sigma = h_a (\Delta T_e - \Delta T_a) + R_h \text{ on } \Gamma_q \]  
(30)

and,

\[ n^T \Delta \sigma = \Delta \bar{T} \text{ on } \Gamma_\sigma \]  
(31)

The residuals are given by,

\[ R_k = (k_a - k_0)(\epsilon_0 + \Delta \epsilon) \]  
(32)

and,
3.4.1 Incremental Rate of Heat Generation

The incremental rate of heat generation is derived by first considering the Arrhenius law of heat generation, which governs the heat generated during cement hydration, in incremental form,

$$
Q_0 + \Delta Q = A_T \left\{ f(\alpha) + f'(\alpha) \Delta \alpha \right\} \exp \left( -\frac{E_a}{RT} \right)
$$

Taking a Taylor expansion,

$$
\exp \left( -\frac{E_a}{RT} \right) = \exp \left( -\beta_0 \right) \left\{ 1 + \beta_0 \frac{\Delta T}{T_0} + \beta_0 R_e \right\}
$$

where,

$$
\beta_0 = \frac{E_a}{RT_0}
$$

and,

$$
R_e = -(1 - \frac{1}{2} \beta_0) \left( \frac{\Delta T}{T_0} \right)^2 + (1 - \beta_0 + \frac{1}{6} \beta_0^2) \left( \frac{\Delta T}{T_0} \right)^3 + \ldots
$$

It follows that,

$$
\Delta Q = \beta_0 Q_0 \left( \frac{\Delta T}{T_0} \right) + \gamma_0 \Delta Q + \beta_0 Q_0 R_e + \beta_0 \gamma_0 R_u
$$

where,

$$
\gamma_0 = \frac{A_T f'(\alpha) \exp (-\beta_0)}{Q_\infty}
$$

and,
\[ R_a = \Delta Q \left( R_e + \frac{\Delta T}{T_0} \right) \]  

(40)

4 Time Integration

The Crank-Nicholson time integration method is employed. Accordingly the temperature and heat are interpolated as follows,

\[ \nu_0 + \Delta \nu = \nu_0 + \alpha_o \Delta t \nu_0 + \alpha \Delta t \left( \nu_0 + \Delta \nu \right) \]  

(41)

where \( \nu \) may be temperature \( T \) or heat \( Q \).

It follows that,

\[ \Delta \nu = i\omega \Delta \nu - \omega^0 \nu_0 \]  

(42)

where,

\[ i\omega = \frac{1}{\alpha \Delta t} \]  

(43)

and,

\[ \omega^0 = 1 + \alpha_o / \alpha \]  

(44)

With \( \alpha = \alpha_0 = \frac{1}{2} \), as stipulated for Crank-Nicholson method, \( i\omega \) and \( \omega^0 \) degenerate to the following,

\[ i\omega = \frac{2}{\Delta t} \]  

(45)

and,

\[ \omega^0 = 2 \]  

(46)
5 System Equations

The finite element system equations are derived using a weighted residual method. Equations will be considered for a structure consisting of one element with convection boundary conditions at both ends. The equations derived can be easily extended to consider a structure of N elements, with both convection and velocity of heat flow boundary conditions. The general descretisation is shown in Fig 1.

5.1 Transient Domain Equilibrium Equations

The element equilibrium equation in incremental form, equation (47), is as follows,

\[
\left( \frac{1}{i\omega} K + C + H_1 + H_2 \right) \Delta \xi_T - A^* \Delta \xi_Q =
\begin{pmatrix}
-R_{Q_0} - \Delta R_Q \\
N_{h_1} - \Delta N_{h_1} + N_{h_2} - \Delta N_{h_2}
\end{pmatrix}
\]  

(47)

It represents the weak enforcement of the incremental form of the domain equilibrium equation, equation (27), for the time integration function, equation (41) and the interpolation functions, equations (15) and (16). It contains the incremental form of the boundary condition, equation (30).

As with all other equations that follow A*, in the most general sense, represents the transpose of the complex conjugate of A.

The matrices and arrays of equation (47) are as follows,

\[
K = \int_0^{L_1} \left( \partial_x T_v \right)^* k_0 \left( \partial_x T_v \right) dx
\]  

(48)

\[
C = \int_0^{L_1} T_v^* \rho c T_v dx
\]  

(49)

\[
A^* = \int_0^{L_1} T_v^* Q_v dx
\]  

(50)
\[ R_{Q0} = \frac{\omega^0}{i\omega} \int_{0}^{L} T_V^* \left( Q_0 - \rho c T_0 \right) dx \]  \hspace{1cm} (51)

\[ \Delta R_Q = \frac{1}{i\omega} \int_{0}^{L} \left( \partial_x T_V \right)^* R_k \, dx \]  \hspace{1cm} (52)

\[ H_1 = \frac{h_1}{i\omega} T_V^* (0) T_V(0) \]  \hspace{1cm} (53)

\[ H_2 = \frac{h_2}{i\omega} T_V^* (L) T_V(L) \]  \hspace{1cm} (54)

\[ N_{h1} = \frac{1}{i\omega} T_V^* (0) h_1 \Delta T_{a1} \]  \hspace{1cm} (55)

\[ N_{h2} = \frac{1}{i\omega} T_V^* (L) h_2 \Delta T_{a2} \]  \hspace{1cm} (56)

\[ \Delta N_{h1} = \frac{1}{i\omega} T_V^* (0) R_{h1} \]  \hspace{1cm} (57)

\[ \Delta N_{h2} = \frac{1}{i\omega} T_V^* (L) R_{h2} \]  \hspace{1cm} (58)

\[ R_{Q0} \] can be shown to take the following form,

\[ R_{Q0} = \frac{\omega^0}{i\omega} \left( K \xi_T + T_V^* (L) \sigma_0 (L) - T_V^* (0) \sigma_0 (0) \right) \]  \hspace{1cm} (59)

5.2 Transient Domain Heat Generation Rate Equations

The transient domain heat generation rate in incremental form, equation (60), is as follows,

\[ DA \Delta \xi_T - A \Delta \xi_T = R_{T} + \Delta R_T \]  \hspace{1cm} (60)

It represents the weak enforcement of the incremental form of the heat generation rate equation, equation (38), the time integration function, equation (41), and the
interpolation function for heat, equation (16).

The matrices and arrays of equation (60) are as follows,

\[ D = \int_{0}^{L} Q V \cdot T_{0} \frac{(i\omega - \gamma_{0})}{\beta_{0} Q_{0}} Q_{V} \, dx \]  \hspace{1cm} (61)

\[ R_{T} = \int_{0}^{L} Q V \cdot T_{0} \frac{T_{0}}{\beta_{0}} \, dx \]  \hspace{1cm} (62)

\[ \Delta R_{T} = \int_{0}^{L} Q V \cdot T_{0} \left( R_{e} + \frac{\gamma_{0}}{Q_{0}} R_{a} \right) \, dx \]  \hspace{1cm} (63)

### 5.3 Finite Element System Matrix

The governing finite element system equation is established by combining the element equilibrium equations in incremental form, equation (47), and the transient domain heat generation rate in incremental form, equation (60), as follows,

\[
\begin{bmatrix}
\frac{1}{i\omega} K + C + H_{1} + H_{2} - A & -A^{\ast} \\
-A & D
\end{bmatrix}
\begin{bmatrix}
\Delta \xi_{T} \\
\Delta \xi_{Q}
\end{bmatrix}
= \begin{bmatrix}
-R_{Q0} - \Delta R_{Q} + (N_{h1} - \Delta N_{h1}) + (N_{h2} - \Delta N_{h2}) \\
R_{T} + \Delta R_{T}
\end{bmatrix}
\]  \hspace{1cm} (64)

### 5.4 Scaling Finite Element System Equations

In order to ensure numerical stability the finite element system variables are scaled. The independent scaling factors taken are \( L_{s}, \ T_{s}, \ Q_{s} \) and \( k_{s} \). \( L_{s} \) is taken as the largest length of the elements. \( T_{s} \) is taken as the maximum anticipated temperature. \( Q_{s} \) is taken as the maximum amount of heat that the concrete elements can liberate. \( k_{s} \) is taken as the maximum conductivity of the concrete elements.

The dependent scaling factors become,
\[ \varepsilon_s = \frac{T_s}{L_s} \]  
(65)

\[ \sigma_s = k_s \varepsilon_s \]  
(66)

\[ t_s = \frac{Q_s L_s}{\sigma_s} \]  
(67)

\[ (\rho c)_s = \frac{Q_s}{T_s} \]  
(68)

\[ (h_f)_s = \frac{k_s}{L_s} \]  
(69)

\[ \beta_s = \frac{E_s}{RT_s} \]  
(70)

\[ (A_f)_s = \frac{k_s T_s}{L_s^2} \exp(\beta_s) \]  
(71)

With these scaling parameters the equation for \( \beta_0 \), equation (36), becomes,

\[ \beta_0 = \frac{\beta_s}{T_0} \]  
(72)

and the equation for \( \gamma_0 \), equation (39), becomes,

\[ \gamma_0 = A_f f(\alpha) \exp\left( \beta_s - \beta_0 \right) \frac{Q}{Q_\infty} \]  
(73)

### 5.5 Nonlinear Solution Procedure for System Equations

Nonlinear solution procedures need to be invoked to solve for the finite element system equations that the problem presents. The terms that induce the nonlinearities are \( k \), the thermal conductivity of concrete, defined in equation (2); \( Q \), the heat generated during hydration, defined in equation (4); and \( h \), the heat flux convection-radiation coefficient.
defined in equation (6).

The procedure used to solve the nonlinear equations is as follows:

1. Define time step

2. Initialize the increments of temperature and heat distributions.

3. Calculate values for the nonlinear terms; \( k \), \( Q \) and \( h_{cr} \) associated with the temperature and heat distributions.

4. Calculate the increment of temperature and heat distribution, \( \Delta T \) and \( \Delta Q \), defined by solving the finite element system equation, equation (64).

5. If the increments of the temperature and heat distributions calculated from equation (63) are the same as those initialised, within an accepted tolerance, the system has converged and the time step considered is completed. If the increments of temperature and heat calculated from equation (64) are not the same as those initialised, then the initialized temperature and heat distributions are updated to \( \Delta T \) and \( \Delta Q \) calculated in step 4, step 3 is then returned to.

6. When the system has converged the next time step is considered. Step 1 is returned to with the updated temperature and heat distributions defined by \( T = T + \Delta T \) and \( Q = Q + \Delta Q \).

**6 Examples**

Two examples are given to illustrate the capabilities of the analysis facility developed. The examples serve to simulate the expected evolution of temperature and hydration degree. As previously mentioned, data used for the thermal properties of cement have been established by an extensive experimental campaign by Azenha (2009).

**6.1 First Example**

The first example models a specimen of concrete held under adiabatic conditions.
Adiabatic tests, where a sample is tested without any heat being expelled, are often used to establish the thermal properties of concrete. The adiabatic condition is simulated by imposing a zero velocity of heat flow on the boundaries of the element.

The specific thermal properties correspond to the cement type denoted CA CEM I 42.5R in Azenha (2009). The model is shown in Fig. 2. Details relating to the thermal properties of the concrete are shown in Table 1. The structure is modelled with one element, with Lagrandre polynomials of order 1 to interpolate temperature and heat. It is possible to model adiabatic conditions with Lagrandre polynomials of order 1 as under these conditions there are no temperature and heat gradients. The time step taken is 1 hour. The results are shown in Figs. 3 and 4, which show the development of the hydration degree and temperature, throughout the element, respectively.

![Figure 2 Structure for First Example](image)

### Table 1 Thermal Properties of Concrete

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity $k_\infty$</td>
<td>$2.6$ W/mK</td>
</tr>
<tr>
<td>Volumetric specific heat $\rho c$</td>
<td>$2400$ kJ/m$^3$K</td>
</tr>
<tr>
<td>Pouring temperature $T$</td>
<td>$17^\circ$C</td>
</tr>
<tr>
<td>Activation energy $E_a$</td>
<td>$43.83$ kJ/mol</td>
</tr>
<tr>
<td>Rate constant $A_T$</td>
<td>$2.150x10^8$ W/kg</td>
</tr>
<tr>
<td>Maximum heat released $Q_\infty$</td>
<td>$355.2$ kJ/kg</td>
</tr>
<tr>
<td>Cement content $cc$</td>
<td>$290$ kg/m$^3$</td>
</tr>
</tbody>
</table>
Figure 3. Development of Hydration Degree

Figure 4. Development of Temperature
6.2 Second Example

The second example models a specimen with convective boundary conditions. The specific thermal properties corresponds to the cement type denoted CA CEM I 52.5R in Azenha (2009). The structure is shown in Figure 5. Details relating to the thermal properties of the concrete are shown in Table 2. The structure is modelled with one element, with Lagrandre polynomials of order 13 to interpolate temperature and heat. The time step taken is 1 hour. The results are shown in Figs. 6, 7 and 8. Figs. 6 and 7 show the development of the hydration degree and temperature, at the mid point of the element, respectively. Fig. 8 shows the temperature profile across the depth of the structure at the instant when the maximum temperature is attained.

Figure 5 Structure for Second Example

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>$k_\infty = 2.6 \text{ W/mK}$</td>
</tr>
<tr>
<td>Volumetric specific heat</td>
<td>$pc = 2400 \text{ kJ/m}^3\text{K}$</td>
</tr>
<tr>
<td>Pouring temperature</td>
<td>$T = 17^\circ\text{C}$</td>
</tr>
<tr>
<td>Activation energy</td>
<td>$E_a = 48.19 \text{ kJ/mol}$</td>
</tr>
<tr>
<td>Rate constant</td>
<td>$A_T = 1.607 \times 10^9 \text{ W/kg}$</td>
</tr>
<tr>
<td>Maximum heat released</td>
<td>$Q_\infty = 386.3 \text{ kJ/kg}$</td>
</tr>
<tr>
<td>Cement content</td>
<td>$cc = 290 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>

Table 2 Thermal Properties of Concrete
Figure 6. Development of Hydration Degree at Mid Point of Element

Figure 7. Development of Temperature at Mid Point of Element
6.3 Discussion on Examples

The examples model the expected development of temperature and hydration degree. It has not been possible to verify the analytical results against experimental test results as data is not available. However, the model results show correct trends, giving confidence in the correctness of the formulation and implementation.

In the first example which simulates concrete held under adiabatic conditions the modelled results show that both the temperature and hydration degree approach asymptotic values. The temperature approaches an asymptotic value of about 60°C and the hydration degree approaches an asymptotic value of 1, indicating that full hydration is reached after about 3.5 days.

In the second example which models a structure with convective boundary condition the hydration degree also asymptotically tends to 1 after about 4.5 days, which indicates full hydration. The evolution of temperature at the mid point of the structure shows that the
simulated temperature increases from the pouring temperature of $17^0\text{C}$ to about $33^0\text{C}$ after about 15 hours, before cooling. The structure cools to reach the imposed ambient temperature of $17^0\text{C}$ after about 4.5 days.

**7 Conclusions**

The early age behaviour of concrete is characterised by heat and strength gains in time as the cement hydrates. An efficient, reliable and accurate one dimensional hybrid-mixed finite element applicable for the numerical simulation of the hydration process in slabs and bridges has been developed. It maybe used to effectively provide useful design and construction information, such as most appropriate cement to be used and curing requirements. Provision of such design guidance on matters relating to the early age behaviour of concrete is becoming increasingly necessary as the use of High Performance Cement (HPS), which have high heat of hydration potentials, is becoming increasingly wide spread.

**8 References**


