Sensitivity Assessment of Hybrid-Trefftz Stress and Displacement Elements for Poroelasticity

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Wave propagation problems defined on biphasic media are solved using the displacement and stress models of the hybrid-Trefftz finite element formulation [1]. The mathematical model considered for the analysis is the solid displacement - fluid seepage (u-w) variant of the Biot Theory of Porous Media [2, 3], assuming an elastic solid phase fully permeated by a liquid phase with a flow governed by Darcy’s law.

The hybrid-Trefftz displacement model is derived directly from the (pure) hybrid displacement formulation [4], by constraining the functions collected in the domain displacement basis to satisfy locally the governing Navier equation. An independent approximation is adopted for the traction (Cauchy stress) field on the Dirichlet and inter-element boundaries and is used to enforce on average the boundary compatibility conditions. The same strategy is adopted for the derivation of the hybrid-Trefftz stress element, which is constructed on the independent approximations of the stress field in the domain and of the displacement field on the Neumann and inter-element boundaries. The functions used for the domain stress approximation are selected from the solution space of the governing Beltrami equation, while the boundary displacement basis is used to enforce weakly the boundary equilibrium. As the approximation functions of either model locally satisfy all the domain equations, they embody the physical substructure of the modelled phenomenon, endorsing high convergence rates under very low h-refinements and causing the finite element solving system to collect exclusively boundary integral terms.

The numerical applications reported herein are designed to assess the ability of the hybrid-Trefftz elements to circumvent problems that typically hinder the application of the conventional finite elements to dynamic poroelasticity problems, such as incompressibility, mesh dependency on the wavelength and gross mesh distortion.

Biphasic problems occurring in geomechanics typically involve a porous solid skeleton, with interconnected pores, filled with water. The high bulk moduli that often characterize one or both constituents, pose the problem of incompressibility [5]. While the fluid incompressibility per se do not pose any numerical difficulties under drained conditions, as the volume change can still occur due to fluid seepage, undrained loading conditions or high values of the Biot coefficient characterizing the coupling between the phases can result in a near-incompressibility of the mixture. Furthermore, the solid skeleton itself may also be almost incompressible, posing the well-known problem of the Poisson coefficient approaching 0.5. Tests on both displacement and stress models show that the results are virtually insensitive to incompressibility in all the these situations, for Poisson coefficients as large as $0.5 - 10^{-12}$.

For dynamic problems, the frequency of the induced excitations is a two-fold concern when conventional elements are used. On one hand, depending on the value of the frequency [6], the coupling between the solid and the liquid phases can range from extremely strong (low frequency range), causing the material to behave as a frozen mixture, to very loose, with consequent increases in seepage and attenuation capacity of the material. This substantial coupling variability affects significantly the behavior of the
mixture, and is capable of causing numerical difficulties. On the other hand, independently of the type of material considered, higher excitation frequencies are a major computational concern, due to to the necessity of calibrating the mesh size to the wavelength of the propagating wave [7]. The ‘thumb rule’ commonly adopted to mitigate the effect of the interpolation error states that at least 6 (but preferably 10) elements per wavelength should be used, and this number is much larger when the pollution effect is also to be avoided. In order to assess the sensitivity of the hybrid-Trefftz elements to large frequency variations, a significant number of tests is reported, covering all coupling ranges and frequencies of practical interest in geomechanics. It is shown that the elements exhibit virtually no sensitivity to major frequency variations, endorsing the use of the same finite element mesh for all problems.

Finally, the sensitivity of the results to gross mesh distortion is addressed, showing that both models are virtually insensitive, even for relatively low degrees of $p$-refinements. All tests are performed on both models, for a comprehensive range of excitation frequencies, designed to cover all types of coupling between the solid and fluid phases. The effect of both $h$- and $p$-refinements on the convergence and robustness properties of the elements is also discussed.

References


